

MODULE-3

Electrical Properties of Materials

Electrical Properties: Review of classical free electron theory, limitations of classical free electron theory. Postulates of quantum free electron theory, Fermi energy, Fermi velocity, Fermi temperature. Fermi factor and its dependence on energy and temperature. Merits of quantum free electron theory. Problems

Semiconductors: Fermi level in Intrinsic & Extrinsic Semiconductor, Expression for concentration of electrons in conduction band & holes concentration in valance band (only mention the expression), Relation between Fermi energy & Energy gap in intrinsic semiconductors (derivation), Law of mass action, Electrical conductivity of a semiconductor (derivation), Hall effect, Expression for Hall coefficient (derivation) and its application. Numerical problems.

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MODULE-3

Electrical Properties of Materials**Classical free electron theory:****Assumptions of Classical Free Electron Theory**

- A metal is imagined as the structure of 3-dimensional array of ions in between which, there are free moving valence electrons confined to the body of the material. Such freely moving electrons cause electrical conduction under an applied field and hence referred to as conduction electrons
- The free electrons are treated as equivalent to gas molecules and they are assumed to obey the laws of kinetic theory of gases. In the absence of the field, the energy associated with each electron at a temperature T is given by $\frac{3}{2} KT$, where K is a Boltzmann constant.

It is related to the kinetic energy.

$$\frac{3}{2} KT = \frac{1}{2} m v_{th}^2$$

Where v_{th} is the thermal velocity same as root mean square velocity.

- The electric potential due to the ionic cores is taken to be essentially constant throughout the body of the metal and the effect of repulsion between the electrons is considered insignificant.
- The electric current in a metal due to an applied field is a consequence of the drift velocity in a direction opposite to the direction of the field.

Drift velocity (v_d):

The average velocity of occupied by the electrons in the steady state in an applied electric field is called drift velocity.

The drift velocity $V_d = \frac{eE\tau}{m}$

where e is the magnitude of charge, E, τ and m represents the electric field, mean collision time and mass of the electron respectively.

Thermal velocity (V_{th}):

The velocity of electrons in random motion due to thermal agitation is called the thermal velocity.

Mean free path (λ):

The average distance travelled by the conduction electrons between any two successive collisions with lattice ions.

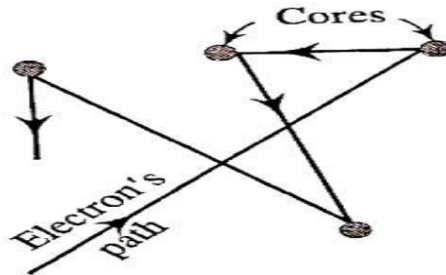


Fig.1 Collision of electrons

Relaxation time (τ_r):

From the instant of sudden disappearance of an electric field across a metal, the average velocity of the conduction electrons decays exponentially to zero, and the time required in this process for the average velocity to reduce to $(1/e)$ times its value is known as Relaxation time.

The relaxation time τ_r and mean collision time τ are related as

$$\tau_r = \frac{\tau}{1 - \langle \cos\theta \rangle}$$

For isotropic scattering or symmetrical scattering $\langle \cos\theta \rangle = 0$ Then $\tau_r = \tau$

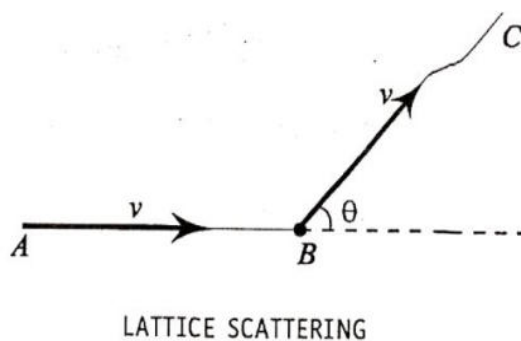


Fig.2 Scattering of electrons

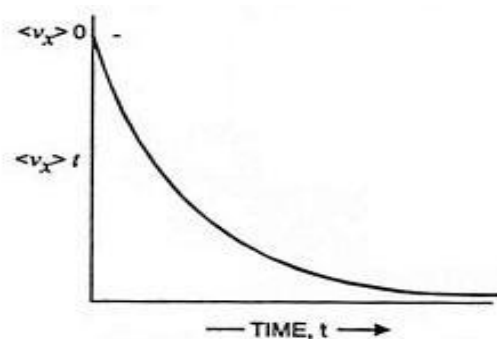


Fig. 3 Graph of Relaxation time

Failures of classical free electron theory:

