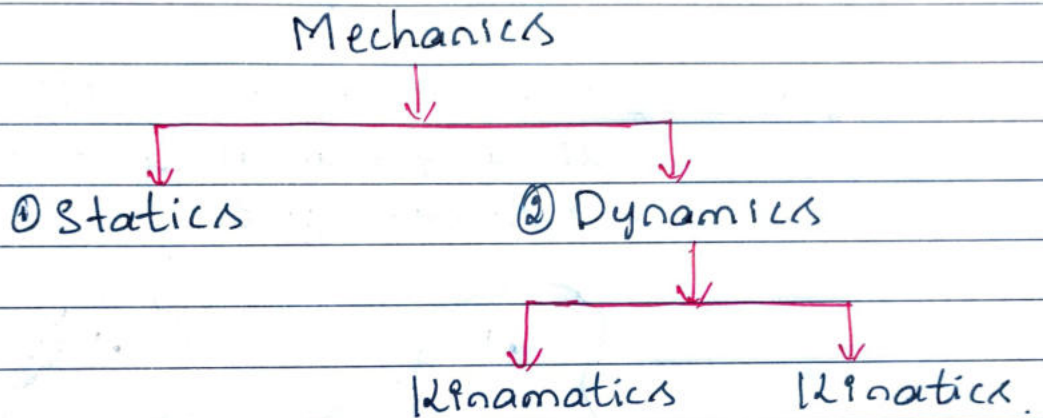


Mechanics and its classification

Mechanics:- It is a branch of physical science which deals with the state of rest (or) motion of a body under the action of forces.



① Statics:- It is a branch of physical science which deals with study of state of rest of a body under the action of external forces.

② Dynamics:- It is a branch of physical science which deals with study of state of motion of a body under the action of external forces.

i) Kinematics:- It is a branch of dynamics in which the force causing motion will not be considered.

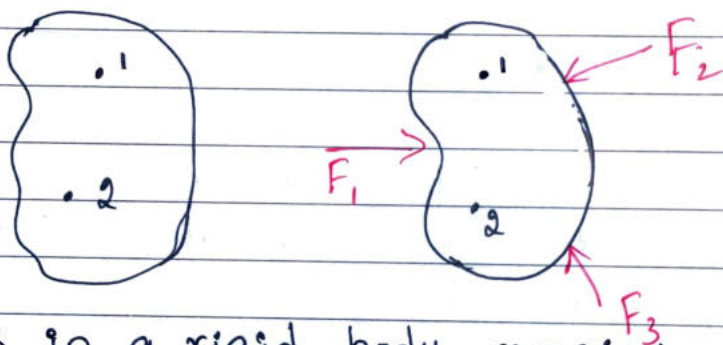
ii) Kinetics:- It is a branch of dynamics in which the force causing motion will be considered.

## Basic definitions / Basic idealizations

Particle:- A body with a infinitely small volume & negligible mass.

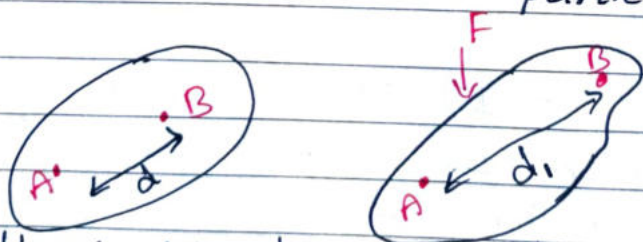
Body:- Assemblage of a number of particle.

Rigid body:- The body with its particles remaining at a fixed distance even after applying external force.



All particles in a rigid body remains at a fixed distance one another even after the application of forces. Thus there is no change in either shape (or) dimensions of the body.

Deformable body:- After applying the external force on a body with its particles the distance between the particles will vary.



If there is change in distance between any two particles of the body after application of external force then it is called as deformable body.

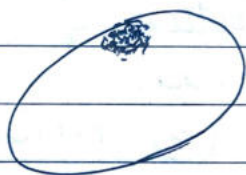
Scalar Quantity:- The Quantity which has only magnitude and no direction.

Eg:- Mass, Density, Volume, Distance.

Vector Quantity:- The Quantity which has both magnitude & direction.

Eg:- Velocity, Acceleration, Displacement, Force etc.,

Continuum:- The continuous distribution of the particles in a body without any intermolecular space.

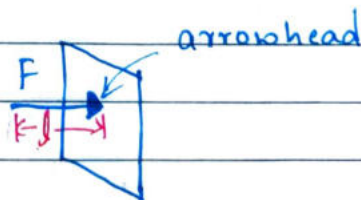
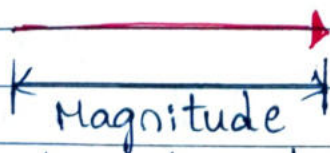


### Force and its characteristics

Force:- It is an external agency which will tend to change of state of rest (or) motion of a body.

(or)

Force is an external agent which changes (or) tends to change the state of rest (or) motion of an object. It is a vector quantity [has both magnitude & direction]. The SI unit of force is Newton (N).



Every force is characterised by its:-

- ① Magnitude:- length of the vector represents the magnitude of force.

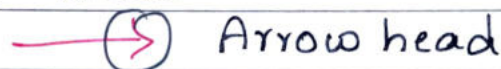
Eg:- If the magnitude of the force is higher then the length of the vector is higher.



If the magnitude of the force is lesser then the length of the vector is lesser.

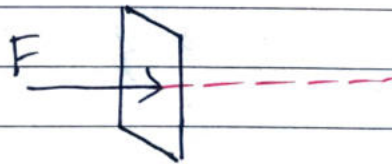


③ Direction:- The direction of the force is represented by an arrow head.



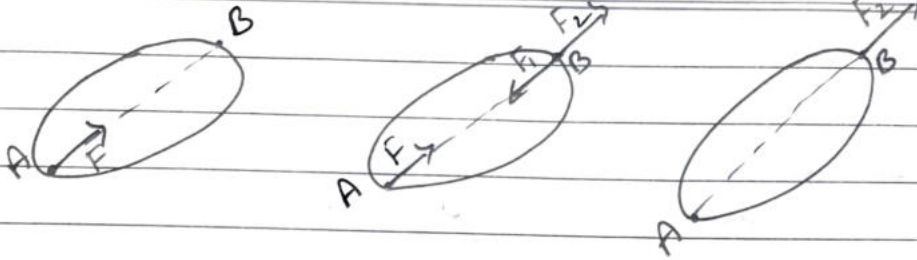
③ Point of application:- Point at which the force <sup>acting</sup> is applied. Both the direction and point of application define its line of action.

④ Line of action:- The line along which the force acts.



Principle of transmissibility of forces

This principle states that a force can be transmitted from one point to another point along the same line of action such that the effect produced by the force on a body remains unchanged.



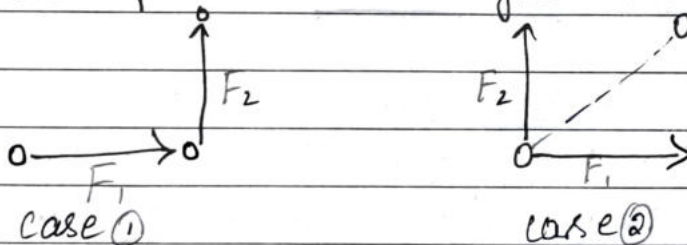
Consider a rigid body subjected to a force  $F$  at point 'A'. According to the principle of transmissibility, the force ' $F$ ' can be transmitted to a new point 'B' along the same line of action such that the net effect on the body remains unchanged.

i.e.,  $F = F_1 = F_2$

∴  $F = F_1 + F_2 = \Sigma F$

### Principle of Superposition of forces

The net effect of a system of forces on a rigid body is equal to the combined effect of individual forces on the rigid body.



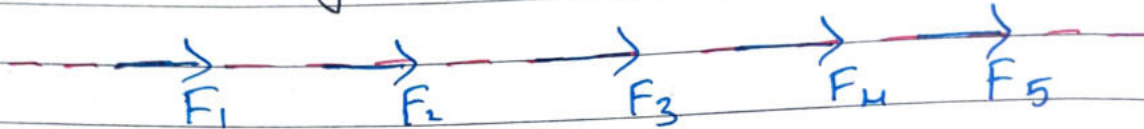
### Classification of force system (or) types of system of forces

If two (or) more forces are acting on a body (or) on a member (or) on a particle constitute a force system.

## Types

### ① Collinear force system

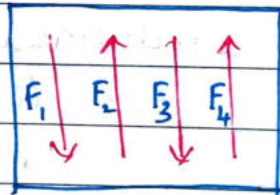
Forces acting in a single plane



i.e., All the forces acting along the same line  
(or) line of action of forces lying in the same plane. line.

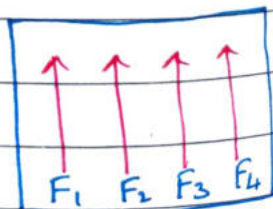
### ② Coplanar parallel force system

All the forces lying in the same plane & all the forces are parallel to each other.



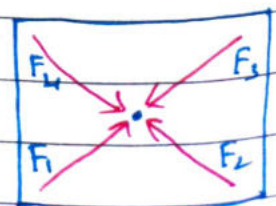
### ③ Coplanar parallel like force system

All the forces lying in the same plane parallel to each other and acting along the same direction.



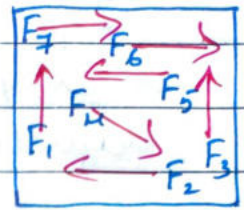
### ④ Coplanar concurrent force system

All the forces are lying in the same plane and line of action of all the forces passing through a single point.



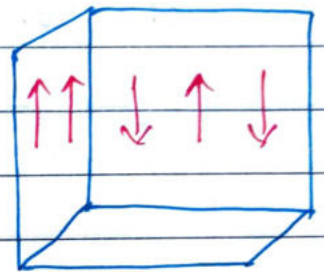
### ⑤ Coplanar non-concurrent force system

All the forces are lying in the same plane & are not parallel to each other and their line of action will not meet at a single point.



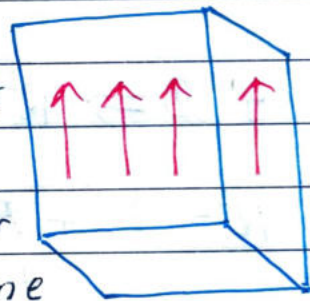
### ⑥ Non-coplanar parallel force system

The forces are lying in different planes but all the forces are parallel to each other.



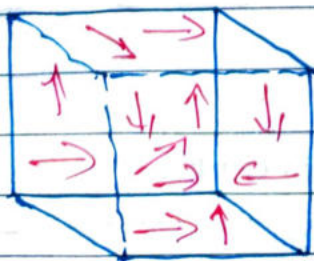
### ⑦ Non-coplanar parallel like force system

The forces are lying in different planes but all the forces are parallel to each other and their line of action is along the same direction.



