Basics of Fluid Mechanics

Compressibility and Compressible flow

- In Fluid mechanics, the Compressibility (β) is defined as a relative change in volume of a fluid element in response to a pressure variation.

$\beta = -\frac{1}{v}\frac{dv}{dp}$ **v is the specific volume**

$\beta = \frac{1}{\rho}\frac{d\rho}{dP}$ **ρ is the density ($\rho = 1/v$)**

Compressibility at standard atmospheric conditions:
Water – $4.55 \times 10^{-10}\ m^2/N$
Air - $7.04 \times 10^{-6}\ m^2/N$

Note :- Pressure is the stress associated with normal compressive force

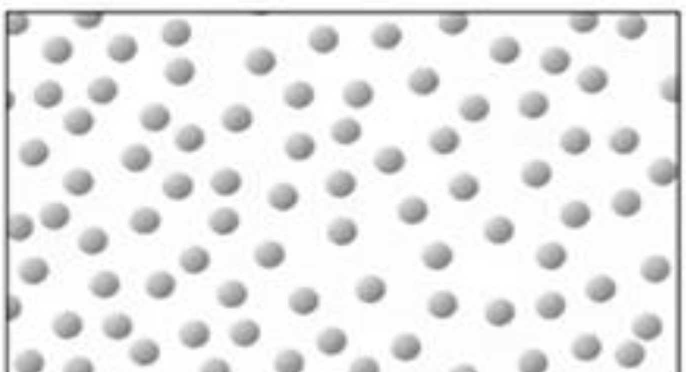
- The thermodynamics process is also an important parameter to calculate compressibility.

$\beta_T = -\frac{1}{v}\left(\frac{dv}{dP}\right)_T$ **isothermal compressibility**

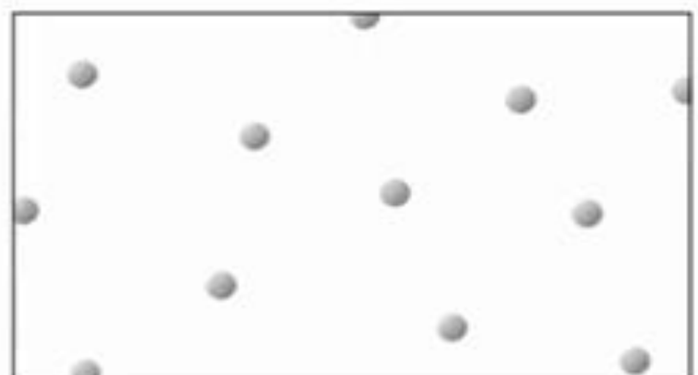
$\beta_s = -\frac{1}{v}\left(\frac{dv}{dP}\right)_s$ **isentropic compressibility**

Continuum

- Fluid dynamics considers macroscopic properties of the flow
- Typical analysis deals with small volumes or fluid parcels
- The distribution of flow field variables velocity, pressure, temperature, density etc. is continuous over space and time.
- In essence continuum is considered.
- This assumption is valid only when there are a large number of molecules in the given small volume.



Schematic of continuum

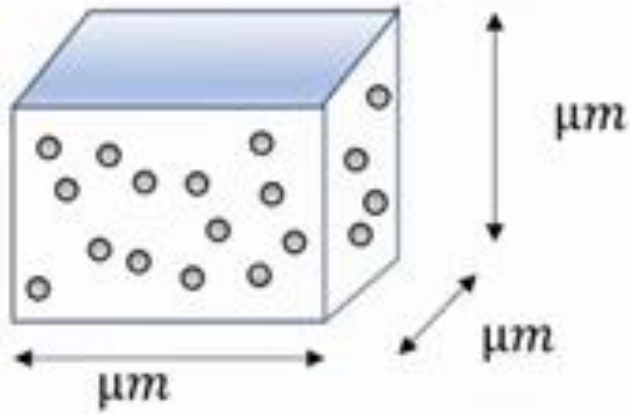


Schematic of rarefied

Continuum

N_2

Mol. Wt = 28.014 g / mol



At Sea level
$P = 10^5\ N/m^2$
$T = 273.15\ K$

Number of molecules =
$2.68 \times 10^7 / \mu m^3$

At 80Km altitude *
$P = 1.1\ N/m^2$
$T = 198.49\ K$

Number of molecules =
$4.01 \times 10^2 / \mu m^3$

- How to decide whether to consider continuum or consider individual molecules ? – importance of Knudsen number.
- **Knudsen number** is the ratio of molecular mean free path to the typical length scale

$$\mathbf{Kn} = \frac{\lambda}{L}$$

λ = Molecular mean free path
L = Typical Length scale

Greek letters

Α α alpha	Β β beta	Γ γ gamma	Δ δ delta	Ε ε epsilon
Ζ ζ zeta	Η η eta	Θ θ theta	Ι ι lota	Κ κ kappa
Λ λ lambda	Μ μ mu	Ν ν nu	Ξ ξ xl	Ο ο omikron
Π π pi	Ρ ρ rho	Σ σ/ς sigma	Τ τ tau	Υ υ upsilon
Φ φ phi	Χ χ hi	Ψ ψ psi	Ω ω omega	

Cont……..

23AEPC203/ FLUID MECHANICS

II YEAR AERO (III SEMESTER)

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UNIT - 1

PROPERTIES OF FLUIDS AND FLOW CHARATERISTICS

FLUID MECHANICS

- Topic:

  - ***Properties of Eluids.***

CONTENTS

- Introduction
- Fluid
- Continuum
- Fluid Properties

INTRODUCTION

- Fluid mechanics is the science that deals with the action of forces on fluids at rest as well as in motion.
- If the fluids are at rest, the study of them is called **fluid statics**.
- If the fluids are in motion, where pressure forces are not considered, the study of them is called **fluid Kinematics**
- If the fluids are in motion and the pressure forces are considered, the study of them is called **fluid dynamics**.

FLUID

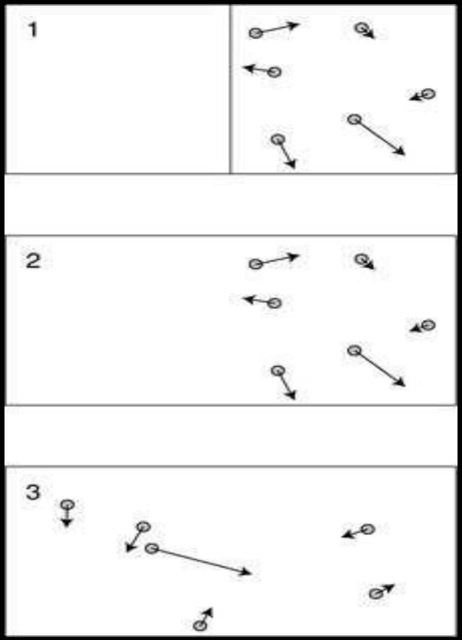
- Matter exists in two states- the solid state and the fluid state. This classification of matter is based on the spacing between different molecules of matter as well as on the behavior of matter when subjected to stresses.
- Because molecules in solid state are spaced very closely, solids possess compactness and rigidity of form. The molecules in fluid can move more freely within the fluid mass and therefore the fluids do not possess any rigidity of form.
- Thus Fluid exist in two form:-
  - Liquid
  - Gas

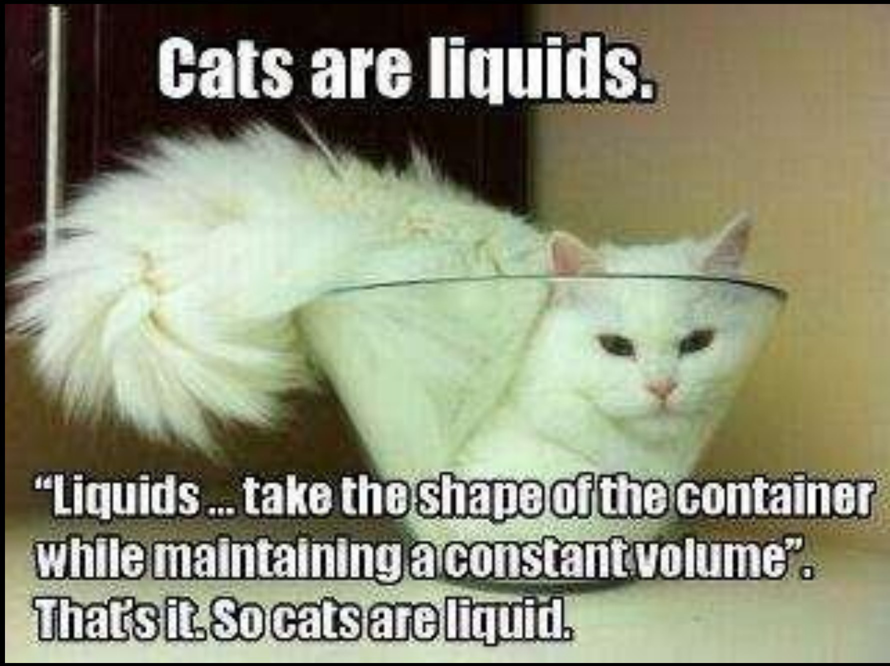
WHAT'S THE DIFFERENCE?

- Liquids flow and take the shape of their container but maintain a constant volume.
- Gases expand to fill the available volume.

•Liquids are incompressible While

the gas are compressible.





FLUID PROPERTIES

- Mass Density
- Specific Weight
- Specific Volume
- Specific Gravity
- Viscosity
- Surface tension
- Vapour Pressure
- Capillarity
- Cavitation

MASS DENSITY / DENSITY ρ

- The "mass per unit volume" is mass density. Hence it has units of kilograms per cubic meter.

- The mass density of water at 4^oC is 1000 kg/m^3 while it is 1.20 kg/m^3 for air at 20^oC at standard pressure.

$$\boxed{D = \frac{m}{V}}$$

Density: Example

A quantity of helium gas at 0°C with a volume of 4.00 m^3 has a mass of 0.712 kg at standard atmospheric pressure.

Determine the density of this sample of helium gas.

$$V = 4.00\,m^3$$

$$m = 0.712\,kg$$

$$D = ?$$

$$D = \frac{m}{V}$$

$$D = \frac{0.712\,kg}{4.00\,m^3} = 0.178\frac{kg}{m^3}$$

SPECIFIC WEIGHT OR WEIGHT DENSITY γ

- It is the ratio between the weight of a fluid to its volume.
- It is also weight per unit volume of a fluid.
- Its unit is N/m^3.
- Water at 20 ^oC has a specific weight of 9.79 kN/m^3
- $\gamma = \rho g$

SPECIFIC VOLUME

- It is defined as the volume of a fluid occupied by a unit mass or volume per unit mass of a fluid is called specific volume.