Electrical and thermal conductivities can be explained from classical free electron theory. It fails to account the facts such as specific heat, temperature dependence of conductivity and dependence of electrical conductivity on electron concentration.

i) Specific heat: The molar specific heat of a gas at constant volume is

$$C_v = \frac{3}{2} R$$

As per the classical free electron theory, free electrons in a metal are expected to behave just as gas molecules. Thus the above equation holds good equally well for the free electrons also.

But experimentally it was found that, the contribution to the specific heat of a metal by its conduction electrons was

$$C_v = 10^{-4} RT$$

which is lower than the expected value. Also according to the theory the specific heat is independent of temperature whereas experimentally specific heat is proportional to temperature.

ii) Temperature dependence of electrical conductivity:

Experimentally, electrical conductivity σ is inversely proportional to the temperature T .

i.e.
$$\sigma_{\text{exp}} \propto \frac{1}{T} \rightarrow (1)$$

According to the assumptions of classical free electron theory

$$\frac{3}{2} KT = \frac{1}{2} m v_{\text{th}}^2$$

$$v_{\text{th}} = \sqrt{\frac{3KT}{m}}$$

i.e.,
$$v_{\text{th}} \propto \sqrt{T}$$

The mean collision time ' τ ' is inversely proportional to the thermal velocity.

i.e.
$$\tau \propto \frac{1}{v_{\text{th}}} \quad \text{or} \quad \tau \propto \frac{1}{\sqrt{T}}$$

$$\text{But } \sigma = \frac{ne^2\tau}{m}$$

$$\sigma \propto \tau$$

$$\text{or } \sigma \propto \frac{1}{\sqrt{T}} \quad \rightarrow (2)$$

From equations (1) & (2) it is clear that the experimental value is not agreeing with the theory.

iii)Dependence of electrical conductivity on electron concentration:

According to classical free electron the theory

$$\sigma = \frac{ne^2\tau}{m}; \text{ where } n \text{ is the electron concentration,}$$

$$\text{Therefore, } \sigma \propto n$$

Experimental results.

Metals	Electron concentration(n)	conductivity (σ)
Copper	$8.45 \times 10^{28}/\text{m}^3$	$5.88 \times 10^7/\Omega\text{m}$
Aluminum	$18 \times 10^{28}/\text{m}^3$	$3.6 \times 10^7/\Omega\text{m}$

From the above data it is clear that copper is having high electrical conductivity than aluminum

But if we go according to CFeT, aluminum should have been better conductor because $n_{\text{al}} > n_{\text{cu}}$

Hence theory fails

Quantum free electron theory:

Assumptions of quantum free electron theory:

- The energy values of the conduction electrons are quantized. The allowed energy values are realized in terms of a set of energy values.
- The distribution of electrons in the various allowed energy levels occur as per Pauli's exclusion principle.
- The electrons travel with a constant potential inside the metal but confined within its boundaries.
- The attraction between the electrons and the lattice ions and the repulsion between the electrons themselves are ignored.

Density of states $g(E)$:

There are large numbers of allowed energy levels for electrons in solid materials. A group of energy levels close to each other is called as energy band. Each energy band is spread over a few electron-volt energy ranges. In 1mm^3 volume of the material, there will be a more than a thousand billion permitted energy levels in an energy range of few electron-volts. Because of this, the energy values appear to be virtually continuous over a band spread. To represent it technically it is stated as density of energy levels. The dependence of density of energy levels on the energy is denoted by $g(E)$. The graph shows variation of $g(E)$ versus E .

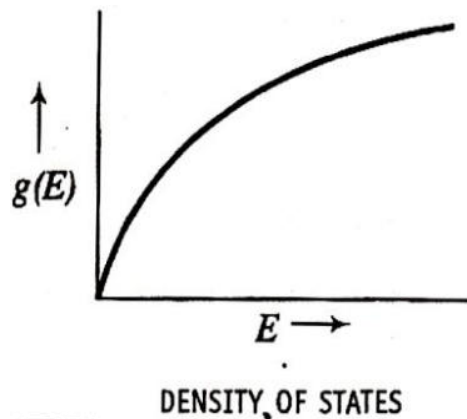


Fig. 4 Graph of Density of States

It is called density of states function. It is the number of allowed energy levels per unit energy interval in the band associated with material of unit volume. In an energy band as E changes $g(E)$ also changes.