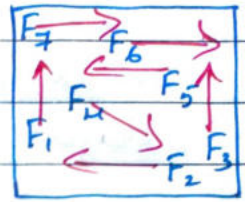
### ⑧ Non-coplanar concurrent force system

The forces will be lying in the different planes, not parallel to each other and not meeting at a single point.



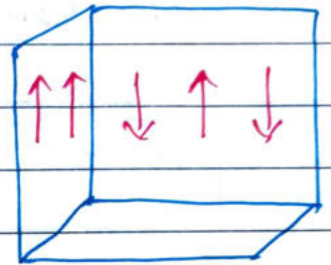
### ⑤ Coplanar non-concurrent force system

All the forces are lying in the same plane & are not parallel to each other and their line of action will not meet at a single point.



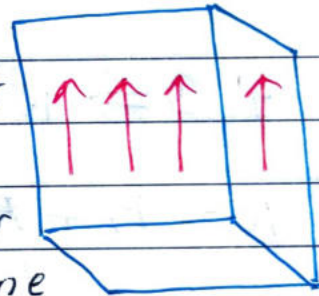
### ⑥ Non-coplanar parallel force system

The forces are lying in different planes but all the forces are parallel to each other.



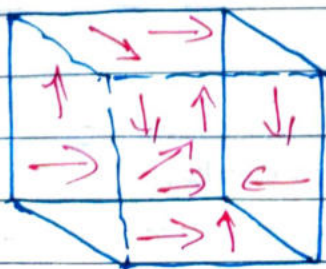
### ⑦ Non-coplanar parallel like force system

The forces are lying in different planes but all the forces are parallel to each other and their line of action is along the same direction.



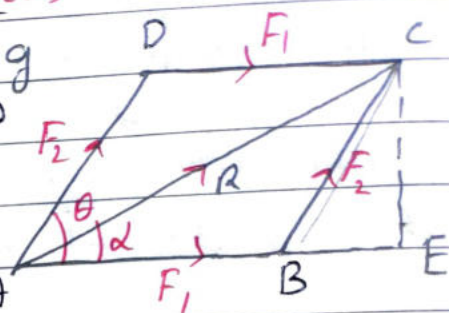
### ⑧ Non-coplanar concurrent force system

The forces will be lying in the different planes, not parallel to each other and not meeting at a single point.



## Parallelogram Law of forces

If two forces  $F_1$  &  $F_2$  acting at a point represented by two adjacent sides of a parallelogram then their resultant is represented by the diagonal of parallelogram passing through the point of intersection of both the forces.



### Magnitude of the resultant force (R)

From  $\Delta^e CEA$ , apply Pythagoras theorem

$$R^2 = AE^2 + CE^2$$

$$R^2 = (AB + BE)^2 + CE^2 \rightarrow \text{①}$$

$$\text{eqn ①} \Rightarrow R^2 = (F_1 + F_2 \cos \theta)^2 + F_2^2 \sin^2 \theta$$

$$R^2 = F_1^2 + F_2^2 \cos^2 \theta + 2F_1 F_2 \cos \theta + F_2^2 \sin^2 \theta$$

$$R^2 = F_1^2 + F_2^2 (\cos^2 \theta + \sin^2 \theta) + 2F_1 F_2 \cos \theta$$

$$R^2 = F_1^2 + F_2^2 + 2F_1 F_2 \cos \theta$$

But from  $\Delta^e CEB$

$$\cos \theta = \frac{BE}{F_2}$$

$$BE = F_2 \cos \theta$$

$$\sin \theta = \frac{CE}{F_2}$$

$$CE = F_2 \sin \theta$$

$$\therefore R = \sqrt{F_1^2 + F_2^2 + 2F_1 F_2 \cos \theta}$$

### Direction of the resultant force (α)

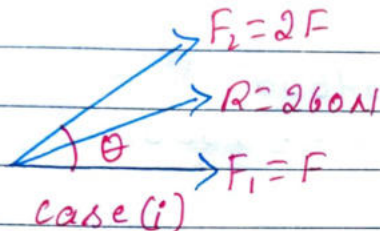
From  $\Delta^e CEA$

$$\tan \alpha = \frac{CE}{AE} = \frac{CE}{AB + BE} = \frac{F_2 \sin \theta}{F_1 + F_2 \cos \theta}$$

$$\therefore \alpha = \tan^{-1} \left[ \frac{F_2 \sin \theta}{F_1 + F_2 \cos \theta} \right]$$

- ① The resultant of two forces, one of which is double the other is 260N. If the direction of the larger force is reversed and the other one remains unaltered, the resultant reduces to 180N. Determine the magnitude of the forces and the angle between the two forces.

Sol<sup>n</sup>:-

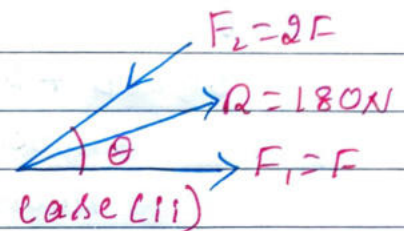


$$F_1 = F = ?$$

$$F_2 = 2F = ?$$

$$\theta = ?$$

$$R = 260 \text{ N}$$



$$F_1 = F = ?$$

$$F_2 = -2F = ?$$

$$\theta = ?$$

$$R = 180 \text{ N}$$

case (i):-

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos \theta}$$

$$260 = \sqrt{F^2 + 2F^2 + 2(F)(2F) \cos \theta}$$

Squaring on both sides

$$260^2 = F^2 + 4F^2 + 4F^2 \cos \theta$$

$$5F^2 + 4F^2 \cos \theta = 67600 \rightarrow \textcircled{1}$$

case (ii):-

$$180 = \sqrt{F^2 + (-2F)^2 + 2F(-2F) \cos \theta}$$

$$180 = \sqrt{F^2 + 4F^2 - 4F^2 \cos \theta}$$

squaring both sides

$$5F^2 - 4F^2 \cos \theta = 32400 \rightarrow \textcircled{2}$$

Adding eq<sup>n</sup> ① & eq<sup>n</sup> ②

$$5F^2 + 4F^2 \cos \theta = 67600$$

$$5F^2 - 4F^2 \cos \theta = 32400$$

$$10F^2 = 10,000$$

$$\therefore F = 1000$$

$$\therefore F = 100 \text{ N}$$

$$F_1 = 100 \text{ N} \quad \& \quad F_2 = 200 \text{ N}$$

Substitute the value of Force 'F' in eq<sup>n</sup> ①

$$5F^2 + 4F^2 \cos \theta = 67600$$

$$5(100)^2 + 4(100)^2 \cos \theta = 67600$$

$$50,000 + 40,000 \cos \theta = 67600$$

$$40000 \cos \theta = 67600 - 50000$$

$$40000 \cos \theta = 17600$$

$$\cos \theta = \frac{17600}{40000}$$

$$\cos \theta = 0.44$$

$$\theta = \cos^{-1}(0.44)$$

$$\theta = 63.9^\circ$$

- ② The sum of two resultant forces P & Q is 500N & their resultant is 400N. If the resultant is perpendicular to 'P', Find P, Q and the angle b/w P & Q.

Sol<sup>n</sup>:-  
 $P + Q = 500 \text{ N} \rightarrow \text{①}$   
 $R = 400 \text{ N}$

From fig.  $Q^2 = P^2 + R^2$

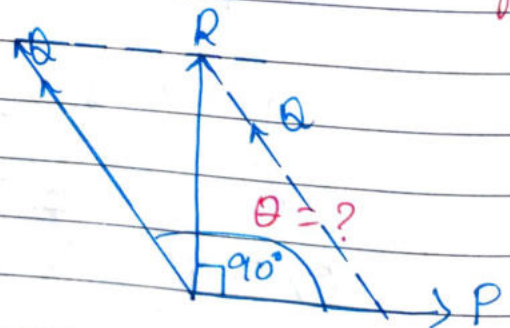
$$\therefore Q^2 - P^2 = R^2$$

$$(Q+P)(Q-P) = R^2$$

$$500(Q-P) = 400^2$$

$$Q-P = \frac{400^2}{500}$$

$$Q-P = 320 \rightarrow \text{②}$$



Adding eqn ① & ②

$$Q + P = 500$$

$$Q - P = 320$$

$$2Q = 820$$

$$Q = 410$$

$$\therefore P = 90 \text{ N}$$

WKT,  $R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta}$

$$400 = \sqrt{90^2 + 410^2 + 2 \times 90 \times 410 \times \cos \theta}$$

$$400 = \sqrt{176200 + 73800 \cos \theta}$$

Square on both sides

$$160000 = 176200 + 73800 \cos \theta$$

$$73800 \cos \theta = -176200 + 160000$$

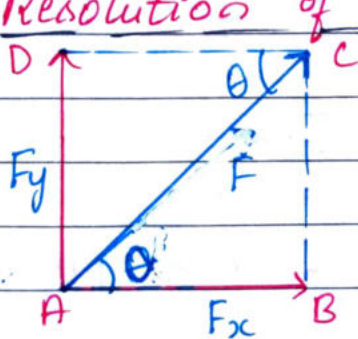
$$\cos \theta = -0.219$$

$$\theta = \cos^{-1}(-0.219)$$

$$\theta = 102.6^\circ$$

## ⑧ Resolution and composition of forces

### ① Resolution of forces



From  $\triangle ABC$

$$\cos \theta = \frac{F_x}{F}$$

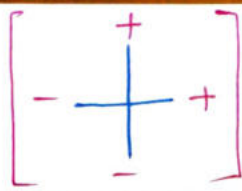
$$F_x = F \cos \theta$$

From  $\triangle ADC$

$$\sin \theta = \frac{F_y}{F}$$

$$F_y = F \sin \theta$$

Splitting up of a force into two rectangular components (H<sup>le</sup> and V<sup>le</sup>) is called resolution of a force.