- Specific Volume = Volume of the Fluid / Mass of the Fluid

  = 1/mass of the fluid/volume of the fluid

  = 1 / ρ

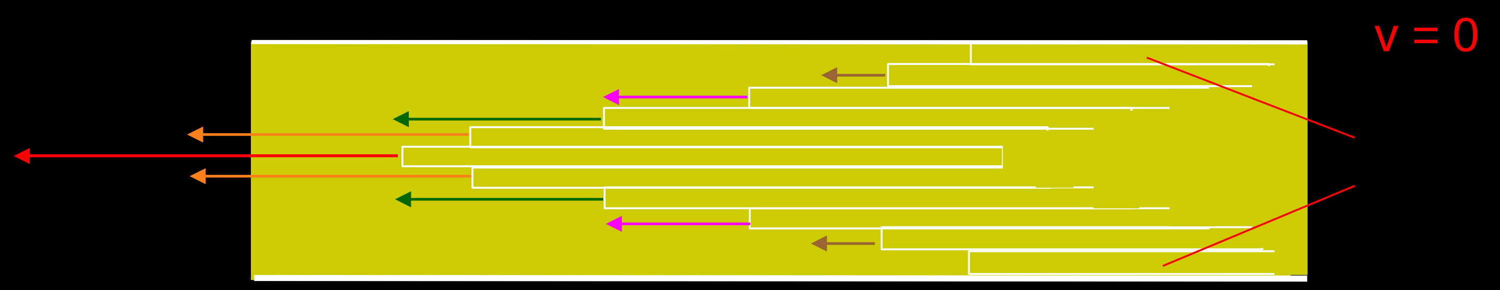
SPECIFIC GRAVITY (S)

- The ratio of specific weight of a given liquid to the specific weight of water at a standard reference temperature (4^oC)is defined as *specific gravity*, S.
- The specific weight of water at atmospheric pressure is 9810 N/m^3.
- The specific gravity of mercury at 20^oC is

$$S_{Hg} = \frac{133\,kN/m^3}{9.81\,kN/m^3} = 13.6$$

Viscosity

- Different kinds of fluids flow more easily than others. Oil, for example, flows more easily than molasses. This is because molasses has a higher viscosity, which is a measure of resistance to fluid flow. Inside a pipe or tube a very thin layer of fluid right near the walls of the tube are motionless because they get caught up in the microscopic ridges of the tube. Layers closer to the center move faster and the fluid sheãrs. The middle layer moves the fastest.

- 

The more viscous a fluid is, the more the layers want to cling together, and the more it resists this shearing. The resistance is due the frictional forces between the layers as the slides past one another. Note, there is no friction occurring at the tube′s surface since the fluid there is essentially still. The friction happens in the fluid and generates heat. The Bernoulli equation applies to fluids with negligible viscosity.

VISCOSITY

- It is defined as the property of a fluid which offers resistance to the movement of one layer of fluid over another adjacent layer of the fluid.
- The viscosity of a fluid is a measure of its "resistance to deformation."

NEWTON'S LAW OF VISCOSITY

- It states that the shear stress on a fluid element layer is directly proportional to the rate of shear strain.
- The constant of proportionality is called the coefficient of viscosity.
  - $\tau = \mu \, du/dy$
    - *Where* τ = shear stress

      du/dy = Velocity Gradient μ

      = coefficient of viscosity

KINEMATIC VISCOSITY

- It is defined as the ratio between the dynamic viscosity and density of the fluid.

$$\nu = \frac{\mu}{\rho} = \frac{N.s/m^2}{kg/m^3} = m^2/s$$

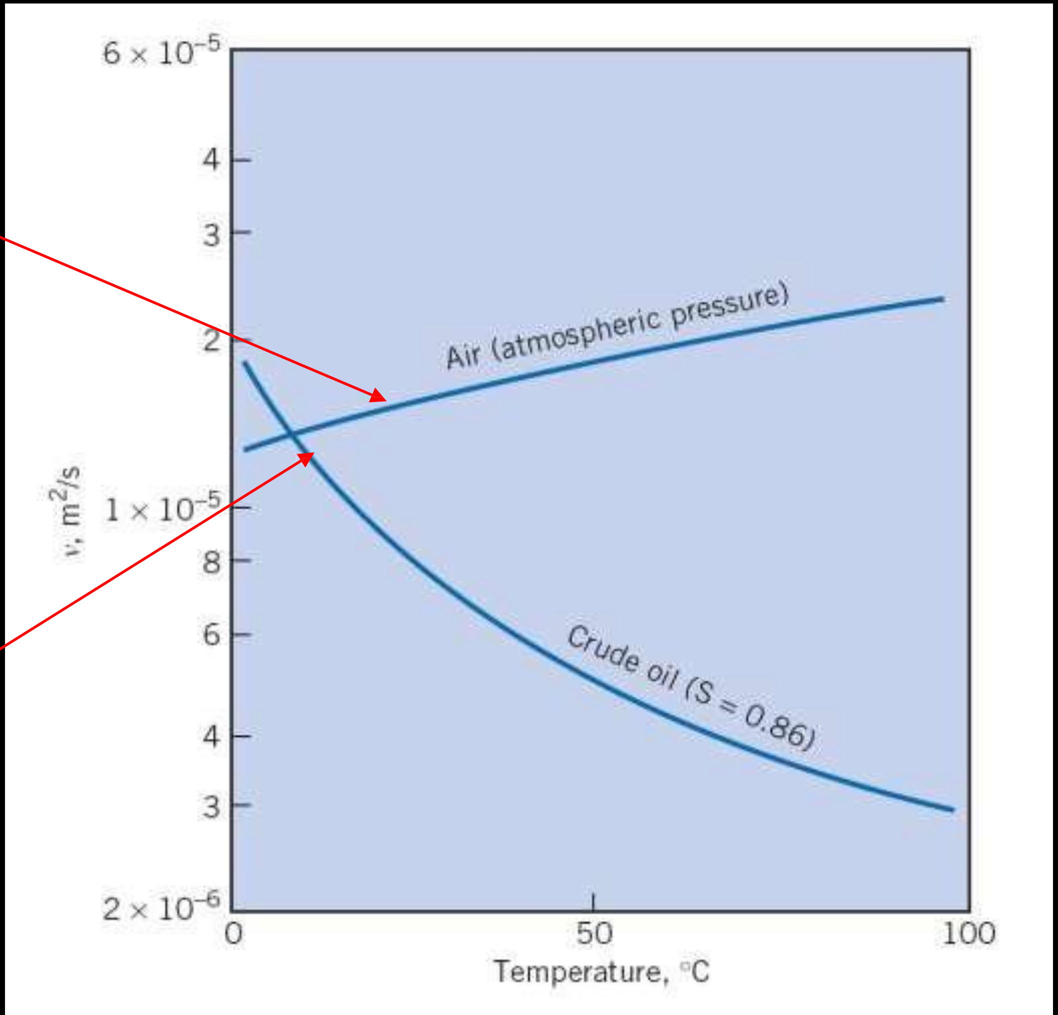
VARIATION OF VISCOSITY WITH TEMPERATURE

- Liquids - cohesion and momentum transfer
  - Viscosity decreases as temperature increases.
- Relatively independent of pressure (incompressible)

- Gases - transfer of molecular momentum
  - Viscosity increases as temperature increases.
  - Viscosity increases as pressure increases

VARIATION OF VISCOSITY WITH TEMPERATURE

Increasing temp → increasing viscosity



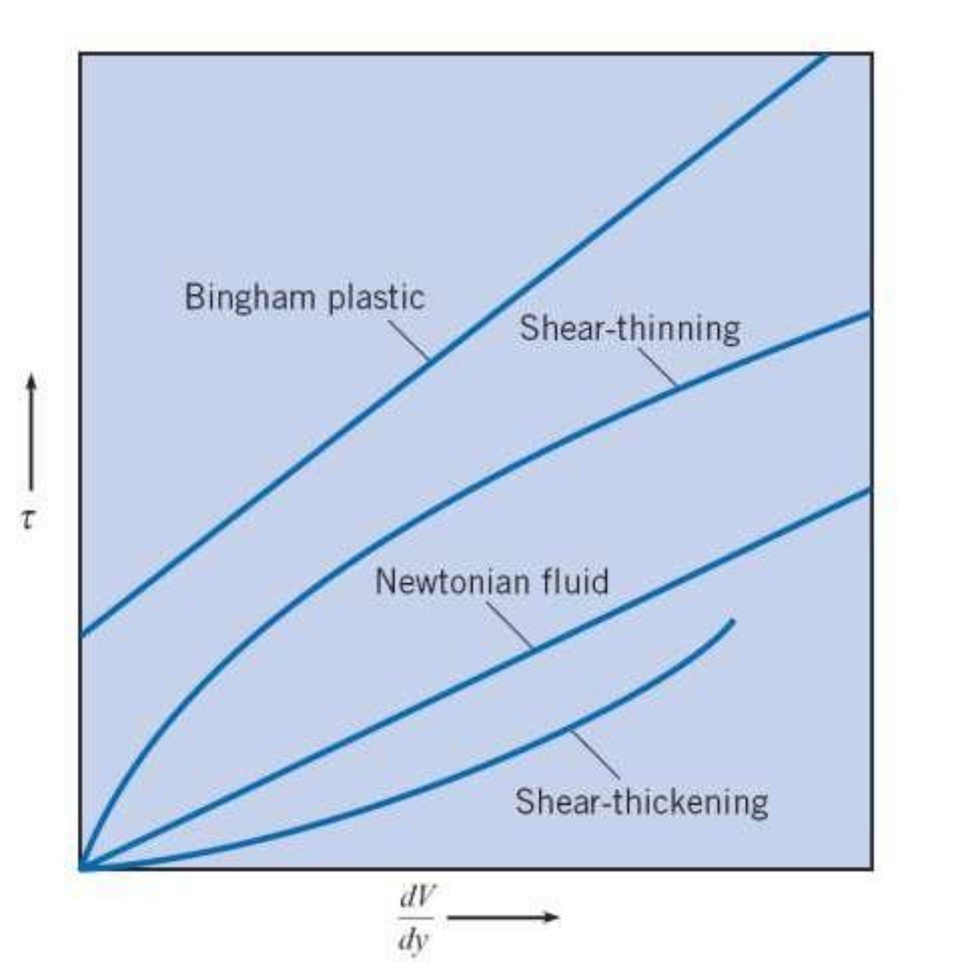
Increasing temp → decreasing viscosity

TYPES OF FLUIDS

- Ideal Fluid
- Real Fluid
- Newtonian Fluid
- Non- Newtonian Fluid
- Ideal Plastic Fluid

TYPES OF FLUID

- *Newtonian fluid*: shear stress is proportional to shear strain
  - Slope of line is dynamic viscosity
- *Shear thinning*: ratio of shear stress to shear strain decreases as shear strain increases (toothpaste, catsup, paint, etc.)
- *Shear thickening*: viscosity increases with shear rate (glass particles in water, gypsum-water mixtures).

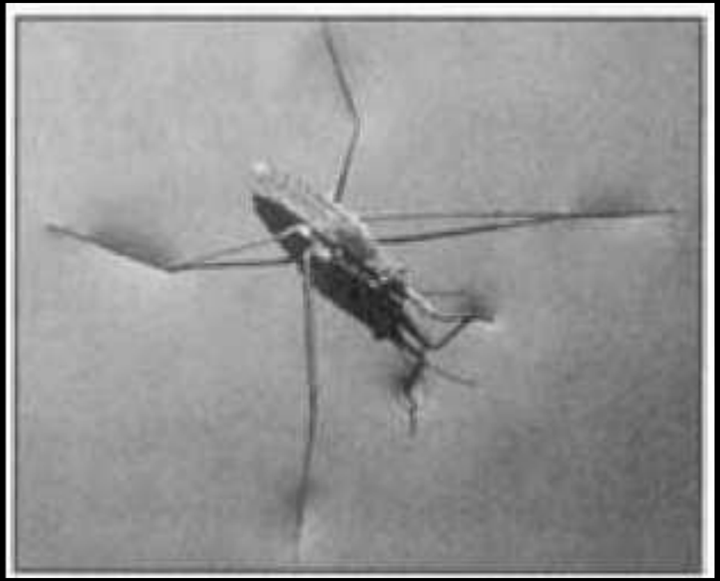


APPLICATION OF VISCOSITY :-

1. Transparent and storing facilities for fluids i.e., pipes, tanks
2. Bitumen used for road construction.
3. Designing of the sewer line or any other pipe flow viscosity play an important role in finding out its flow behaviour.
4. Drilling for oil and gas requires sensitive viscosity.
5. To maintain the performance of machine and automobiles by determining thickness of lubricating oil or motor oil.