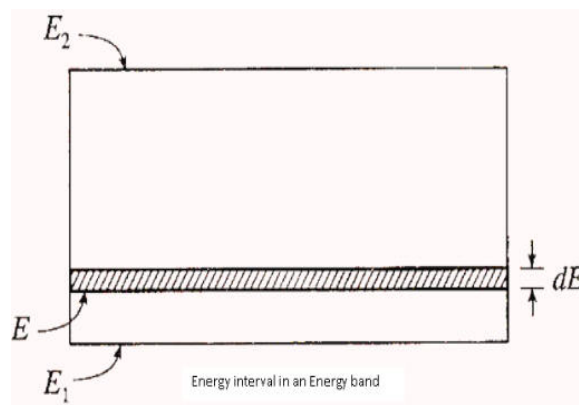


Fig. 4 Energy intervals in energy bands

Consider an energy band spread in an energy interval between E_1 and E_2 . Below E_1 and above E_2 there are energy gaps. $g(E)$ represents the density of states at E . As dE is small, it is assumed

that $g(E)$ is constant between E and $E+dE$. The density of states in range E and $(E+dE)$ is denoted by $g(E)dE$.

$$\text{i.e. } g(E)dE = \left[\frac{8\sqrt{2\pi m^{\frac{3}{2}}}}{h^3} \right] E^{\frac{1}{2}} dE$$

It is clear $g(E)$ is proportional to $\sqrt{E}$ in the interval dE

Fermi energy:

In a metal having N atoms, there are N allowed energy levels in each band. In the energy band the energy levels are separated by energy differences. It is characteristic of the material. According to Pauli's exclusion principle, each energy level can accommodate a maximum of two electrons, one with spin up and the other with spin down. The filling up of energy levels occurs from the lowest level. The next pair of electrons occupies the next energy level and so on till all the electrons in the metal are accommodated. Still number of allowed energy levels, are left vacant. This is the picture when there is no external energy supply for the electrons. The energy of the highest occupied level at absolute zero temperature (0K) is called the Fermi energy and the energy level is called Fermi level. It is denoted by ' E_f '

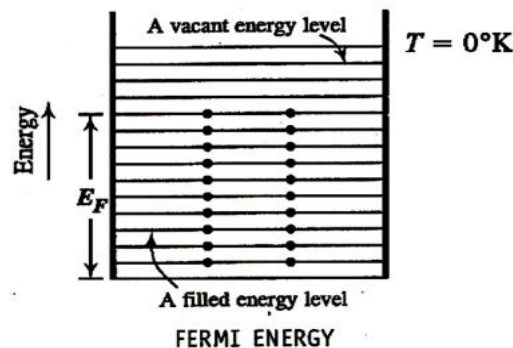


Fig. 5 Energy levels.

Fermi-Dirac statistics:

At thermal equilibrium the free electrons acquire energy obeying a statistical rule called Fermi-Dirac statistics. Fermi-Dirac statistics is applicable to the assembly of particles which obeys Pauli's exclusion principle, they must also be identical particles of spin $\frac{1}{2}$ and are indistinguishable.

Fermi-Dirac statistics permits to evaluation of probability of finding the electrons occupying energy levels in a certain energy range and is given by a function called Fermi factor $f(E)$.

If $g(E)$ is the density of energy states, $f(E)$ is the probability of occupation of energy states. Then the number of electrons distributed in the energy range is called Fermi-Dirac distribution and given by

$$N(E) dE = g(E) f(E) dE$$

Fermi factor:

The electrons in the energy levels for below Fermi level cannot absorb the energy above absolute zero temperature. At ordinary temperature because there are no vacant energy levels above Fermi level into which electrons could get into after absorbing the thermal energy. Though the excitations are random, the distributions of electrons in various energy levels will be systematically governed by a statistical function at the steady state.

The probability $f(E)$ that a given energy state with energy E is occupied by the electrons at a steady temperature T is given by

$$f(E) = \frac{1}{e^{\frac{(E-E_F)}{kT}} + 1}$$

$f(E)$ is called the fermi factor.