Sign convention

## ② Composition of forces

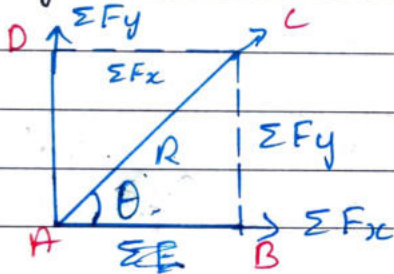
Combining 2 or more forces into a single resultant force is called composition of forces.

$$\Sigma F_{xc} = F_1 + F_2 \cos \theta_2 - F_3 \cos(90 - \theta_3) - F_4 \cos \theta_4$$

$$\Sigma F_{xc} = F_1 + F_2 \cos \theta_2 - F_3 \sin \theta_3 - F_4 \cos \theta_4$$

$$\Sigma F_y = -F_2 \sin \theta_2 - F_3 \sin(90 - \theta_3) + F_4 \sin \theta_4 + F_5$$

$$\Sigma F_y = -F_2 \sin \theta_2 - F_3 \cos \theta_3 + F_4 \sin \theta_4 + F_5$$



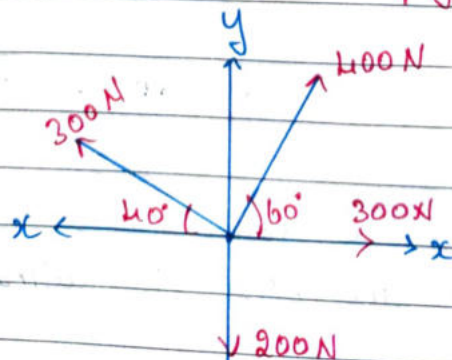
$$R^2 = \Sigma F_x^2 + \Sigma F_y^2$$

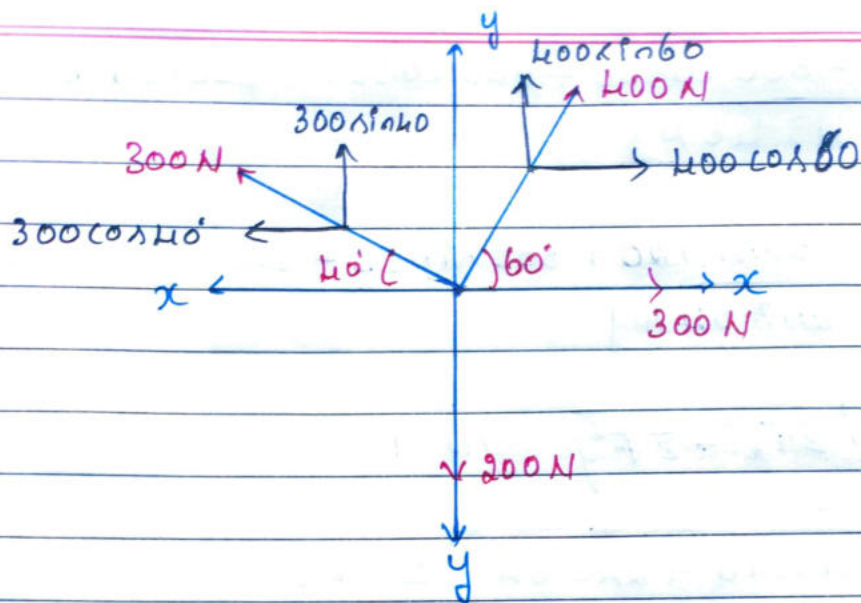
$$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}$$

$$\tan \theta = \frac{\Sigma F_y}{\Sigma F_x}$$

$$\theta = \tan^{-1} \left[ \frac{\Sigma F_y}{\Sigma F_x} \right]$$

① Find the resultant of the coplanar concurrent force system shown in figure.





$$\Sigma F_x = -300 \cos 40^\circ + 400 \cos 60^\circ + 300$$

$$\boxed{\Sigma F_x = 270.187 \text{ N}}$$

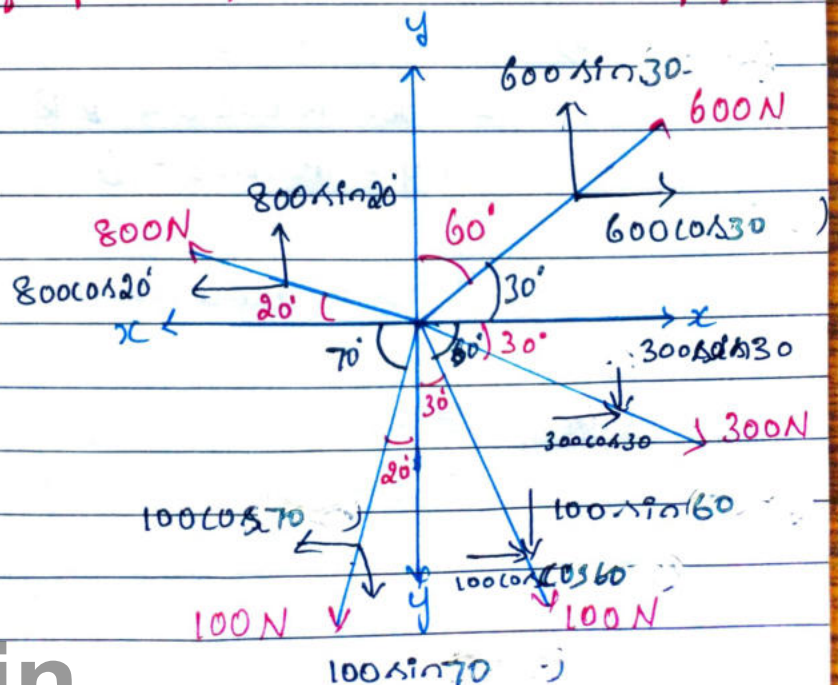
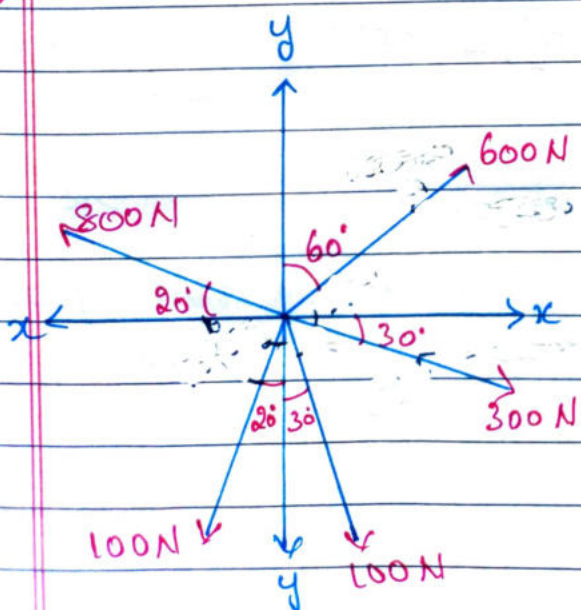
$$\Sigma F_y = 300 \sin 40^\circ + 400 \sin 60^\circ - 200$$

$$\boxed{\Sigma F_y = 339.246 \text{ N}}$$

$$\therefore R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2} = \sqrt{270.187^2 + 339.246^2} = \underline{\underline{433.7 \text{ N}}}$$

$$\theta = \tan^{-1} \left[ \frac{\Sigma F_y}{\Sigma F_x} \right] = \tan^{-1} \left[ \frac{339.246}{270.187} \right] = \underline{\underline{51.47^\circ}}$$

② Find the resultant of force system shown in figure.



$$\Sigma F_x = -800 \cos 20 + 600 \cos 30 + 300 \cos 30 + 100 \cos 60 - 100 \cos 70$$

$$\Sigma F_x = 43.46 \text{ N}$$

$$\Sigma F_y = 800 \sin 20 + 600 \sin 30 - 300 \sin 30 - 100 \sin 60 - 100 \sin 70$$

$$\Sigma F_y = 243.04 \text{ N}$$

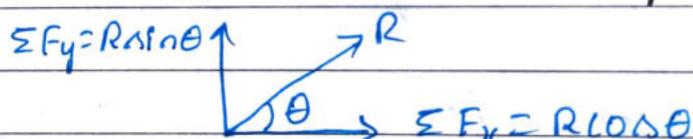
$$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}$$

$$R = \sqrt{43.46^2 + 243.04^2} = \underline{\underline{246.9 \text{ N}}}$$

$$\theta = \tan^{-1} \left[ \frac{\Sigma F_y}{\Sigma F_x} \right] = \tan^{-1} \left[ \frac{243.04}{43.46} \right] = \underline{\underline{80^\circ}}$$

Note:-

1) If the magnitude <sup>& direction</sup> of the <sup>is</sup> given in the problem then  $\Sigma F_x = R \cos \theta$  &  $\Sigma F_y = R \sin \theta$



If 'R' makes an angle 'theta' with x-axis then  $\Sigma F_x = R \cos \theta$  &  $\Sigma F_y = R \sin \theta$

2) If the resultant is given along the x-axis.

$$\Sigma F_x = R \cos 0 = \underline{\underline{R}}$$

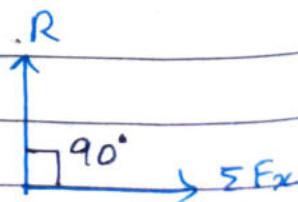
$$\Sigma F_y = R \sin 0 = \underline{\underline{0}}$$



3) If the resultant is given along the y-axis

$$\Sigma F_x = R \cos 90 = \underline{\underline{0}}$$

$$\Sigma F_y = R \sin 90 = \underline{\underline{R}}$$



- ③ Four coplanar forces acting at a point are shown in figure. One of the forces is unknown & its magnitude is shown by 'P'. The resultant has a magnitude of 500 N and is acting along x-axis. Determine the unknown force P and its inclination with the x-axis.

Sol<sup>n</sup>:- Resultant lies in x-axis

$$\therefore \Sigma F_x = R ; \Sigma F_y = 0 ;$$

$$\therefore \Sigma F_x = R = 500 \text{ N}$$

$$\Sigma F_y = 0$$

$$\Sigma F_x = 500 \text{ N}$$

$$- P \cos \theta + 200 \cos 45^\circ - 500 \cos 30^\circ = 500 \text{ N}$$

$$P \cos \theta = -791.6 \text{ N} \rightarrow \text{①}$$

$$\Sigma F_y = 0$$

$$- 500 \sin 30^\circ + P \sin \theta + 200 \sin 45^\circ - 200 = 0$$

$$P \sin \theta = 308.58 \text{ N} \rightarrow \text{②}$$

Squaring and adding eq<sup>n</sup> ① & ②

$$P^2 \sin^2 \theta + P^2 \cos^2 \theta = 308.58^2 + (-791.6)^2$$

$$P^2 (\sin^2 \theta + \cos^2 \theta) = 721852.17$$

$$P = 849.61 \text{ N}$$

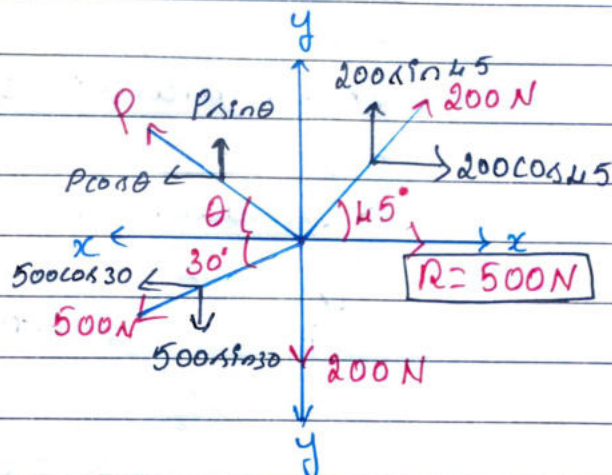
Substitute 'P' value in eq<sup>n</sup> ①