SURFACE TENSION

- Surface tension is a contractive tendency of the surface of a fluid that allows it to resist an external force. Surface tension is an important property that mark ably influences the ecosystems.

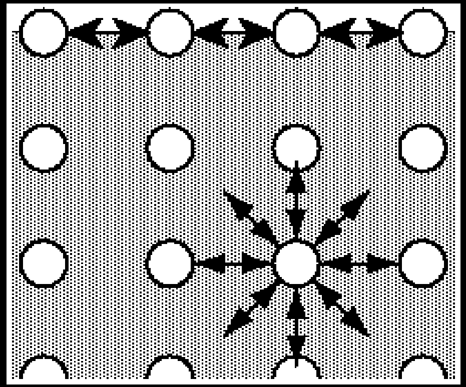


What’s happening here?

- *Bug is walking on water*

SURFACE TENSION

- A molecules in the interior of a liquid is under attractive force in all direction.
- However, a molecule at the surface of a liquid is acted on by a net inward cohesive force that is perpendicular to the surface.
- Hence it requires work to move molecules to the surface against this opposing force and surface molecules have more energy than interior ones
- Higher forces of attraction at surface
- Creates a "stretched membrane effect"



SURFACE TENSION

- *Surface tension*, σ_s: the force resulting from molecular attraction at liquid surface [N/m]

$F_s = \sigma_s L$

$F_s =$ *surface tension force [N]*

$\sigma_s =$ *surface tension [N/m]*

$L =$ *length over which the surface tension acts [m]*

Example: Surface Tension

- Estimate the difference in pressure (in Pa) between the inside and outside of a bubble of air in 20ºC water. The air bubble is 0.3 mm in diameter.

R = 0.15 x 10^{-3} m

σ = 0.073 N/m

$$p = \frac{2\sigma}{R}$$

$$p = \frac{2(0.073\ \text{N/m})}{0.15\times10^{-3}\,\text{m}}$$

p= 970 Pa

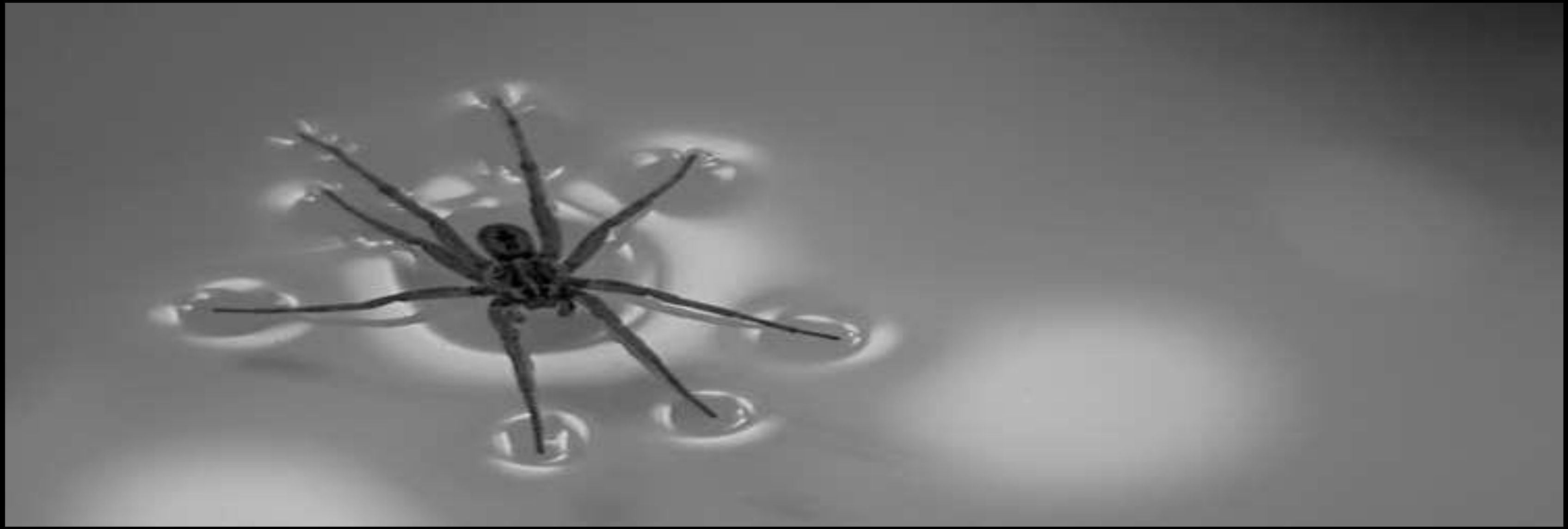
Statics !

$$p = \gamma h$$

$$h = \frac{p}{\gamma} = \frac{974Pa}{9806N/m^3} = 0.1\,\text{m water}$$

APPLICATION OF SURFACE TENSION :-

- A water strider can walk on water.
- Some tent are made impermeable of the rain but they are not really impermeable, but if water is placed on it then the water doesn′t pass through the fine small pores of the tent cover. But as you touch the cover while water is on it, you break surface tension and water passes through.

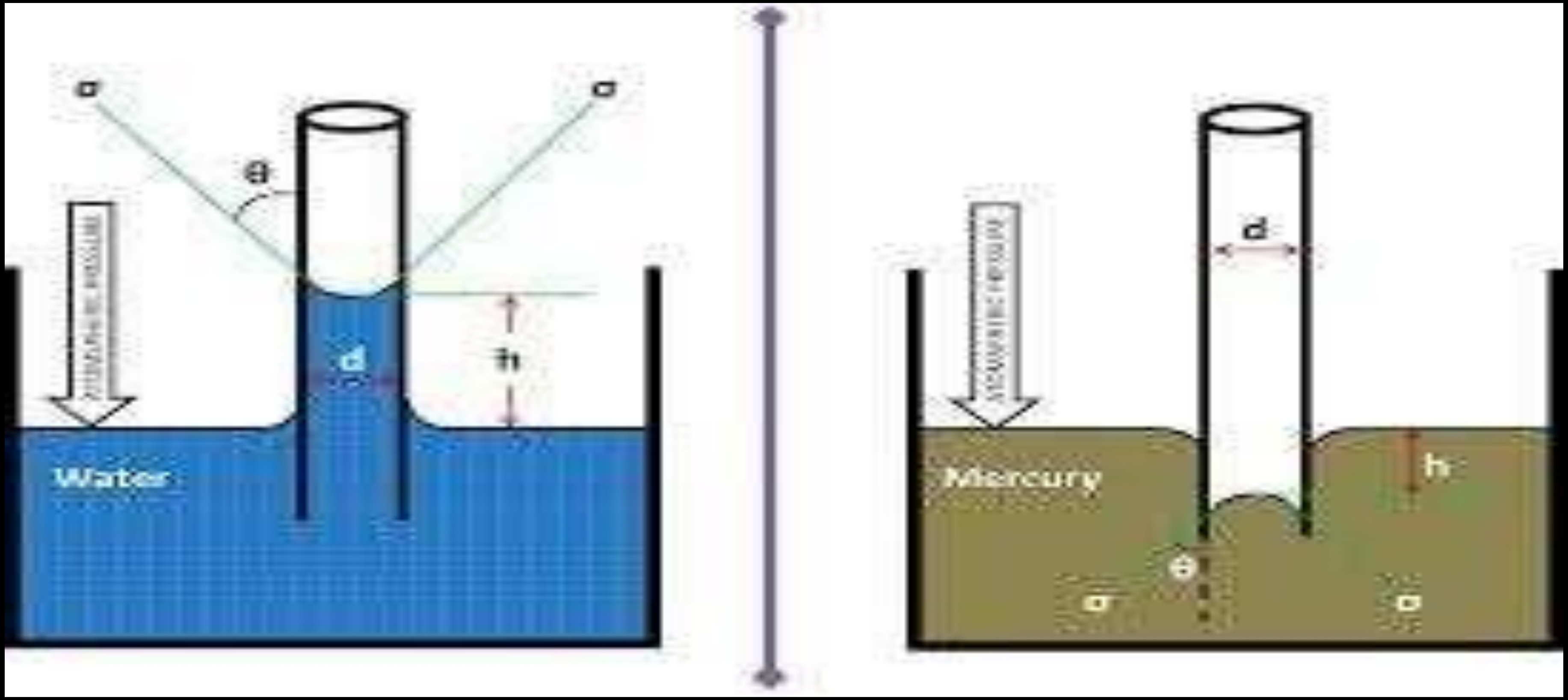


CAPILLARY ACTION

- How do trees pump water hundreds of feet from the ground to their highest leaves? Why do paper towels soak up spills? Why does liquid wax rise to the tip of a candle wick to be burned? Why must liquids on the space shuttle be kept covered to prevent them from crawling right out of their containers?! These are all examples of *capillary action*--the movement of a liquid up through a thin tube. It is due to adhesion and cohesion.
- Capillary action is a result of adhesion and cohesion. A liquid that adheres to the material that makes up a tube will be drawn inside. Cohesive forces between the molecules of the liquid will "connect" the molecules that aren't in direct contact with the inside of the tube. In this way liquids can crawl up a tube. In a pseudo-weightless environment like in the space shuttle, the "weightless" fluid could crawl right out of its container.

CAPILLARY ACTION :-

- Capillary action is the ability of a fluid to flow in narrow spaces without the assistance of, and in opposition to, external forces like gravity.



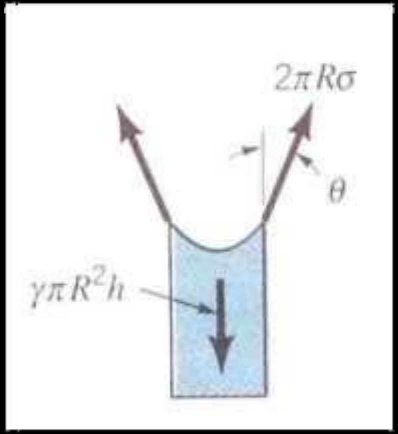
DIFFERENCES BETWEEN ADHESIVE & COHESIVE

- The force of or attraction between unlike charges in the atoms molecules of substances are responsible for cohesion and adhesion.
- *Cohesion* is the clinging together of molecules/atoms within a substance. Ever wonder why rain falls in drops rather than individual water molecules? It's because water molecules cling together to form drops.
- *Adhesion* is the clinging together of molecules/atoms of two different substances. Adhesive tape gets its name from the adhesion between the tape and other objects. Water molecules cling to many other materials besides clinging to themselves.

CAPILLARY EFFECT

- h=height of capillary rise (or depression)
- σ=surface tension
- θ=wetting angle
- G=specific weight
- R=radius of tube
- If the tube is clean, θ is 0 for water

$$F_{\sigma,z} - W = 0$$



$$2\pi R\sigma Cos\theta = \pi R^2 h\gamma$$

$$h = \frac{2\sigma Cos\theta}{\gamma r}$$

APPLICATION OF CAPILLARY ACTION :-

- Capillary action is found in thermometer where fluid used in it automatically rises when comes in contact with higher temperature or falls down with lower ones.