Fermi factor is the probability of occupation of a given energy state by the electrons in a material at thermal equilibrium.

Dependence of Fermi factor with temperature

The dependence of Fermi factor on temperature and energy is as shown in the figure.

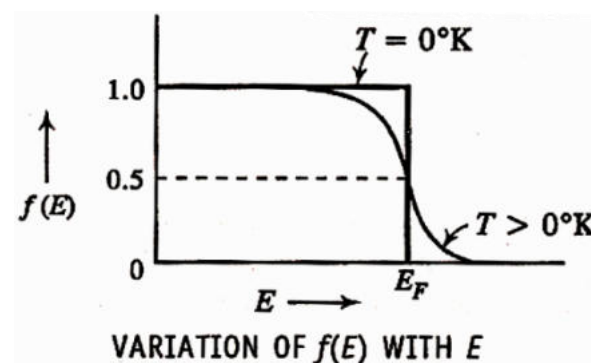


Fig. 6 Graph for dependence of Fermi factor on temperature and energy

i) Probability of occupation for $E < E_F$ at $T = 0K$:

When $T = 0K$ and $E < E_F$

$$f(E) = \frac{1}{e^{-\infty} + 1} = \frac{1}{0 + 1} = 1$$

The probability of occupation of energy state is 100%

$$f(E)=1 \text{ for } E < E_f.$$

$f(E)=1$ means the energy level is certainly occupied and $E < E_f$ applies to all energy levels below E_f . Therefore, at $T=0K$ all the energy levels below the Fermi level are occupied.

ii) Probability of occupation for $E > E_f$ at $T=0K$:

When $T=0K$ and $E > E_f$

$$f(E) = \frac{1}{e^{\infty} + 1} = \frac{1}{\infty} = 0$$

The probability of occupation of energy state is 0%

$$f(E)=0 \text{ for } E > E_f$$

$\therefore$ At $T=0K$, all the energy levels above Fermi levels are unoccupied. Hence at $T=0K$ the variation of $f(E)$ for different energy values, becomes a step function as shown in the above figure.

iii) The probability of occupation at ordinary temperature (for $E \approx E_f$ at $T > 0K$)

At ordinary temperatures though the value of probability remains 1, for $E < E_f$ it starts reducing from 1 for values of E close to but lesser than E_f as in the figure.

The values of $f(E)$ become $\frac{1}{2}$ at $E=E_f$

This is because for $E=E_f$

$$e^{(E-E_f)/kT} = e^0 = 1$$

$$\therefore f(E) = \frac{1}{e^{(E-E_f)/kT} + 1} = \frac{1}{1 + 1} = \frac{1}{2}$$

The probability of occupation of energy state is 50%

Further for $E > E_f$ the probability value falls off to zero rapidly. Hence, the Fermi energy is the most probable or the average energy of the electrons across which the energy transitions occur at temperature above zero degree absolute.

Merits of Quantum free electron theory:

Quantum free electron theory has successfully explained following observed experimental facts whereas the classical free electron theory failed.

i) Specific heat:

According to quantum free electron theory, it is only those electrons that are occupying energy levels close to E_F , which are capable of absorbing the heat energy to get excited to higher energy levels.

Hence only a small percentage of the conduction electrons are capable of receiving the thermal energy input, thus the specific heat value becomes very small for the metal.

According to quantum free electron theory, it can be shown

$$C_V = \left(\frac{2k}{E_F} \right) RT$$

Considering $E_F = 5 \text{ eV} \left(\frac{2k}{E_F} \right) \approx 10^{-4}$

$\therefore C_V = 10^{-4} RT$ which is close to experimental value.

ii) Temperature depends on electrical conductivity:

Experimentally electrical conductivity $\sigma_{\text{expt}} \propto \frac{1}{T}$

Electrical conductivity according to classical theory $\sigma_{CFT} = \frac{ne^2\tau}{m}$

Where m^* is called effective mass of an electron.