$$\text{eq<sup>n</sup> ①} \Rightarrow 849.61 \cos \theta = -791.6$$

$$\cos \theta = -0.931$$

$$\theta = \cos^{-1}(-0.931)$$

$$\theta = 21.40^\circ$$



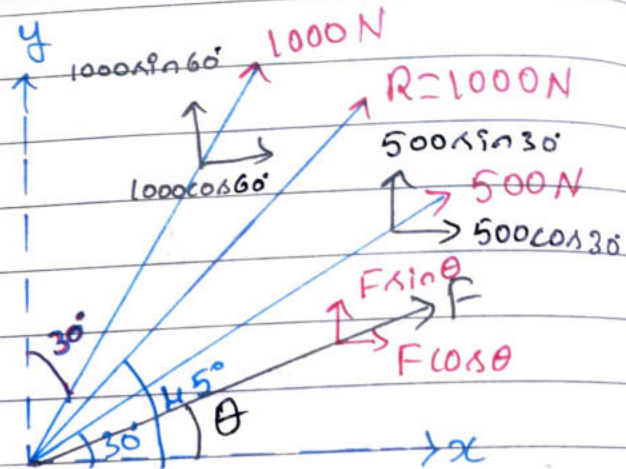
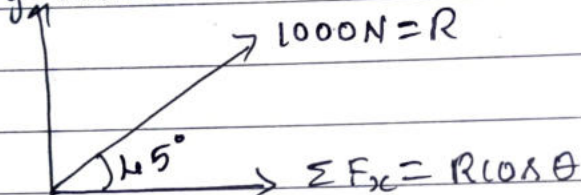
- ④ Two forces acting on a body are 500N & 1000N as shown in figure. Determine the 3<sup>rd</sup> force 'F' such that the resultant of all the three forces is 1000N directed at  $45^\circ$  to the x-axis

Soln:-

Assume,

Let 'F' be the 3<sup>rd</sup> force which makes an angle ' $\theta$ ' with the x-axis as shown in figure.

$$\Sigma F_y = R \sin \theta$$



$$\Sigma F_x = R \cos \theta = 1000 \cos 45 = 707.10 \text{ N}$$

$$\Sigma F_y = R \sin \theta = 1000 \sin 45 = 707.10 \text{ N}$$

consider

$$\Sigma F_x = 707.10 \text{ N}$$

$$F \cos \theta + 500 \cos 30 + 1000 \cos 60 = 707.10$$

$$F \cos \theta = -225.91 \rightarrow \textcircled{1}$$

consider

$$\Sigma F_y = 707.10 \text{ N}$$

$$F \sin \theta + 500 \sin 30 + 1000 \sin 60 = 707.10$$

$$F \sin \theta = -409 \rightarrow \textcircled{2}$$

Squaring & adding eqn ① & ②

$$F^2 \cos^2 \theta + F^2 \sin^2 \theta = 225.91^2 + 409^2$$

$$F^2 (\sin^2 \theta + \cos^2 \theta) = 218316.32$$

$$F = 467.24 \text{ N}$$

Divide eq<sup>n</sup> ② by ①

$$\frac{F \sin \theta}{F \cos \theta} = \frac{409}{225.91}$$

$$\tan \theta = \frac{409}{225.91}$$

$$\theta = \tan^{-1}(1.81)$$

$$\theta = 61.08^\circ$$

- ⑤ 2 kN force is the resultant of system of forces acting up along y-axis as shown in fig. Determine the value of  $F$  &  $\theta$ .

Sol<sup>n</sup>:-

$$\uparrow R \quad \Sigma F_y = 2 \text{ kN}$$

$$\square 90^\circ \rightarrow \Sigma F_x = 0$$

As Resultant lies in y-axis

$$\Sigma F_y = 2 \text{ kN} \quad \& \quad \Sigma F_x = 0$$

considering,  $\Sigma F_y = 2 \text{ kN}$

$$\therefore -2.4 \sin 30^\circ + F \sin \theta = 2$$

$$F \sin \theta = 3.2 \rightarrow \text{①}$$

Taking  $\Sigma F_x = 0$

$$-5 + 2.4 \cos 30^\circ + F \cos \theta = 0$$

$$F \cos \theta = 2.92 \text{ kN} \rightarrow \text{②}$$

Squaring & adding eq<sup>n</sup> ① & ②

$$F^2 \sin^2 \theta + F^2 \cos^2 \theta = 3.2^2 + 2.92^2$$

$$F^2 (\sin^2 \theta + \cos^2 \theta) = 18.76$$

$$F = 4.33 \text{ kN}$$

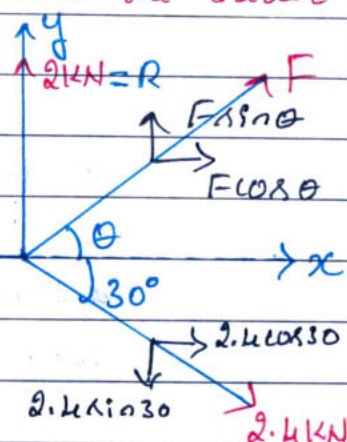
Divide eq<sup>n</sup> ① by ②

$$\frac{F \sin \theta}{F \cos \theta} = \frac{3.2}{2.92}$$

$$\tan \theta = \frac{3.2}{2.92}$$

$$\theta = \tan^{-1}(3.2/2.92)$$

$$\theta = 47.605^\circ$$

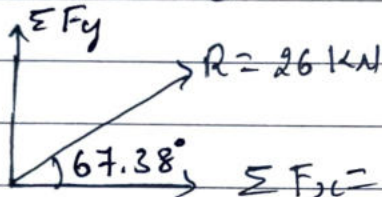


- ⑥ 26 kN force is the resultant of two forces, one of which is as shown in figure. Determine the other force.

Sol<sup>n</sup> :- From figure

$$\theta_1 = \tan^{-1} (12/5) = 67.38^\circ$$

$$\theta_2 = \tan^{-1} (3/4) = 36.87^\circ$$



$$\Sigma F_x = R \cos \theta$$

$$= 26 \cos 67.38^\circ$$

$$\Sigma F_x = 10 \text{ kN}$$

$$\Sigma F_y = R \sin \theta = 26 \sin 67.38^\circ = 24 \text{ kN}$$

Considering,  $\Sigma F_x = 10 \text{ kN}$

$$10 \cos 36.87^\circ + P \cos \theta = 10$$

$$P \cos \theta = 2 \rightarrow \textcircled{1}$$

Taking,  $\Sigma F_y = 24 \text{ kN}$

$$10 \sin 36.87^\circ + P \sin \theta = 24$$

$$P \sin \theta = 18 \rightarrow \textcircled{2}$$

Squaring & adding eq<sup>n</sup> ① & ②

$$P^2 \cos^2 \theta + P^2 \sin^2 \theta = 2^2 + 18^2$$

$$P^2 (\sin^2 \theta + \cos^2 \theta) = 328$$

$$P = 18.11 \text{ kN}$$

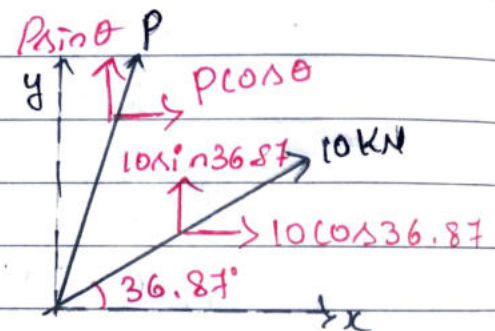
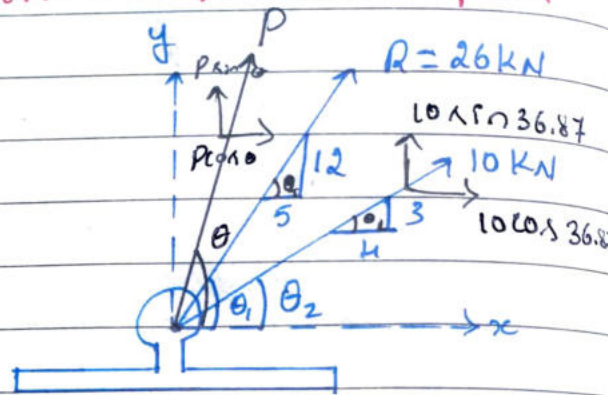
Divided eq<sup>n</sup> ② by ①

$$\frac{P \sin \theta}{P \cos \theta} = \frac{18}{2}$$

$$\tan \theta = 9$$

$$\theta = \tan^{-1} (9)$$

$$\theta = 83.66^\circ$$



- ⑦ The resultant of a force system on a bracket as shown in figure is acting vertically upwards. If the angle  $30^\circ$  between the two like forces is fixed, find the angle ' $\theta$ ' & magnitude of the force.

Sol:- As resultant lies in y-axis

$$R = \Sigma F_y$$

$$\Sigma F_x = 0$$

considering  $\Sigma F_x = 0$

$$-2 \cos 60 + 4 \cos (30 + \theta) + 4 \cos \theta = 0$$

$$-1 + 4 \cos (30 + \theta) + 4 \cos \theta = 0$$

$$4 [\cos (30 + \theta) + \cos \theta] = 1$$

$$\cos (30 + \theta) + \cos \theta = \frac{1}{4}$$

$$2 \cos \left( \frac{30 + \theta + \theta}{2} \right) \cos \left( \frac{30 + \theta - \theta}{2} \right) = \frac{1}{4}$$

$$\left[ \because \cos A + \cos B = 2 \cos \left( \frac{A+B}{2} \right) \cos \left( \frac{A-B}{2} \right) \right]$$

$$2 \cos \left( \frac{30 + 2\theta}{2} \right) \cos 15 = \frac{1}{4}$$

$$1.931 \cos \left( \frac{30 + 2\theta}{2} \right) = \frac{1}{4}$$

$$1.931 \cos (15 + \theta) = \frac{1}{4}$$

$$\cos (15 + \theta) = 0.129$$

$$15 + \theta = 82.56^\circ$$

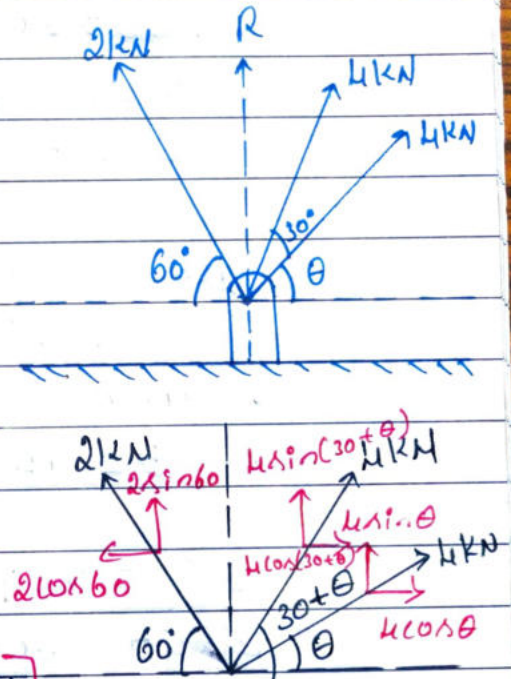
$$\theta = 67.5^\circ$$

Taking  $\Sigma F_y = R$

$$2 \sin 60 + 4 \sin (30 + 67.5) + 4 \sin 67.5 = R$$

$$1.73 + 3.96 + 3.69 = R$$

$$R = 9.39 \text{ kN}$$



- ⑧ Determine the resultant force acting on the structure at point 'O' both in magnitude and direction.