- Capillary action can be performed to transfer fluid from one vessel to another on its own.

VAPOR PRESSURE

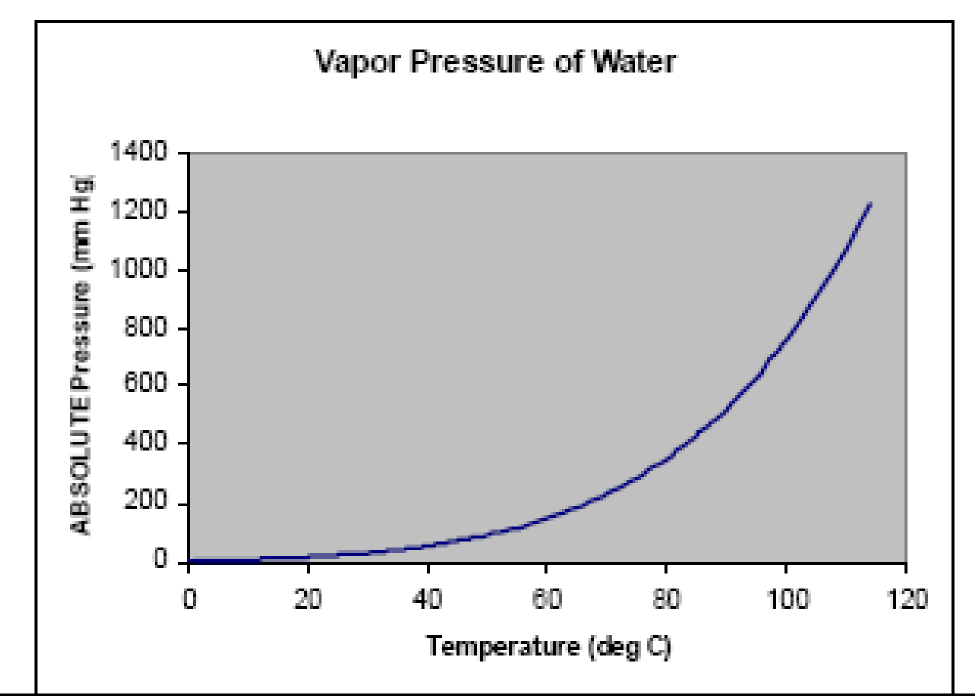
- *Vapor pressure: the* pressure at which a liquid will boil.

➢ Vapor pressure ↑ when temperature increases

➢ At atmospheric pressure, water at 100 °C will boil

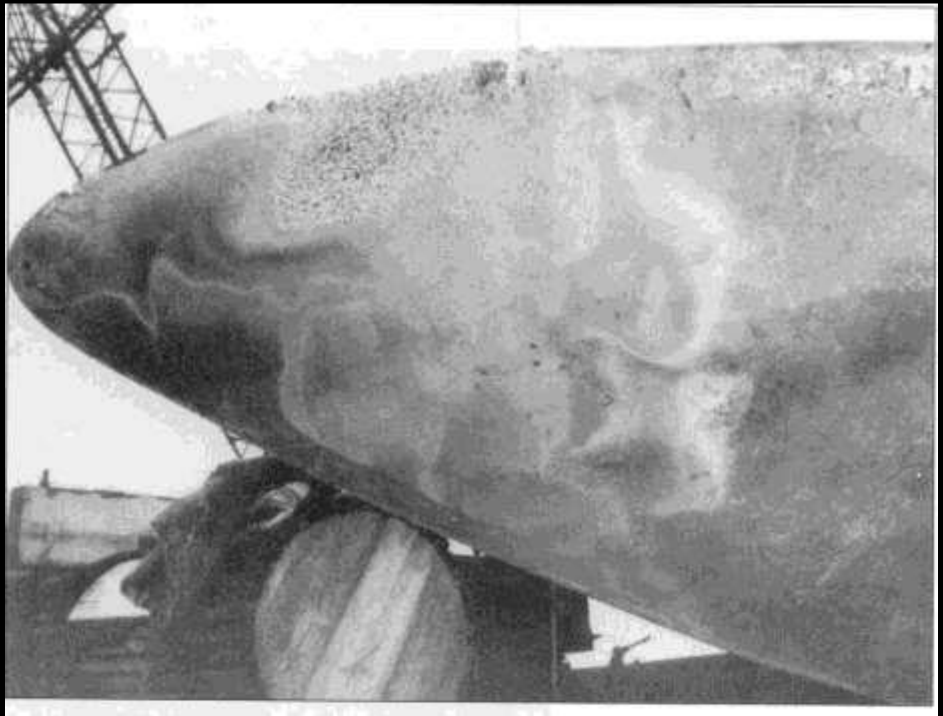
➢ Water can boil at lower temperatures if the pressure is lower

  - When vapor pressure > the liquid’s actual pressure
  - It will boil.



CAVITATION

- It is the phenomenon of formation of vapour bubbles of a flowing liquid in a region where the pressure of the liquid falls below the vapour pressure and sudden collapsing of these vapour bubbles in a region of a higher pressure.



Classification of Fluid Flows

1) Uniform flow; steady flow
2) If we look at a fluid flowing under normal circumstances - a river for example - the conditions (e.g. velocity, pressure) at one point will vary from those at another point, then we have non-uniform flow.
3) If the conditions at one point vary as time passes, then we have unsteady flow.
4) Uniform flow: If the flow velocity is the same magnitude and direction at every point in the flow it is said to be uniform. That is, the flow conditions DO NOT change with position.
5) Non-uniform: If at a given instant, the velocity is not the same at every point the flow is non-uniform.

Classification of Fluid Flows

Steady: A steady flow is one in which the conditions (velocity, pressure and cross-section) may differ from point to point but DO NOT change with time.

Unsteady: If at any point in the fluid, the conditions change with time, the flow is described as unsteady.

Combining the above we can classify any flow in to one of four types

Steady uniform flow. Conditions do not change with position in the stream or with time. An example is the flow of water in a pipe of constant diameter at constant velocity.

Classification of Fluid Flows

Steady non-uniform flow:
Conditions change from point to point in the stream but do not change with time. An example is flow in a tapering pipe with constant velocity at the inlet - velocity will change as you move along the length of the pipe toward the exit.

Unsteady uniform flow: At a given instant in time the conditions at every point are the same, but will change with time.
An example is a pipe of constant diameter connected to a pump pumping at a constant rate which is then switched off.

Unsteady non-uniform flow: Every condition of the flow may change from point to point and with time at every point. An example is surface waves in an open channel.

Classification of Fluid Flows

Typical examples are fully-developed flows in long uniform pipes and open-channels.

One-dimensional flow problems will require only elementary analysis, and can be solved analytically in most cases.

One-dimensional ideal flow along a pipe, where the velocity is uniform across the pipe section.

MITE

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DEPARTMENT OF AERONAUTICAL ENGINEERING

COURSE MATERIAL

23AEPC206 FLUID MECHANICS

II YEAR - III SEMESTER

Fluids Mechanics

What is fluid mechanics?

As its name suggests it is the branch of applied mechanics concerned with the statics and dynamics of fluids - both liquids and gases. The analysis of the behavior of fluids is based on the fundamental laws of mechanics which relate continuity of mass and energy with force and momentum together with the familiar solid mechanics properties.

Objectives of this section

- Define the nature of a fluid.
- Show where fluid mechanics concepts are common with those of solid mechanics and indicate some fundamental areas of difference.
- Introduce viscosity and show what are Newtonian and non-Newtonian fluids
- Define the appropriate physical properties and show how these allow differentiation between solids and fluids as well as between liquids and gases.

Fluids

There are two aspects of fluid mechanics which make it different to solid mechanics:

1. The nature of a fluid is much different to that of a solid
2. In fluids we usually deal with continuous streams of fluid without a beginning or end. In solids we only consider individual elements.

We normally recognize three states of matter: solid; liquid and gas. However, liquid and gas are both fluids: in contrast to solids they lack the ability to resist deformation. Because a fluid cannot resist the deformation force, it moves, it flows under the action of the force. Its shape will change continuously as long as the force is applied. A solid can resist a deformation force while at rest, this force may cause some displacement but the solid does not continue to move indefinitely.

The deformation is caused by shearing forces which act tangentially to a surface. Referring to the figure below, we see the force F acting tangentially on a rectangular (solid lined) element ABDC. This is a shearing force and produces the (dashed lined) rhombus element A'B'DC.

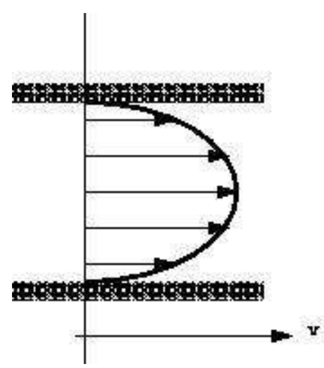
$$\sigma = \frac{\rho_{substance}}{\rho_{H_2O(at\,4^\circ c)}}$$

We can then say

> A Fluid is a substance which deforms continuously, or flows, when subjected to shearing force, and conversely this definition implies the very important point that
>
> If a fluid is at rest there are no shearing forces acting. All forces must be perpendicular to the planes which they are acting.

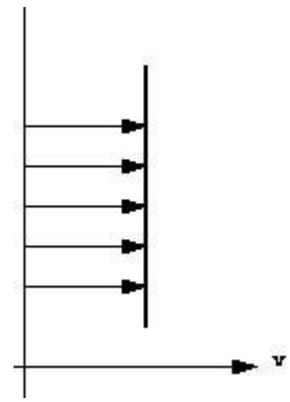
When a fluid is in motion shear stresses are developed if the particles of the fluid move relative to one another. When this happens adjacent particles have different velocities. If fluid velocity is the same at every point then there is no shear stress produced: the particles have zero relative velocity.

Consider the flow in a pipe in which water is flowing. At the pipe wall the velocity of the water will be zero. The velocity will increase as we move toward the centre of the pipe. This change in velocity across the direction of flow is known as velocity profile and shown graphically in the figure below:



Velocity profile in a pipe.

Because particles of fluid next to each other are moving with different velocities there **are** shear forces in the moving fluid i.e. shear forces are **normally** present in a moving fluid. On the other hand, if a fluid is a long way from the boundary and all the particles are travelling with the same velocity, the velocity profile would look something like this:

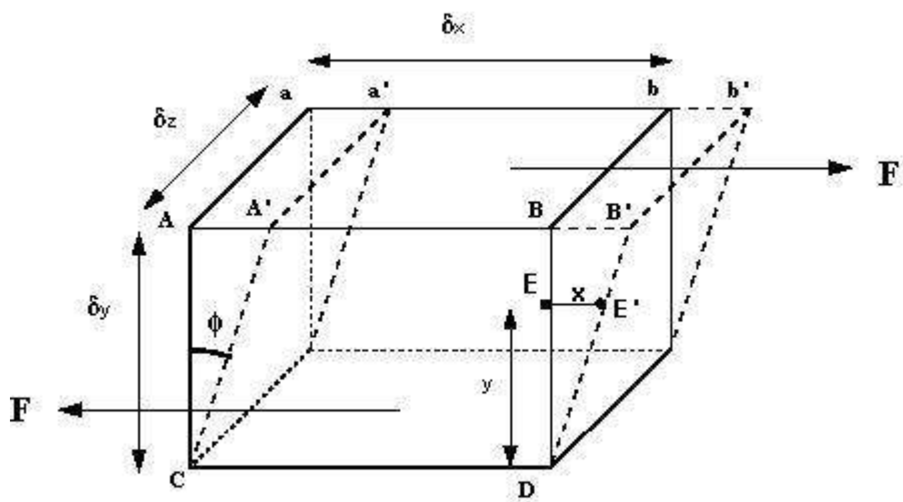


Velocity profile in uniform flow

and there will be no shear forces present as all particles have zero relative velocity. In practice we are concerned with flow past solid boundaries; aeroplanes, cars, pipe walls, river channels etc. and shear forces will be present.

Newton's Law of Viscosity

We can start by considering a 3d rectangular element of fluid, like that in the figure below.



Fluid element under a shear force

The shearing force F acts on the area on the top of the element. This area is given by $A = \delta z \times \delta x$.

We can thus calculate the shear stress which is equal to force per unit area i.e.

$$\text{shear stress, } \tau = \frac{F}{A}$$

The deformation which this shear stress causes is measured by the size of the angle ϕ and is know as shear strain.

In a solid shear strain, ϕ, is constant for a fixed shear stress τ.

In a fluid ϕ increases for as long as τ is applied - the fluid flows.

It has been found experimentally that the rate of shear stress (shear stress per unit time, /time) is directly proportional to the shear stress.

If the particle at point E (in the above figure) moves under the shear stress to point E' and it takes time t to get there, it has moved the distance x. For small deformations we can write

shear strain $\phi = \frac{x}{y}$

$$\text{rate of shear strain} = \frac{\phi}{t}$$

$$= \frac{x}{ty} = \frac{x}{t}\frac{1}{y}$$

$$= \frac{u}{y}$$ where $\frac{x}{t} = u$ is the velocity of the particle at E.

Using the experimental result that shear stress is proportional to rate of shear strain then

$$\tau = \text{Constant} \times \frac{u}{y}$$

The term $\frac{u}{y}$ is the change in velocity with y, or the velocity gradient, and may be written in the differential form $\frac{du}{dy}$. The constant of proportionality is known as the dynamic viscosity, μ, of the fluid, giving $\tau = \mu \frac{du}{dy}$

This is known as **Newton's law of viscosity**.

Fluids and Solids

In the above we have discussed the differences between the behaviour of solids and fluids under an applied force. Summarising, we have;

- For a **solid** the strain is a function of the applied stress (providing that the elastic limit has not been reached). For a **fluid**, the rate of strain is proportional to the applied stress.

- The strain in a **solid** is independent of the time over which the force is applied and (if the elastic limit is not reached) the deformation disappears when the force is removed. A **fluid** continues to flow for as long as the force is applied and will not recover its original form when the force is removed.

It is usually quite simple to classify substances as either solid or liquid. Some substances, however, (e.g. pitch or glass) appear solid under their own weight. Pitch will, although appearing solid at room temperature, deform and spread out over days - rather than the fraction of a second it would take water.

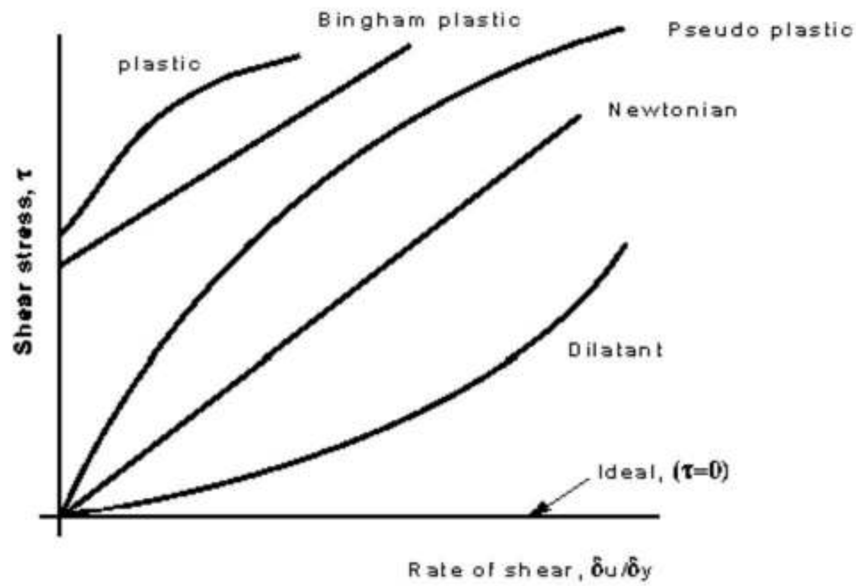
As you will have seen when looking at properties of solids, when the elastic limit is reached they seem to flow. They become plastic. They still do **not** meet the definition of true fluids as they will only flow after a certain minimum shear stress is attained.

Newtonian / Non-Newtonian Fluids

Even among fluids which are accepted as fluids there can be wide differences in behaviour under stress. Fluids obeying Newton's law where the value of μ is constant are known as **Newtonian** fluids. If μ is constant the shear stress is linearly dependent on velocity gradient. This is true for most common fluids.

Fluids in which the value of μ is not constant are known as **non-Newtonian** fluids. There are several categories of these, and they are outlined briefly below.

These categories are based on the relationship between shear stress and the velocity gradient (rate of shear strain) in the fluid. These relationships can be seen in the graph below for several categories



Shear stress vs. Rate of shear strain $\delta u/\delta y$

Each of these lines can be represented by the equation

$$\tau = A + B\left(\frac{\delta u}{\delta y}\right)^n$$

where A, B and n are constants. For Newtonian fluids A = 0, B = μ and n = 1.

Below are brief description of the physical properties of the several categories:

- Plastic: Shear stress must reach a certain minimum before flow commences.
- Bingham plastic: As with the plastic above a minimum shear stress must be achieved. With this classification n = 1. An example is sewage sludge. Pseudo-plastic: No
- minimum shear stress necessary and the viscosity decreases with rate of shear, e.g. colloidial substances like clay, milk and cement.
- Dilatant substances; Viscosity increases with rate of shear e.g. quicksand.
- Thixotropic substances: Viscosity decreases with length of time shear force is applied e.g. thixotropic jelly paints.
- Rheopectic substances: Viscosity increases with length of time shear force is applied

- Viscoelastic materials: Similar to Newtonian but if there is a sudden large change in shear they behave like plastic.

There is also one more - which is not real, it does not exist - known as the **ideal fluid**. This is a fluid which is assumed to have no viscosity. This is a useful concept when theoretical solutions are being considered - it does help achieve some practically useful solutions.

Liquids and Gasses

Although liquids and gasses behave in much the same way and share many similar characteristics, they also possess distinct characteristics of their own. Specifically

- A liquid is difficult to compress and often regarded as being incompressible. A gas is easily to compress and usually treated as such - it changes volume with pressure. A given
- mass of liquid occupies a given volume and will occupy the container it is in and form a free surface (if the container is of a larger volume). A gas has no fixed volume, it changes volume to expand to fill the containing vessel. It will completely fill the vessel so no free surface is formed.

Causes of Viscosity in Fluids

Viscosity in Gasses

The molecules of gasses are only weakly kept in position by molecular cohesion (as they are so far apart). As adjacent layers move by each other there is a continuous exchange of molecules. Molecules of a slower layer move to faster layers causing a drag, while molecules moving the other way exert an acceleration force. Mathematical considerations of this momentum exchange can lead to Newton law of viscosity.

If temperature of a gas increases the momentum exchange between layers will increase thus increasing viscosity.

Viscosity will also change with pressure - but under normal conditions this change is negligible in gasses.

Viscosity in Liquids

There is some molecular interchange between adjacent layers in liquids - but as the molecules are so much closer than in gasses the cohesive forces hold the molecules in place much more rigidly. This cohesion plays an important roll in the viscosity of liquids.

Increasing the temperature of a fluid reduces the cohesive forces and increases the molecular interchange. Reducing cohesive forces reduces shear stress, while increasing molecular interchange increases shear stress. Because of this complex interrelation the effect of temperature on viscosity has something of the form:

$$\mu_T = \mu_0 (1 + AT + BT)$$

Where μ_T is the viscosity at temperature TC, and μ_0 is the viscosity at temperature 0C. A and B are constants for a particular fluid.

High pressure can also change the viscosity of a liquid. As pressure increases the relative movement of molecules requires more energy hence viscosity increases.

Properties of Fluids

The properties outlines below are general properties of fluids which are of interest in engineering. The symbol usually used to represent the property is specified together with some typical values in SI units for common fluids. Values under specific conditions (temperature, pressure etc.) can be readily found in many reference books. The dimensions of each unit is also give in the MLT system (see later in the section on dimensional analysis for more details about dimensions.)

Density

The density of a substance is the quantity of matter contained in a unit volume of the substance. It can be expressed in three different ways.

Mass Density

Mass Density, ρ, is defined as the mass of substance per unit volume.

Units: Kilograms per cubic metre, kg/m^3 (or kgm^{-3})

Dimensions: ML^{-3}

Typical values:

Water = 1000 kgm^{-3}, Mercury = 13546 kgm^{-3} Air = 1.23 kgm^{-3}, Paraffin Oil = 800 kgm^{-3}.

(at pressure =1.013 $\times 10^{-5}\ Nm^{-2}$ and Temperature = 288.15 K.)

Specific Weight

Specific Weight ω, (sometimes , and sometimes known as specific gravity) is defined as the weight per unit volume. or

The force exerted by gravity, g, upon a unit volume of the substance.

The Relationship between g and ω can be determined by Newton's 2nd Law,

since weight per unit volume = mass per unit volume g $\omega = \rho g$

Units: Newton's per cubic metre, N/m^3 (or Nm^{-3})

Dimensions: $ML^{-2}T^{-2}$.

Typical values:

Water =9814 $N m^{-3}$, Mercury = 132943 $N m^{-3}$, Air =12.07 $N m^{-3}$, Paraffin Oil =7851 $N m^{-3}$

Relative Density

Relative Density, σ, is defined as the ratio of mass density of a substance to some standard mass density. For solids and liquids this standard mass density is the maximum mass density for water (which occurs at 4° c) at atmospheric pressure.

$$\sigma = \frac{\rho_{substance}}{\rho_{H_2O(at\,4^\circ c)}}$$

Units: None, since a ratio is a pure number.

Dimensions: 1.

Typical values: Water = 1, Mercury = 13.5, Paraffin Oil =0.8.

Viscosity

Viscosity, μ is the property of a fluid, due to cohesion and interaction between molecules, which offers resistance to sheer deformation. Different fluids deform at different rates under the same shear stress. Fluid with a high viscosity such as syrup, deforms more slowly than fluid with a low viscosity such as water.

All fluids are viscous, "Newtonian Fluids" obey the linear relationship

given by Newton's law of viscosity. $\tau = \mu \frac{du}{dy}$, which we saw earlier.

where is the shear stress,

Units $N m^{-2}$; $kg\, m^{-1}s^{-2}$

Dimensions $ML^{-1}T^{-2}$.

$\frac{du}{dy}$ is the velocity gradient or rate of shear strain, and has

Units: $radians s^{-1}$,

Dimensions t^{-1}

μ is the "coefficient of dynamic viscosity" - see below.