Sol<sup>n</sup>:- From  $\Delta^{le} OBC$

$$\cos 60 = BC/6$$

$$BC = 6 \cos 60 = \underline{3\text{ m}}$$

$$\sin 60 = CO/6$$

$$CO = 6 \sin 60 = \underline{5.196\text{ m}}$$

From figure  $AC = AB + BC = 3 + 3 = \underline{6\text{ m}}$

From  $\Delta^{le} OCA$

$$\tan \theta = 5.196/6$$

$$\theta = \tan^{-1} (5.196/6) = \underline{40.9^\circ}$$

$$\Sigma F_x = 800 - 600 \cos 40.9 = \underline{346.48\text{ N}}$$

$$\Sigma F_y = -600 \sin 40.9 = \underline{-392.84\text{ N}}$$

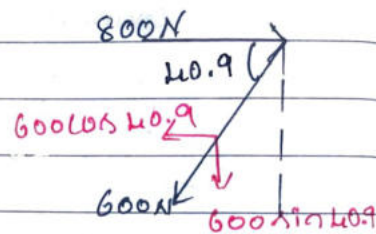
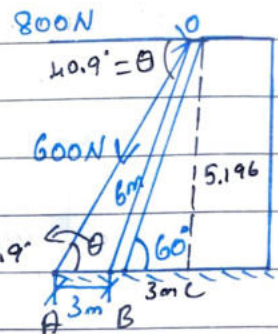
$$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2} = \sqrt{346.48^2 + 392.84^2}$$

$$\boxed{R = 523.708\text{ N}}$$

$$\theta = \tan^{-1} \left[ \frac{\Sigma F_y}{\Sigma F_x} \right]$$

$$\theta = \tan^{-1} \left[ \frac{392.84}{346.48} \right]$$

$$\boxed{\theta = 48.58^\circ}$$



- ⑨ Three forces acting at the centre of gravity of a block are shown in figure. The direction of 300 N forces may vary but the angle b/w them is always  $120^\circ$ . Determine the value of ' $\theta$ ' for which the resultant of the three forces is directed parallel to the plane. also find resultant of force.

Sol<sup>n</sup>: As resultant lies in x-axis

$$\Sigma F_x = R \quad \& \quad \Sigma F_y = 0$$

considering  $\Sigma F_y = 0$

$$500 \sin 30 + 300 \sin (40 + \theta) + 300 \sin \theta = 0$$

$$500 \sin 30 + 300 [\sin (40 + \theta) + \sin \theta] = 0$$

$$300 [\sin (40 + \theta) + \sin \theta] = -250$$

$$\sin (40 + \theta) + \sin \theta = -0.833$$

$$2 \sin \left( \frac{40 + \theta + \theta}{2} \right) \cos \left( \frac{40 + \theta - \theta}{2} \right) = -0.833$$

$$\sin (A+B) = 2 \sin \left( \frac{A+B}{2} \right) \cos \left( \frac{A-B}{2} \right)$$

$$\sin \left[ \frac{(40 + 2\theta)}{2} \right] \cos (20) = -0.416$$

$$\sin \left( \frac{2(20 + \theta)}{2} \right) \cos 20 = -0.416$$

$$\sin (20 + \theta) \cos 20 = -0.416$$

$$\sin (20 + \theta) \times 0.94 = -0.416$$

$$\sin (20 + \theta) = -0.443$$

$$20 + \theta = \sin^{-1}(-0.443)$$

$$20 + \theta = -26.3$$

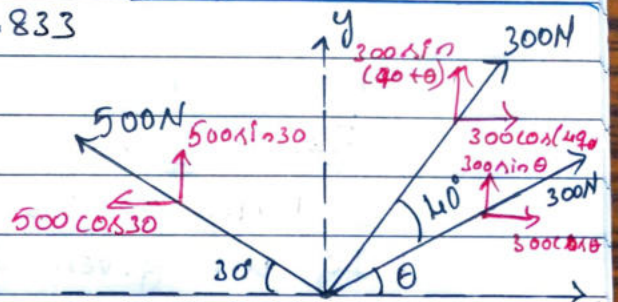
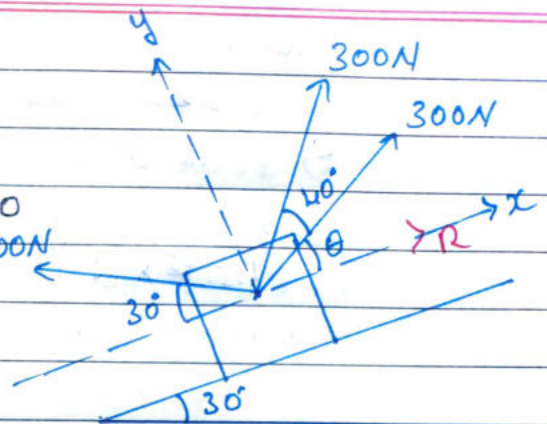
$$\theta = -46.3^\circ$$

Taking  $\Sigma F_x = R$

$$-500 \cos 30 + 300 \cos (40 - 46.3) + 300 \cos (-46.3) = R$$

$$-433.01 + 298.18 + 207.26 = R$$

$$R = 72.43 \text{ N}$$



## Moment of force [M]

The measure of rotational effect of a force on a body is called moment of force.

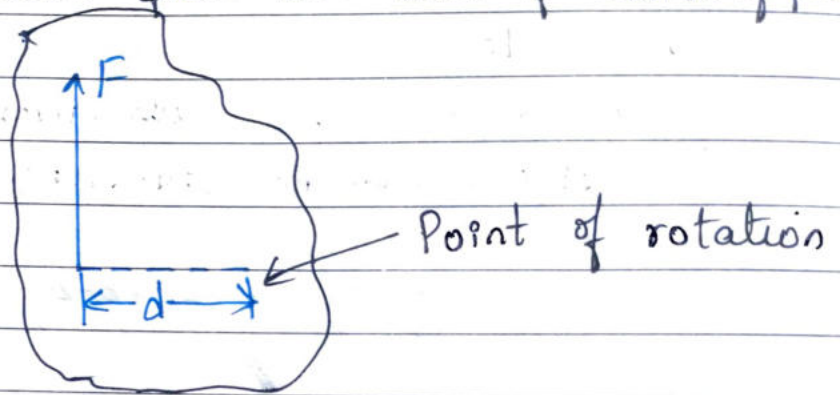
It is calculated as the product of the magnitude of the force & the perpendicular distance b/w the line of action of the forces and the point (or) axis of rotation.



Moment of force depends on two main factors

- 1) Magnitude of the force  $[F]$
- 2) Perpendicular distance from the line of action of force  $[d]$

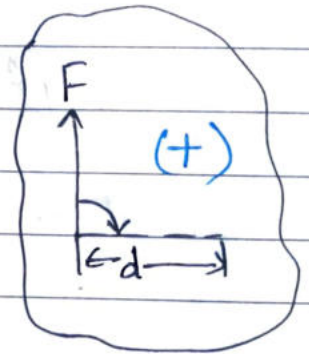
$$\therefore M = F \times d$$



### Types of moment

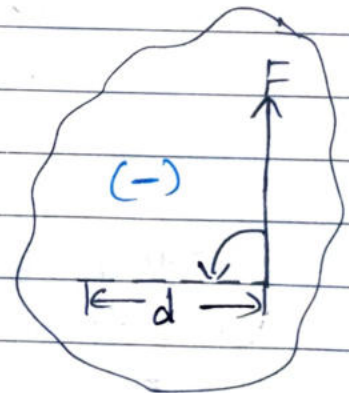
#### ① Clockwise moment

If the given force tends to rotate the body in clockwise direction, then it is said to be a clockwise moment and taken to be +ve.

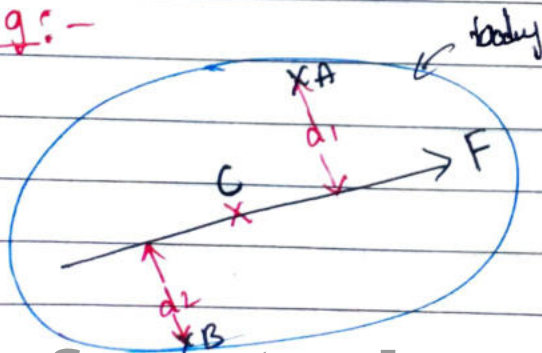


#### ② Anticlockwise moment

If the given force tends to rotate the body in anticlockwise direction, then it is said to be an anticlockwise moment and taken to be -ve.



E.g:-



$$M_A = -Fd$$

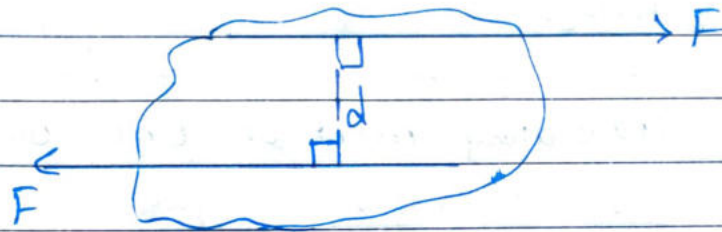
$$M_B = +Fd$$

$$M_C = F \times 0 = 0$$

## Couple

Two parallel forces equal in magnitude and opposite direction and separated by a definite distance, constitutes a couple.

The sum of the forces forming a couple in any direction is zero. i.e., The translatory effect of the couple is zero.