Coefficient of Dynamic Viscosity

The Coefficient of Dynamic Viscosity, μ, is defined as the shear force, per unit area, (or shear stress τ), required to drag one layer of fluid with unit velocity past another layer a unit distance away.

$$\mu = \tau \bigg/ \frac{du}{dy} = \frac{\text{Force}}{\text{Area}} \bigg/ \frac{\text{Velocity}}{\text{Distance}} = \frac{\text{Force} \times \text{Time}}{\text{Area}} = \frac{\text{Mass}}{\text{Length} \times \text{Area}}$$

Units: Newton seconds per square metre, $N s m^{-2}$ or Kilograms per meter per second, $kg m^{-1} s^{-1}$.

(Although note that μ is often expressed in Poise, P, where 10 P = 1 $kg m^{-1} s^{-1}$.)

Typical values:

Water =1.14 $\times 10^{-3}$ $kg m^{-1} s^{-1}$, Air =1.78 $\times 10^{-5}$ $kg m^{-1} s^{-1}$, Mercury =1.552 $kg m^{-1} s^{-1}$, Paraffin Oil =1.9 $kg m^{-1} s^{-1}$.

Kinematic Viscosity

Kinematic Viscosity, ν, is defined as the ratio of dynamic viscosity to mass density.

$$\nu = \frac{\mu}{\rho}$$

Units: square metres per second, $m^2 s^{-1}$

(Although note that ν is often expressed in Stokes, St, where 10^4 St = 1 $m^2 s^{-1}$.)

Dimensions: $L^2 T^{-1}$.

Typical values:

Water =1.14 $\times 10^{-6}$ $m^2 s^{-1}$, Air =1.46 $\times 10^{-5}$ $m^2 s^{-1}$, Mercury =1.145 $\times 10^{-4}$ $m^2 s^{-1}$, Paraffin Oil =2.375 $\times 10^{-3}$ $m^2 s^{-1}$.

Example - Problems

1. Explain why the viscosity of a liquid decreases while that of a gas increases with a temperature rise. The following is a table of measurement for a fluid at constant temperature. Determine the dynamic viscosity of the fluid.

du/dy (s^{-1})	*0.0*	*0.2*	*0.4*	*0.6*	*0.8*
τ ($N\ m^{-2}$)	0.0	1.0	1.9	3.1	4.0

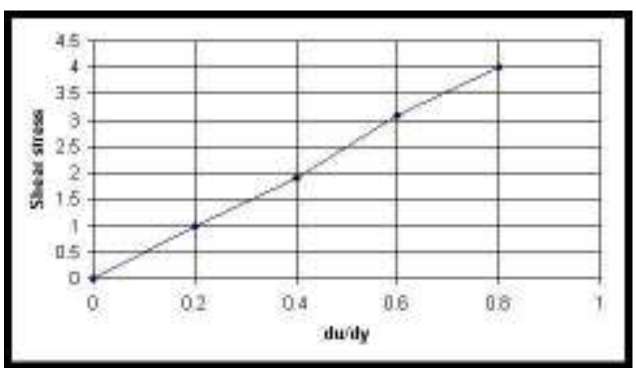
Using Newton's law of viscocity

$$\tau = \mu \frac{\partial u}{\partial y}$$

where μ is the viscosity. So viscosity is the gradient of a graph of shear stress against vellocity gradient of the above data, or

$$\mu = \frac{\tau}{\frac{\partial u}{\partial y}}$$

Plot the data as a graph:



Calculate the gradient for each section of the line

du/dy (s^{-1})	*0.0*	*0.2*	*0.4*	*0.6*	*0.8*
τ ($N\ m^{-2}$)	0.0	1.0	1.9	3.1	4.0
Gradient	-	5.0	4.75	5.17	5.0

Thus the mean gradient = viscosity = 4.98 N s / m^2

2. The density of oil is 850 kg/m^3. Find its relative density and Kinematic viscosity if the dynamic viscosity is 5 $\times$ 10^{-3} kg/ms.

ρ_{oil} = 850 kg/m^3 ρ_{water} = 1000 kg/m^3

γ_{oil} = 850 / 1000 = 0.85

Dynamic viscosity = μ = 5 $\times$ 10^{-3} kg/ms

Kinematic viscosity = ν = μ / ρ

$$\nu = \frac{\mu}{\rho} = \frac{5\times10^{-3}}{1000} = 5\times10^{-6}\, m^2 s^{-1}$$

3. The velocity distribution of a viscous liquid (dynamic viscosity = 0.9 Ns/m^2) flowing over a fixed plate is given by u = 0.68y - y^2 (u is velocity in m/s and y is the distance from the plate in m).
What are the shear stresses at the plate surface and at y=0.34m?

$$u = 0.68y - y^2$$
$$\frac{\partial u}{\partial y} = 0.68 - 2y$$

At the plate face y = 0m,

$$\frac{\partial u}{\partial y} = 0.68$$

Calculate the shear stress at the plate face

$$\tau = \mu \frac{\partial u}{\partial y} = 0.9\times0.68 = 0.612\, N/m^2$$

At y = 0.34m,

$$\frac{\partial u}{\partial y} = 0.68 - 2\times0.34 = 0.0$$

As the velocity gradient is zero at y=0.34 then the shear stress must also be zero.

4. 5.6m^3 of oil weighs 46 800 N. Find its mass density, ρ, and relative density, .

Weight 46 800 = mg γ

Mass m = 46 800 / 9.81 = 4770.6 kg

Mass density ρ = Mass / volume = 4770.6 / 5.6 = 852 kg/m^3

Relative density $\gamma = \frac{\rho}{\rho_{water}} = \frac{852}{1000} = 0.852$

5. From table of fluid properties the viscosity of water is given as 0.01008 poises. What is this value in Ns/m^2 and Pa s units?

μ = 0.01008 poise

1 poise = 0.1 Pa s = 0.1 Ns/m^2

μ = 0.001008 Pa s = 0.001008 Ns/m^2

6. In a fluid the velocity measured at a distance of 75mm from the boundary is 1.125m/s. The fluid has absolute viscosity 0.048 Pa s and relative density 0.913. What is the velocity gradient and shear stress at the boundary assuming a linear velocity distribution.

μ = 0.048 Pa s

γ = 0.913

$$\frac{\partial u}{\partial y} = \frac{1.125}{0.075} = 15 s^{-1}$$

$$\tau = \mu \frac{\partial u}{\partial y}$$

$$= 0.048 \times 15 = 0.720\, Pa s$$

Units

System of units

As any quantity can be expressed in whatever way you like it is sometimes easy to become confused as to what exactly or how much is being referred to. This is particularly true in the field of fluid mechanics. Over the years many different ways have been used to express the various quantities involved. Even today different countries use different terminology as well as different units for the same thing - they even use the same name for different things e.g. an American pint is 4/5 of a British pint!

To avoid any confusion on this course we will always used the SI (metric) system - which you will already be familiar with. It is essential that all quantities be expressed in the same system or the wrong solution will results. Despite this warning you will still find that that this is the most common mistake when you attempt example questions.

The SI System of units

The SI system consists of six **primary** units, from which all quantities may be described. For convenience **secondary** units are used in general practise which are made from combinations of these primary units.

Primary Units

The six **primary** units of the SI system are shown in the table below:

Quantity	SI Unit	Dimension
Length	Metre, m	L
Mass	Kilogram, kg	M
Time	Second, s	T
Temperature	Kelvin, K	
Current	Ampere, A	I
Luminosity	Candela	Cd

In fluid mechanics we are generally only interested in the top four units from this table.

Notice how the term 'Dimension' of a unit has been introduced in this table. This is not a property of the individual units, rather it tells what the unit represents. For example a metre is a length which has a dimension L but also, an inch, a mile or a kilometre are all lengths so have dimension of L.

(The above notation uses the MLT system of dimensions, there are other ways of writing dimensions - we will see more about this in the section of the course on dimensional analysis.)

Derived Units

There are many **derived** units all obtained from combination of the above **primary** units. Those most used are shown in the table below:

Quantity	SI Unit		Dimension
velocity	m/s	ms^{-1}	LT^{-1}
acceleration	m/s^2	ms^{-2}	LT^{-2}
force	N $kg\ m/s^2$	$kg\ ms^{-2}$	$M\ LT^{-2}$
energy (or work)	Joule J N m, $kg\ m^2/s^2$	$kg\ m^2 s^{-2}$	ML^2T^{-2}
power	Watt W N m/s $kg\ m^2/s^3$	Nms^{-1} $kg\ m^2 s^{-3}$	ML^2T^{-3}
pressure (or stress)	Pascal P, N/m^2, $kg/m/s^2$	Nm^{-2} $kg\ m^{-1}s^{-2}$	$ML^{-1}T^{-2}$
density	kg/m^3	$kg\ m^{-3}$	ML^{-3}
specific weight	N/m^3 $kg/m^2/s^2$	$kg\ m^{-2}s^{-2}$	$ML^{-2}T^{-2}$
relative density	a ratio no units		1 no dimension
viscosity	$N\ s/m^2$ kg/m s	$N\ sm^{-2}$ $kg\ m^{-1}s^{-1}$	$M\ L^{-1}T^{-1}$
surface tension	N/m $kg\ /s^2$	Nm^{-1} $kg\ s^{-2}$	MT^{-2}

The above units should be used at all times. Values in other units should NOT be used without first converting them into the appropriate SI unit. If you do not know what a particular unit means find out, else your guess will probably be wrong. One very useful tip is to write down the units of any equation you are using. If at the end the units do not match you know you have made a mistake. For example is you have at the end of a calculation, 30 kg/m s = 30 m

You have certainly made a mistake - checking the units can often help find the mistake. More on this subject will be seen later in the section on dimensional analysis and similarity.

Units

A water company wants to check that it will have sufficient water if there is a prolonged drought in the area. The region it covers is 500 square miles and the following consumption figures have been sent in by various different offices. There is sufficient information to calculate the amount of water available, but unfortunately it is in several different units.

Of the total area 100 000 acres is rural land and the rest urban. The density of the urban population is50 per square kilometre. The average toilet cistern is sized 200mm by 15in by 0.3m and on average each person uses this 3 time per day. The density of the rural population is 5 per square mile. Baths are taken twice a week by each person with the average volume of water in the bath being 6 gallons. Local industry uses 1000 m3 per week. Other uses are estimated as 5 gallons per person per day. A US air base in the region has given water use figures of 50 US gallons per person per day.

The average rain fall in 1in per month (28 days). In the urban area all of this goes to the river while in the rural area 10% goes to the river, 85% is lost (to the aquifer) and the rest goes to the one reservoir which supplies the region. This reservoir has an average surface area of 500 acres and is at a depth of 10 fathoms. 10% of this volume can be used in a month.

1. What is the total consumption of water per day in cubic meters?
2. If the reservoir was empty and no water could be taken from the river, would there be enough water if available if rain fall was only 10% of average?

Fluid Statics

This section will study the forces acting on or generated by fluids at rest.

Objectives

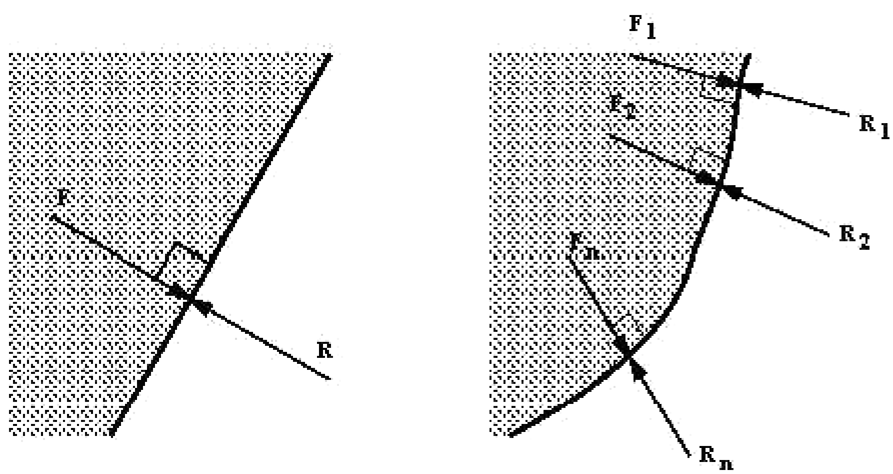
- Introduce the concept of pressure;
- Prove it has a unique value at any particular elevation;
- Show how it varies with depth according to the hydrostatic equation and
- Show how pressure can be expressed in terms of head of fluid.

This understanding of pressure will then be used to demonstrate methods of pressure measurement that will be useful later with fluid in motion and also to analyse the forces on submerges surface/structures.

Fluids statics

The general rules of statics (as applied in solid mechanics) apply to fluids at rest. From earlier we know that:

- a static fluid can have **no shearing force** acting on it, and that
- any force between the fluid and the boundary must be acting at right angles to the boundary.



Pressure force normal to the boundary

Note that this statement is also true for curved surfaces, in this case the force acting at any point is normal to the surface at that point. The statement is also true for any imaginary plane in a static fluid. We use this fact in our analysis by considering elements of fluid bounded by imaginary planes.

We also know that:

- For an element of fluid at rest, the element will be in equilibrium - the sum of the components of forces in any direction will be zero.
- The sum of the moments of forces on the element about any point must also be zero.

It is common to test equilibrium by resolving forces along three mutually perpendicular axes and also by taking moments in three mutually perpendicular planes an to equate these to zero.

Pressure

As mentioned above a fluid will exert a normal force on any boundary it is in contact with. Since these boundaries may be large and the force may differ from place to place it is convenient to work in terms of pressure, p, which is the force per unit area.

If the force exerted on each unit area of a boundary is the same, the pressure is said to be uniform.

$$\text{pressure} = \frac{\text{Force}}{\text{Area over which the force is applied}}$$

$$p = \frac{F}{A}$$

Units:Newton's per square metre, $N m^{-2}$, $kg m^{-1} s^{-2}$.

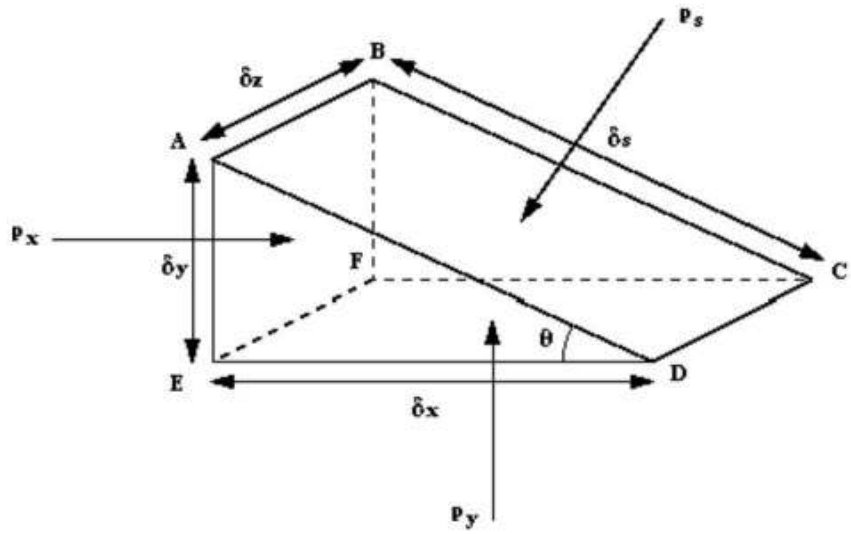
(The same unit is also known as a Pascal, Pa, i.e. 1Pa = 1 $N m^{-2}$)

(Also frequently used is the alternative SI unit the bar, where $1 bar = 10^5 N m^{-2}$)

Dimensions: $M L^{-1} T^{-2}$.

Pascal's Law

By considering a small element of fluid in the form of a triangular prism which contains a point P, we can establish a relationship between the three pressures p_x in the x direction, p_y in the y direction and p_s in the direction normal to the sloping face.



Triangular prismatic element of fluid

The fluid is a rest, so we know there are no shearing forces, and we know that all force are acting at right angles to the surfaces .i.e.

p_s acts perpendicular to surface ABCD,

p_x acts perpendicular to surface ABFE and

p_y acts perpendicular to surface FECD.

And, as the fluid is at rest, in equilibrium, the sum of the forces in any direction is zero.

Summing forces in the x-direction:

Force due to p_x ,

$$F_{x_x} = p_x \times Area_{ABFE} = p_x \delta x \, \delta y$$

Component of force in the x-direction due to p_s,

$$\begin{aligned} F_{x_s} &= -p_s \times Area_{ABCD} \times \sin\theta \\ &= -p_s \delta s \, \delta z \frac{\delta y}{\delta s} \\ &= -p_s \delta y \, \delta z \end{aligned}$$

($\sin\theta = \delta y / \delta s$)

Component of force in x-direction due to p_y ,

$$F_{x_y} = 0$$

To be at rest (in equilibrium)

$$\begin{aligned} F_{x_x} + F_{x_s} + F_{x_y} &= 0 \\ p_x \delta x \delta y + \left(-p_s \delta y \delta z\right) &= 0 \\ p_x &= p_s \end{aligned}$$

Similarly, summing forces in the y-direction. Force due to p_y ,

$$F_{y_y} = p_y \times Area_{EFCD} = p_y \delta x \delta z$$

Component of force due to p_s ,

$$\begin{aligned} F_{y_s} &= -p_s \times Area_{ABCD} \times \cos\theta \\ &= -p_s \delta s \delta z \frac{\delta x}{\delta s} \\ &= -p_s \delta x \delta z \end{aligned}$$

($\cos\theta = \delta x / \delta s$)

Component of force due to p_x ,

$$F_{y_x} = 0$$

Force due to gravity,

$$\begin{aligned} \text{weight} &= -\text{specific weight} \times \text{volume of element} \\ &= -\rho g \times \frac{1}{2} \delta x \delta y \delta z \end{aligned}$$

To be at rest (in equilibrium)

$$F_{y_y} + F_{y_s} + F_{y_x} + \text{weight} = 0$$

$$p_y \delta x \delta y + \left(-p_s \delta x \delta z\right) + \left(-\rho g \frac{1}{2} \delta x \delta y \delta z\right) = 0$$

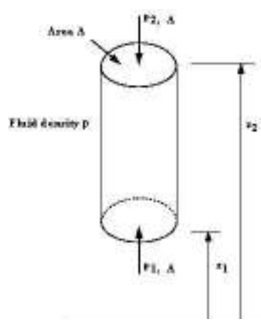
The element is small i.e. δx , δy and are small, and so $\delta x \delta y \delta z$ is very small and considered negligible, hence

$$p_x = p_y = p_s$$

Considering the prismatic element again, p_s is the pressure on a plane at any angle θ, the x, y and z directions could be any orientation. The element is so small that it can be considered a point so the derived expression $p_x = p_y = p_s$. indicates that pressure at any point is the same in all directions. (The proof may be extended to include the z axis).

Pressure at any point is the same in all directions. This is known as **Pascal's Law** and applies to fluids at rest.

Variation of Pressure Vertically In A Fluid Under Gravity



Vertical elemental cylinder of fluid

In the above figure we can see an element of fluid which is a vertical column of constant cross sectional area, A, surrounded by the same fluid of mass density ρ. The pressure at the bottom of the cylinder is p_1 at level z_1, and at the top is p_2 at level z_2. The fluid is at rest and in equilibrium so all the forces in the vertical direction sum to zero. i.e. we have

$$\begin{aligned} \text{Force due to } p_1 \text{ on A (upward)} &= p_1 A \\ \text{Force due to } p_2 \text{ on A (downward)} &= p_2 A \\ \text{Force due to weight of element (downward)} &= mg \\ &= \text{mass density} \times \text{volume} = \rho g A (z_2 - z_1) \end{aligned}$$

Taking upward as positive, in equilibrium we have

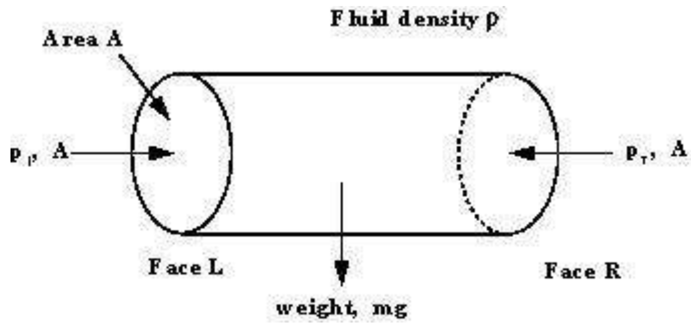
$$p_1 A - p_2 A - \rho g A (z_2 - z_1) = 0$$

$$p_2 - p_1 = -\rho g (z_2 - z_1)$$

Thus in a fluid under gravity, pressure decreases with increase in height $z = (z_2 - z_1)$.

Equality of Pressure At The Same Level In A Static Fluid

Consider the horizontal cylindrical element of fluid in the figure below, with cross-sectional area A, in a
fluid of density ρ, pressure p_1 at the left hand end and pressure p_2 at the right hand end.



Horizontal elemental cylinder of fluid

The fluid is at equilibrium so the sum of the forces acting in the x direction is zero.

$$p_l A = p_r A$$

$$p_l = p_r$$

Pressure in the horizontal direction is constant.

This result is the same for any continuous fluid. It is still true for two connected tanks which appear not to have any direct connection, for example consider the tank in the figure below.

We have shown above that $p_l = p_r$ and from the equation for a vertical pressure change we have

$$p_l = p_p + \rho g z$$

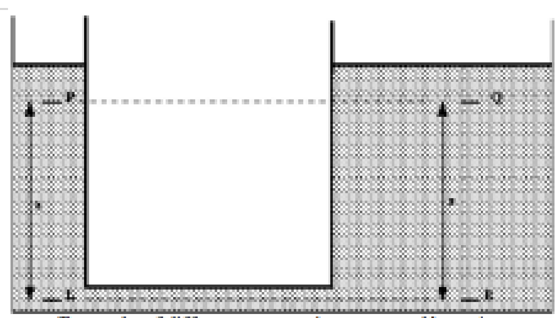
and

$$p_r = p_q + \rho g z$$

so

$$p_p + \rho g z = p_q + \rho g z$$

$$p_p = p_q$$

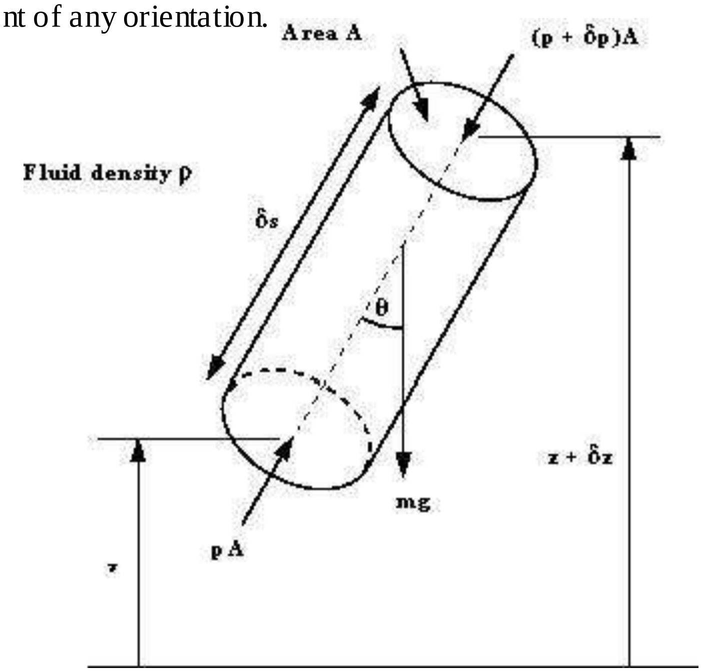


Two tanks of different cross-section connected by a pipe

This shows that the pressures at the two equal levels, P and Q are the same.