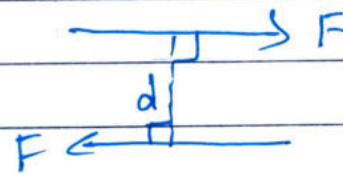


## Moment of a couple

magnitude of

The moment of a couple is the product of, one of the forces and the perpendicular distance between the forces.

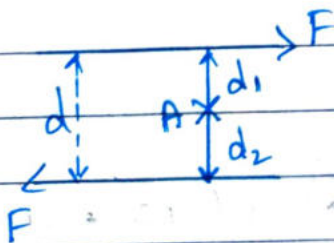
$$\text{i.e., } M = F \times d$$



S.I. Unit:- N-m; kN-m;

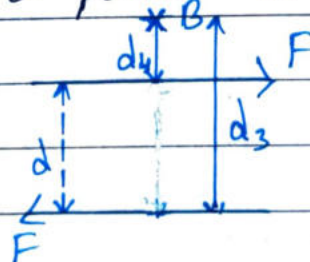
N-m; kN-mm;

Consider two forces of magnitude 'F' and the perpendicular distance between two forces be 'd'. Now, moment of these forces about a point is given by.



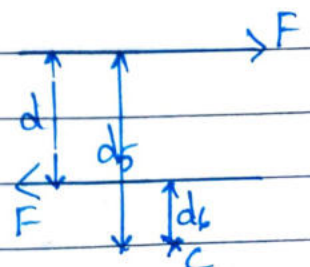
$$M_A = Fd_1 + Fd_2 \\ = F(d_1 + d_2)$$

$$M_A = Fd$$



$$M_B = Fd_3 - Fd_4 \\ = F(d_3 - d_4)$$

$$M_B = Fd$$



$$M_C = Fd_5 - Fd_6 \\ = F(d_5 - d_6)$$

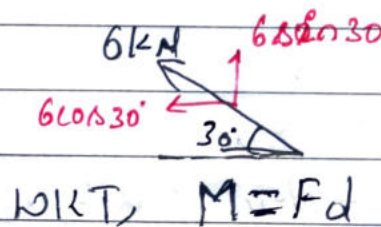
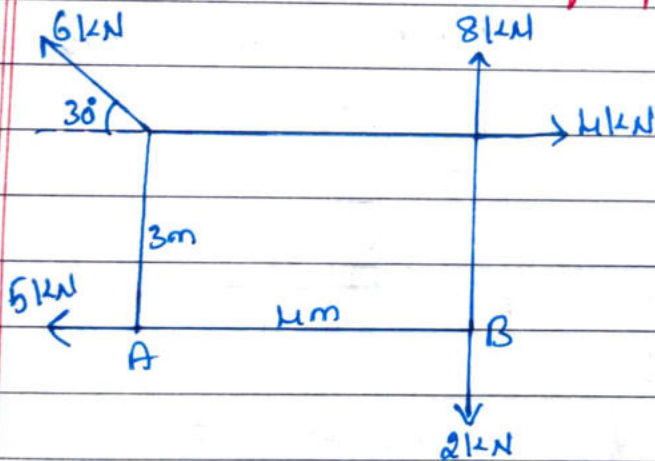
$$M_C = Fd$$

## Characteristics of Couple

- ① Two equal & opposite parallel forces are required to form a couple.
- ② The magnitude of the moment of the couple = product of the magnitude of one of the forces and moment arm [Perpendicular distance between the two forces].
- ③ Resultant of the forces of the couple is zero.
- ④ The translatory effect of the couple is zero.

## Problems on moment of force

- ① Determine moment of forces at points 'A' & 'B'.



Sol<sup>n</sup>:- Moment about 'A'

$$M_A = -6 \cos 30^\circ \times 3 + 6 \sin 30^\circ \times 4 - (8 \times 4) + (4 \times 3) + (2 \times 4) - (5 \times 0)$$

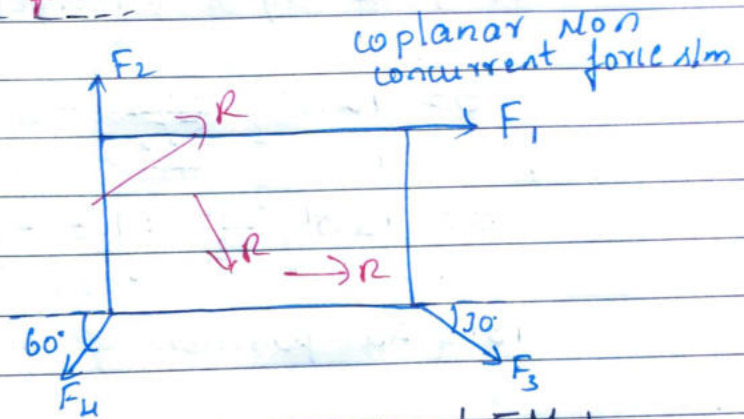
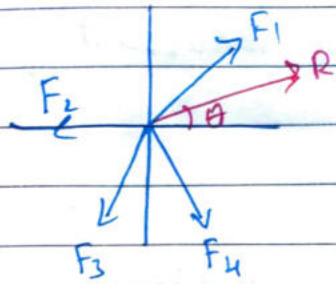
$$M_A = -27.58 \text{ kN-m}$$

Moment about 'B'

$$M_B = -6 \cos 30^\circ \times 3 + 6 \sin 30^\circ \times 4 + (8 \times 0) + (4 \times 3) + (5 \times 0) + (2 \times 0)$$

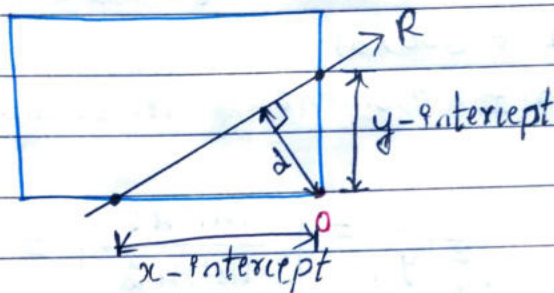
$$M_B = 8.411 \text{ kN-m}$$

## Position of the resultant forces



$$x\text{-intercept} = \left| \frac{\sum M}{\sum F_y} \right|$$

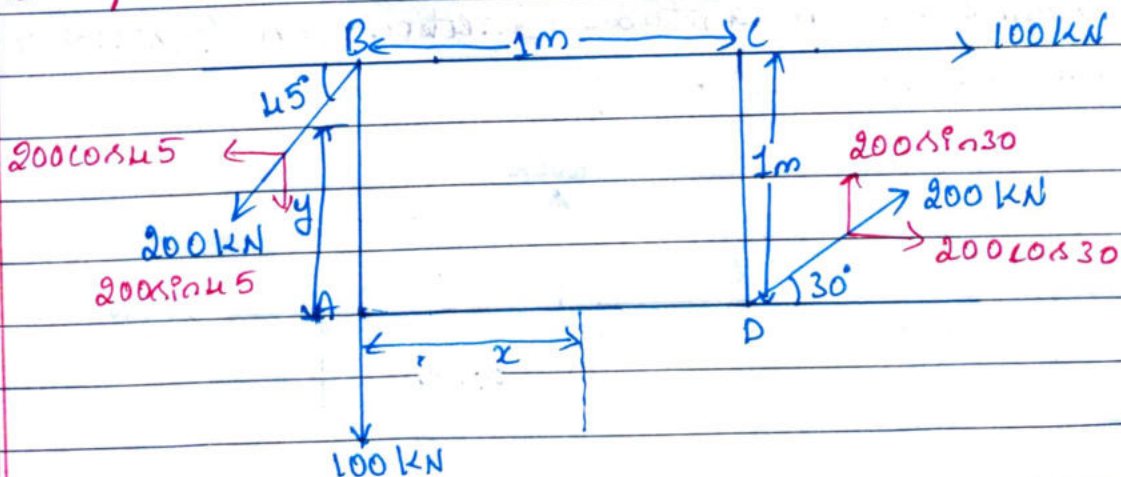
$$y\text{-intercept} = \left| \frac{\sum M}{\sum F_x} \right|$$



$$d = \left| \frac{\sum M}{R} \right| \quad \text{Mode}$$

## Problems on coplanar non-concurrent system of forces

- ① A rigid plate ABCD is subjected to forces as shown in figure. Compute the magnitude, direction and line of action of the resultant of the system with reference to the point A!



Soln:-

$$\sum F_x = -200 \cos 45^\circ + 100 + 200 \cos 30^\circ = 131.78 \text{ kN}$$

$$\sum F_y = -200 \sin 45^\circ + 200 \sin 30^\circ - 100 = -141.42 \text{ kN}$$

$$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2} = \sqrt{131.78^2 + 141.42^2} = \underline{193.3 \text{ kN}} = \text{Magnitude of resultant of force}$$

$$\theta = \tan^{-1} \left[ \frac{\Sigma F_y}{\Sigma F_x} \right] = \text{direction of resultant of forces}$$

$$\theta = \tan^{-1} \left[ \frac{-141.42}{131.78} \right] = \underline{-47.02^\circ}$$

To find position of resultant of system of forces

$$\Sigma M_A = -200 \cos 45^\circ \times 1 + 200 \sin 45^\circ \times 0 + (100 \times 1) + 200 \cos 30^\circ \times 0 - 200 \sin 30^\circ \times 1 + (100 \times 0)$$

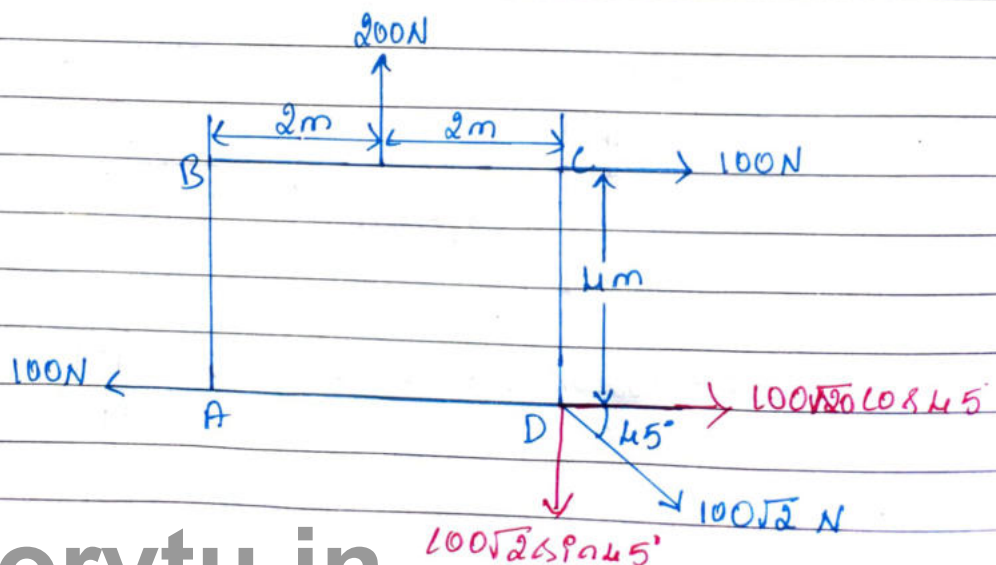
$$\Sigma M_A = -141.42 \text{ kN-m} = \text{Moment of resultant of forces.}$$

$$\text{WKT, } x\text{-intercept} = \left| \frac{\Sigma M}{\Sigma F_y} \right| = \left| \frac{-141.42}{141.42} \right| = \underline{1 \text{ m}}$$

$$y\text{-intercept} = \left| \frac{\Sigma M}{\Sigma F_x} \right| = \left| \frac{-141.42}{131.78} \right| = \underline{1.073 \text{ m}}$$

$$d = \left| \frac{\Sigma M}{R} \right| = \left| \frac{-141.42}{193.3} \right| = \underline{0.732 \text{ m}}$$

- ② For the non-concurrent coplanar system shown in fig. determine the magnitude, direction and position of the resultant force with reference to A.