General Equation for Variation of Pressure in a Static Fluid

Here we show how the above observations for vertical and horizontal elements of fluids can be generalised for an element of any orientation.



A cylindrical element of fluid at an arbitrary orientation.

Consider the cylindrical element of fluid in the figure above, inclined at an angle θ to the vertical, length δs, cross-sectional area A in a static fluid of mass density ρ. The pressure at the end with height z is p and at the end of height $z+\delta z$ is $p+\delta p$.

The forces acting on the element are

$$pA \quad \text{acting at right-angles to the end of the face at } z$$

$$(p+\delta p)A \quad \text{acting at right-angles to the end of the face at } z+\delta z$$

$$\begin{aligned} mg &= \text{the weight of the element acting vertically down} \\ &= \text{mass density} \times \text{volume} \times \text{gravity} \\ &= \rho A \delta s g \end{aligned}$$

There are also forces from the surrounding fluid acting normal to these sides of the element.

For equilibrium of the element the resultant of forces in any direction is zero.

Resolving the forces in the direction along the central axis gives

$$pA-(p+\delta p)A-\rho g A\delta s \cos\theta = 0$$

$$\delta p = -\rho g \delta s \cos\theta$$

$$\frac{\delta p}{\delta s} = -\rho g \cos\theta$$

Or in the differential form

$$\frac{dp}{ds} = -\rho g \cos\theta$$

If $\theta = 90^\circ$ then s is in the x or y directions, (i.e. horizontal),so

$$\left(\frac{dp}{ds}\right)_{\theta=90^\circ} = \frac{dp}{dx} = \frac{dp}{dy} = 0$$

Confirming that pressure on any horizontal plane is zero.

If $\theta = 0^\circ$ then s is in the z direction (vertical) so

$$\left(\frac{dp}{ds}\right)_{\theta=0^\circ} = \frac{dp}{dz} = -\rho g$$

Confirming the result

$$\frac{p_2 - p_1}{z_2 - z_1} = -\rho g$$

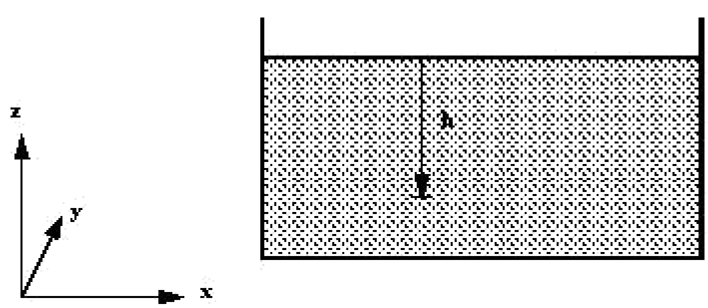
$$p_2 - p_1 = -\rho g(z_2 - z_1)$$

Pressure and Head

In a static fluid of constant density we have the relationship $\frac{dp}{dz} = -\rho g$, as shown above. This can be integrated to give

$$p = -\rho g z + \text{constant}$$

In a liquid with a free surface the pressure at any depth z measured from the free surface so that z = -h (see the figure below)



Fluid head measurement in a tank.

This gives the pressure

$$p = \rho g h + \text{constant}$$

At the surface of fluids we are normally concerned with, the pressure is the atmospheric pressure, $p_{atmospheri}$. So

$$p = \rho g h + p_{atmospheri}$$

As we live constantly under the pressure of the atmosphere, and everything else exists under this pressure, it is convenient (and often done) to take atmospheric pressure as the datum. So we quote pressure as above or below atmospheric.

Pressure quoted in this way is known as gauge pressure i.e.

Gauge pressure is

$$p_{gauge} = \rho g h$$

The lower limit of any pressure is zero - that is the pressure in a perfect vacuum. Pressure measured above this datum is known as absolute pressure i.e.

Absolute pressure is

$$p_{absolute} = \rho g h + p_{atmospheric}$$

$$\text{Absolute pressure} = \text{Gauge prs sure} + \text{Atmospheric pressure}$$

As g is (approximately) constant, the gauge pressure can be given by stating the vertical height of any fluid of density ρ which is equal to this pressure.

$$p = \rho g h$$

This vertical height is known as **head** of fluid.

Note: If pressure is quoted in head, the density of the fluid must also be given.

Example:

We can quote a pressure of $500\,K\,Nm^{-2}$ in terms of the height of a column of water of density, $\rho = 1000 kgm^{-3}$. Using $p = \rho g h$,

$$h = \frac{p}{\rho g} = \frac{500 \times 10^3}{1000 \times 9.81} = 50.95m \text{ of water}$$

And in terms of Mercury with density, $\rho = 13.6 \times 10^3 kgm^{-3}$.

$$h = \frac{500 \times 10^3}{13.6 \times 10^3 \times 9.81} = 3.75m \text{ of Mercury}$$

Pressure Measurement

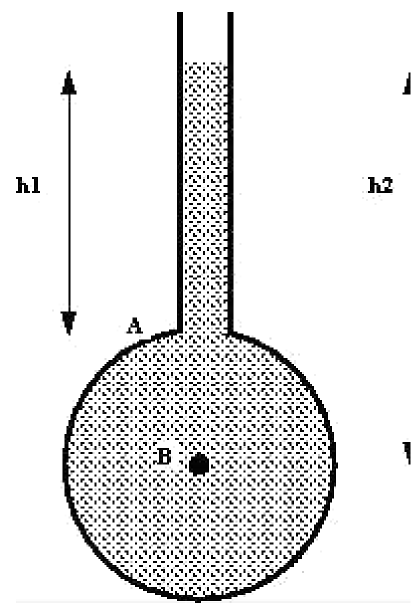
The relationship between pressure and head is used to measure pressure with a manometer (also known as a liquid gauge).

Objective:

- To demonstrate the analysis and use of various types of manometers for pressure measurement.

The Piezometer

The simplest manometer is a tube, open at the top, which is attached to the top of a vessel containing liquid at a pressure (higher than atmospheric) to be measured. An example can be seen in the figure below. This simple device is known as a Piezometer tube. As the tube is open to the atmosphere the pressure measured is relative to atmospheric so is **gauge pressure**.



A simple piezometer tube manometer

pressure at A = pressure due to column of liquid above A

$$p_A = \rho g h_1$$

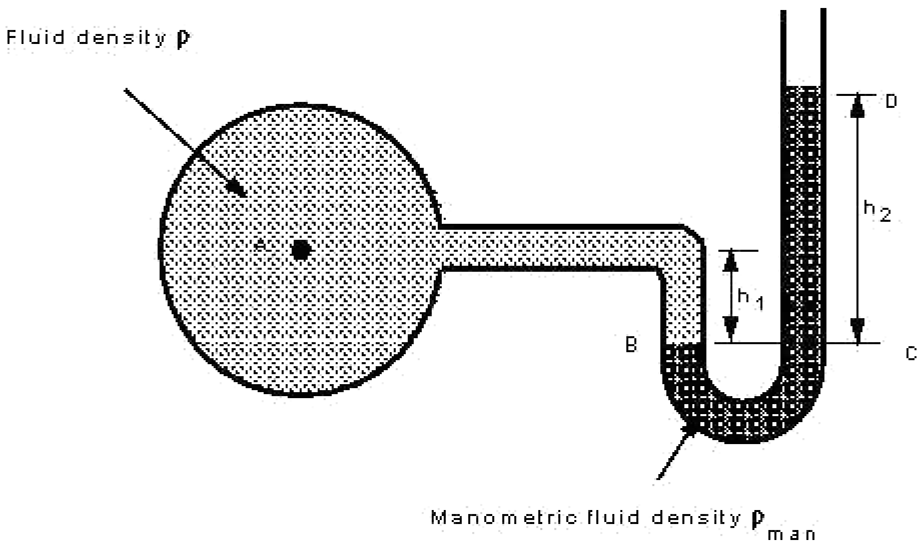
pressure at B = pressure due to column of liquid above B

$$p_B = \rho g h_2$$

This method can only be used for liquids (i.e. **not** for gases) and only when the liquid height is convenient to measure. It must not be too small or too large and pressure changes must be detectable.

The "U"-Tube Manometer

Using a "U"-Tube enables the pressure of both liquids and gases to be measured with the same instrument. The "U" is connected as in the figure below and filled with a fluid called the manometric fluid. The fluid whose pressure is being measured should have a mass density less than that of the manometric fluid and the two fluids should not be able to mix readily - that is, they must be immiscible.



A "U"-Tube manometer

Pressure in a continuous static fluid is the same at any horizontal level so,

$$\text{pressure at B} = \text{pressure at C}$$
$$p_B = p_C$$

For the **left hand** arm

$$\text{pressure at B} = \text{pressure at A} + \text{pressure due to height } h_1 \text{ of fluid being measured}$$
$$p_B = p_A + \rho g h_1$$

For the **right hand** arm

$$\text{pressure at C} = \text{pressure at D} + \text{pressure due to height } h_2 \text{ of manometric fluid}$$
$$p_C = p_{Amospheric} + \rho_{man} g h_2$$

As we are measuring gauge pressure we can subtract $p_{Atmospheric}$ giving

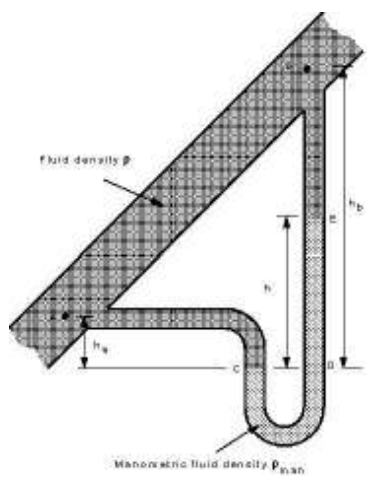
$$p_B = p_C$$
$$p_A = \rho_{man} g h_2 - \rho g h_1$$

If the fluid being measured is a gas, the density will probably be very low in comparison to the density of the manometric fluid i.e. $\rho_{man} >> \rho$. In this case the term $\rho g h_1$ can be neglected, and the gauge pressure give by

$$p_A = \rho_{man} g h_2$$

Measurement of Pressure Difference

If the "U"-tube manometer is connected to a pressurised vessel at two points the pressure difference between these two points can be measured.



Pressure difference measurement by the "U"-Tube manometer

If the manometer is arranged as in the figure above, then

$$\text{pressure at C} = \text{pressure at C}$$
$$p_C = p_D$$
$$p_C = p_A + \rho g h_a$$
$$p_D = p_B + \rho g(h_b - h) + \rho_{man} g h$$
$$p_A + \rho g h_a = p_B + \rho g(h_b - h) + \rho_{man} g h$$

Giving the pressure difference