Sol<sup>n</sup>:-  $\Sigma F_x = -100 + 100 + 100\sqrt{2} \cos 45^\circ = 100 \text{ N}$   
 $\Sigma F_y = 200 - 100\sqrt{2} \sin 45^\circ = 100 \text{ N}$

$$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2} = \sqrt{100^2 + 100^2} = 141.42 \text{ N}$$

$$\theta = \tan^{-1} \left[ \frac{\Sigma F_y}{\Sigma F_x} \right] = \tan^{-1} \left[ \frac{100}{100} \right] = 45^\circ$$

To find the position of the resultant force at point 'A'

$$\Sigma M_A = (100 \times 0) - (200 \times 2) + (100 \times 4) + (100\sqrt{2} \cos 45^\circ \times 0) + (100\sqrt{2} \sin 45^\circ \times 4)$$

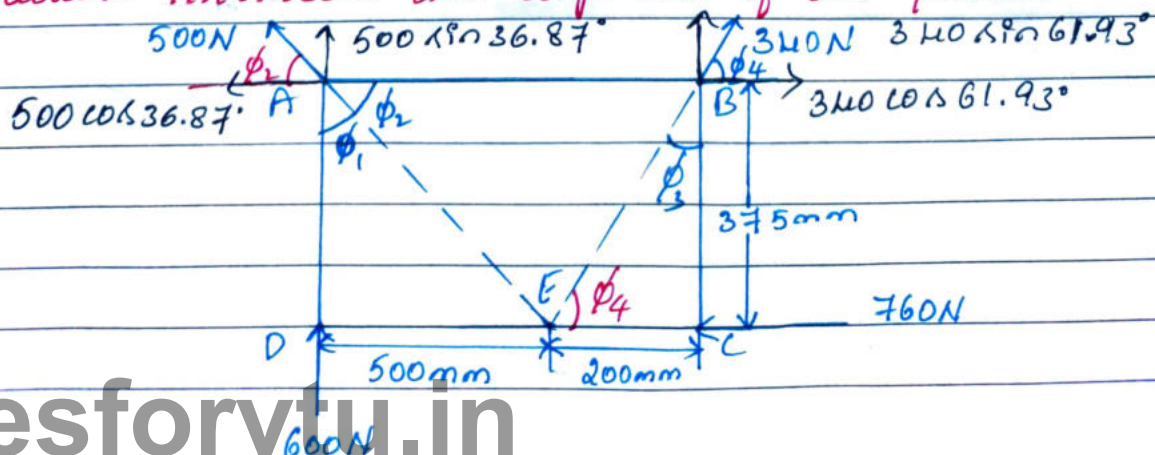
$$\Sigma M_A = 400 \text{ N-m}$$

$$\text{WKT, } x\text{-intercept} = \left| \frac{\Sigma M_A}{\Sigma F_y} \right| = \left| \frac{400}{100} \right| = 4 \text{ m}$$

$$y\text{-intercept} = \left| \frac{\Sigma M_A}{\Sigma F_x} \right| = \left| \frac{400}{100} \right| = 4 \text{ m}$$

$$d = \left| \frac{\Sigma M_O}{R} \right| = \left| \frac{400}{141.42} \right| = 2.82 \text{ m}$$

- ③ Four forces act on a 700mm x 375mm plate as shown in figure. Find the resultant of these forces, and locate the point where the line of action of the resultant intersects the edge AB of the plate.



1) :- From  $\Delta^{\text{Ic}} BFC \Rightarrow \tan \phi_4 = \frac{375}{200} \Rightarrow \phi_4 = \tan^{-1} \left( \frac{375}{200} \right)$

From  $\Delta^{\text{Ic}} ADE \Rightarrow \phi_2 = \tan^{-1} \left( \frac{375}{500} \right) = 36.87^\circ$

$$\Sigma F_x = -500 \cos 36.87 + 340 \cos 61.93 - 760 = -1000 \text{ N}$$

$$\Sigma F_y = 600 + 500 \sin 36.87 + 340 \sin 61.93 = 1200 \text{ N}$$

$$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2} = \sqrt{1000^2 + 1200^2} = 1562 \text{ N}$$

$$\theta = \tan^{-1} \left[ \frac{\Sigma F_y}{\Sigma F_x} \right] = \tan^{-1} \left[ \frac{1200}{-1000} \right] = -50.19^\circ$$

W.K.T,  $\Sigma M_A = 0$

$$\Sigma M_A = (600 \times 0) + (500 \times 0) + (340 \cos 61.93 \times 0) - [340 \sin 61.93 \times 700] + (760 \times 375)$$

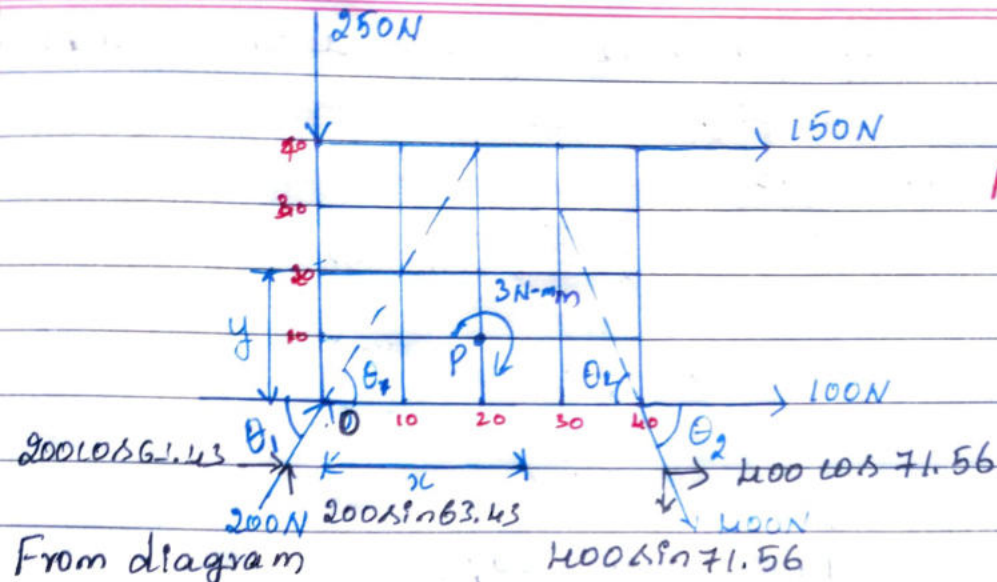
$$\Sigma M_A = 74,995 \text{ N-m}$$

$$x\text{-intercept} = \left| \frac{\Sigma M}{\Sigma F_y} \right| = \left| \frac{74995}{1200} \right| = 62.496 \text{ mm}$$

$$y\text{-intercept} = \left| \frac{\Sigma M}{\Sigma F_x} \right| = \left| \frac{74995}{1000} \right| = 74.995 \text{ mm}$$

$$d = \left| \frac{\Sigma M}{R} \right| = \left| \frac{74995}{1562} \right| = 48.01 \text{ m}$$

Determine the position of resultant of a force system if the external moment  $M = 3 \text{ N-m}$  at the point  $P(20, 10)$  as shown in figure.



$$\theta_2 = \tan^{-1} \left( \frac{30}{10} \right) = 71.56$$

$$\theta_1 = \tan^{-1} \left( \frac{40}{10} \right) = 63.43^\circ$$

$$\Sigma F_x = 200 \cos 63.43 + 150 + 100 + 400 \cos 71.56 = 465.98 \text{ N}$$

$$\Sigma F_y = 200 \sin 63.43 - 250 - 400 \sin 71.56 = -450.58 \text{ N}$$

$$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2} = \sqrt{(465.98)^2 + (-450.58)^2} = 648.19 \text{ N}$$

$$\theta = \tan^{-1} \left[ \frac{\Sigma F_y}{\Sigma F_x} \right] = \tan^{-1} \left[ \frac{-450.58}{465.98} \right] = -44.03^\circ$$

$$\Sigma M_o = (200 \times 0) + (250 \times 0) + (150 \times 40) + (100 \times 0) + (400 \cos 71.56 \times 0) + (400 \sin 71.56 \times 40) + 3$$

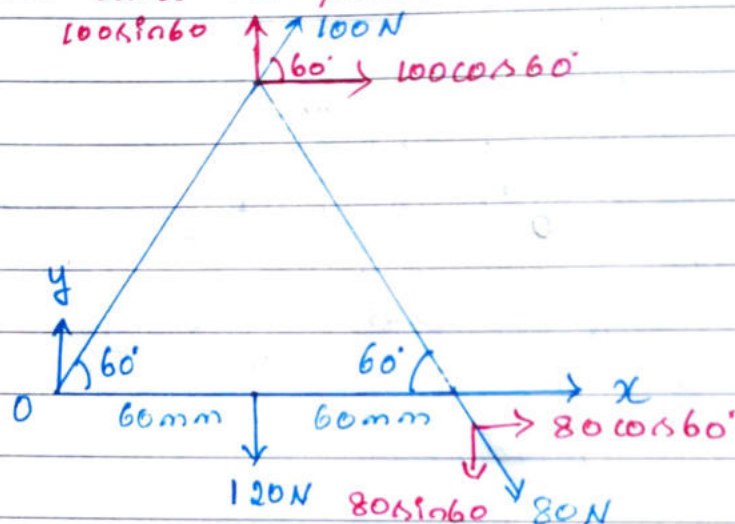
$$\Sigma M_o = 21181.48 \text{ N-mm}$$

$$\text{WKT, } x\text{-intercept} = \left| \frac{\Sigma F_y}{\Sigma F_x} \right| = \left| \frac{450.58}{465.98} \right| = 0.967 \text{ mm}$$

$$y\text{-intercept} = \left| \frac{\Sigma M}{\Sigma F_x} \right| = \left| \frac{7520}{465.98} \right| = 16.13 \text{ mm}$$

$$d = \left| \frac{\Sigma M}{R} \right| = \left| \frac{7520}{648.19} \right| = 11.60 \text{ mm}$$

- ⑤ The system of forces acting on a triangular plate is as shown in figure. Determine the magnitude, direction of the resultant and its position with respect to 'O'.



Sol<sup>n</sup>:- WKT,

$$\Sigma F_x = 100 \cos 60^\circ + 80 \cos 60^\circ = 90 \text{ N}$$

$$\Sigma F_y = 100 \sin 60^\circ - 80 \sin 60^\circ - 120 = -102.68 \text{ N}$$

$$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2} = \sqrt{90^2 + 102.68^2} = 136.54 \text{ N}$$

$$\theta = \tan^{-1} \left[ \frac{\Sigma F_y}{\Sigma F_x} \right] = \tan^{-1} \left[ \frac{-102.68}{90} \right] = -48.57^\circ$$

$$\Sigma M_O = 100 \sin 60^\circ \times 60 + 100 \cos 60^\circ \times 60 + (80 \cos 60^\circ \times 0) + (80 \sin 60^\circ \times 120) + (120 \times 60) + (100 \times 0)$$

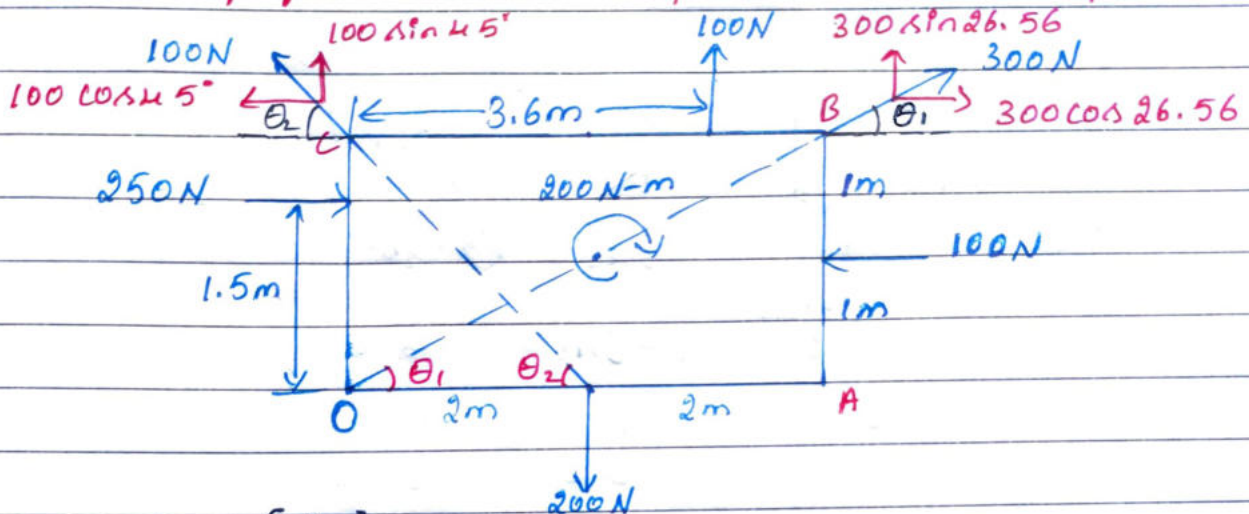
$$\Sigma M_O = 15,517.7 \text{ N-mm}$$

$$x\text{-intercept} = \left| \frac{\Sigma M_O}{\Sigma F_y} \right| = \left| \frac{15517.7}{102.68} \right| = 151.12 \text{ mm}$$

$$y\text{-intercept} = \left| \frac{\Sigma M_O}{\Sigma F_x} \right| = \left| \frac{15517.7}{90} \right| = 172.41 \text{ mm}$$

$$d = \left| \frac{\Sigma M_O}{R} \right| = \left| \frac{15517.7}{136.54} \right| = 113.64 \text{ mm}$$

Determine magnitude, direction & point of application of the resultant force for the system of coplanar forces shown in figure. Locate the position with respect to 'O'.



$$\theta_1 = \tan^{-1} \left[ \frac{2}{4} \right] = 26.56^\circ$$

$$\theta_2 = \tan^{-1} \left[ \frac{2}{2} \right] = 45^\circ$$

$$\Sigma F_x = 250 - 100 \cos 45 + 300 \cos 26.56 - 100 = 347.6 \text{ N}$$

$$\Sigma F_y = 100 \sin 45 + 100 + 300 \sin 26.56 - 200 = 104.85 \text{ N}$$

$$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2} = \sqrt{347.6^2 + 104.85^2} = 363 \text{ N}$$

$$\theta = \tan^{-1} \left[ \frac{\Sigma F_y}{\Sigma F_x} \right] = \tan^{-1} \left[ \frac{104.85}{347.6} \right] = 16.78^\circ$$

$$\Sigma M_o = (250 \times 1.5) - (100 \cos 45 \times 2) - (100 \times 3.6) - (300 \times 0) - (100 \times 1) + 200 + (200 \times 2) - (100 \sin 45 \times 0) - (300 \sin 26.56 \times 4) + (300 \cos 26.56 \times 2)$$

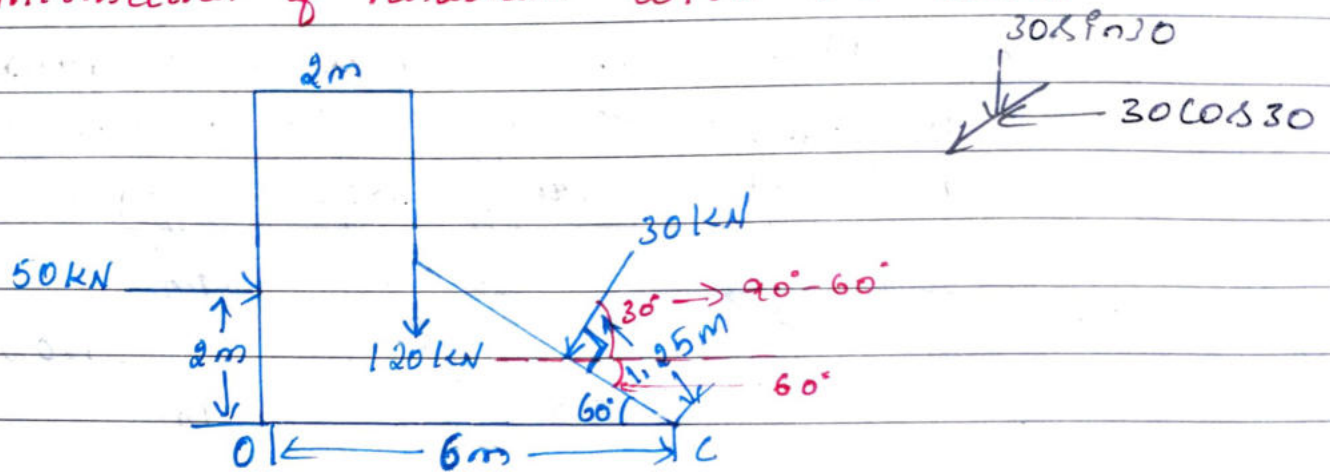
$$\Sigma M_o = 373.7 \text{ N-m}$$

$$x\text{-intercept} = \left| \frac{\Sigma M_o}{\Sigma F_y} \right| = \left| \frac{373.7}{104.85} \right| = 3.56 \text{ m}$$

$$y\text{-intercept} = \left| \frac{\Sigma M_o}{\Sigma F_x} \right| = \left| \frac{373.7}{347.6} \right| = 1.07 \text{ m}$$

$$d = \left| \frac{\Sigma M_o}{R} \right| = \left| \frac{373.7}{363} \right| = 1.02 \text{ m}$$

- ⑦ A dam section is shown in figure. Determine the point of intersection of resultant with the base.



$$\Sigma F_x = 50 - 30 \cos 30 = 24 \text{ kN}$$

$$\Sigma F_y = -120 - 30 \sin 30 = -135 \text{ kN}$$

$$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2} = \sqrt{24^2 + 135^2} = 137.11 \text{ kN}$$

$$\theta = \tan^{-1} \left( \frac{\Sigma F_y}{\Sigma F_x} \right) = \tan^{-1} \left( \frac{-135}{24} \right) = 79.91^\circ$$

Locate the point of intersection

$$\Sigma M_c = (50 \times 2) - (120 \times 4) - (30 \times 1.25) = -417.5 \text{ kN-m}$$

$$x\text{-intercept} = \left| \frac{\Sigma M_c}{\Sigma F_y} \right| = \left| \frac{-417.5}{-135} \right| = 3.09 \text{ m}$$

$$y\text{-intercept} = \left| \frac{\Sigma M_c}{\Sigma F_x} \right| = \left| \frac{-417.5}{24} \right| = 17.39 \text{ m}$$

$$d = \left| \frac{\Sigma M}{R} \right| = \left| \frac{-417.5}{137.11} \right| = 3.04 \text{ m}$$

Imp

Varignons theorem of momentsStatement:-

The algebraic sum of moments of system of forces about a point is equal to moment of their resultant about the same point.

Proof:-

$$F_1 d_1 + F_2 d_2 = R d$$

$$R d = F_1 d_1 + F_2 d_2$$

From  $\triangle OBA$ 

$$\cos \theta = \frac{d}{AO} \Rightarrow d = AO \cdot \cos \theta$$

$$\therefore R (AO \cdot \cos \theta) = R d$$

$$R d = AO \cdot (R \cos \theta)$$

$$R d = AO \cdot (R_x) \rightarrow (1)$$

III<sup>ly</sup>

$$F_1 d_1 = AO [F_{1x}] \rightarrow (2)$$

$$F_2 d_2 = AO [F_{2x}] \rightarrow (3)$$

Add eq<sup>n</sup> (2) & (3)

$$\Rightarrow F_1 d_1 + F_2 d_2 = AO [F_{1x}] + AO [F_{2x}]$$

$$= AO [F_{1x} + F_{2x}]$$

$$F_1 d_1 + F_2 d_2 = AO \cdot (R_x) \rightarrow (4)$$

equate eq<sup>n</sup> (1) & (4)

$$R d = F_1 d_1 + F_2 d_2$$

$F_{1x} + F_{2x} = R_x$   
 $\therefore$  Sum of H<sup>le</sup> components of system of forces is equal to H<sup>le</sup> component of resultant force.