According to quantum free electron theory τ is $\tau = \frac{\lambda}{v_F}$

$$\therefore \sigma_{QFT} = \frac{ne^2\lambda}{m^*v_F} \text{-----(1)}$$

According to quantum free electron theory E_F and v_F are independent of temperature. The dependence of λ & T is as follows

Conduction electrons are scattered by the vibrating ions of the lattice. The vibration occurs such that the displacement of ions takes place equally in all directions. If 'r' is the amplitude of vibrations then the ions can be considered to present effectively a circular cross section of area πr^2 that blocks the path of the electron irrespective of direction of approach. Since the vibrations cover larger area of cross section should scatter more number of electrons, it results in the reduction of mean free path of the electron.

$$\therefore \lambda \propto \frac{1}{\pi r^2}$$

Considering the facts

The energy of vibrating body is proportional to the square of amplitude.

The energy of ions is due to the thermal energy.

The thermal energy is proportional to the temperature 'T'.

We can write $r^2 \propto T$

$\therefore \lambda \propto 1/T$ substituting for λ in eqn (1)

Then $\sigma_{QFT} \propto 1/T$

Thus $\sigma_{QFT} \propto 1/T$ is correctly explained by quantum free electron theory.

iii) **Electrical conductivity and electron concentration:**

As per quantum free electron the electrical conductivity is

$$\sigma_{QFT} = \frac{ne^2 \lambda}{m^* v_F}$$

i.e $\sigma \propto (n)$ and $\sigma \propto (\lambda/v_F)$

The value of (n) for aluminum is 2.13 times higher than that of copper. But the value of (λ/v_F) for copper is about 3.73 times higher than that of aluminum. Thus, the conductivity of copper exceeds that of aluminum.

Semiconducting materials:

Semiconductors are materials whose electrical conductivity lies between conductors and insulators. The conductivity of semiconductors is in the order of 10^4 to 10^{-4} mho/m.

Valance band: Energy level occupied by valance electrons is called valance band. Valance band may be fully filled or partially filled at 0 K

Conduction band: Energy level occupied by conduction electrons (free electrons) is called conduction band. The conduction band is an empty band at 0 K

Forbidden energy gap: The separation between valance band and conduction band and valance band is known as forbidden energy gap. The energy gap is measured in electron volt (eV).

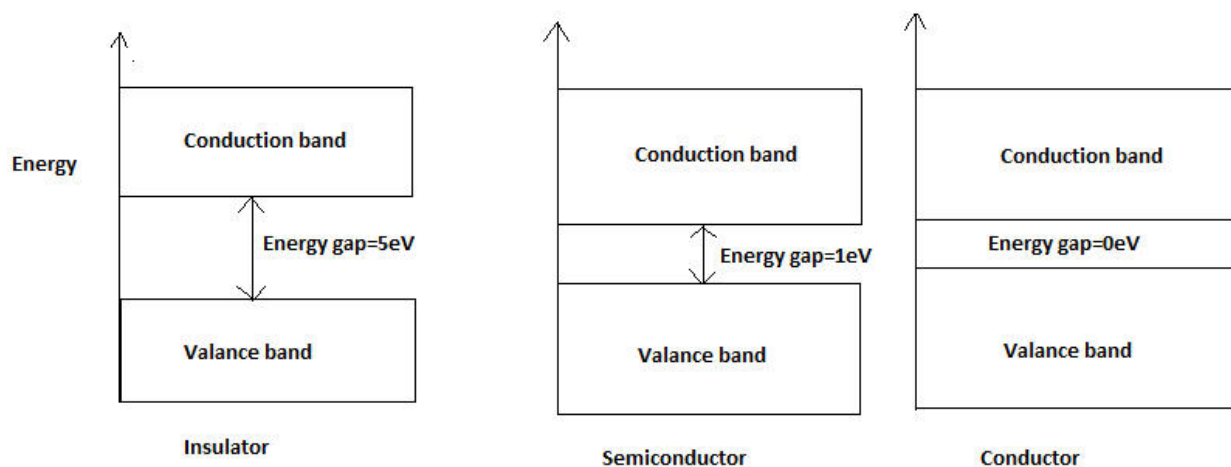


Fig. 7 Energy Bandgap diagram

In semiconductors both electrons and holes are responsible for electrical conduction. At ordinary temperature an intrinsic semiconductor contains some free electrons in conduction band and equal number of holes in valance band due to thermal agitation and lattice vibrations. These electrons and holes move in all possible directions, hence the net current is zero.

When the electric field is applied electrons move towards positive and holes move towards negative direction of the field. The velocity acquired by the charge carriers called drift velocity depends on strength of the applied field.

$$\text{i.e. } V_d \propto E \quad \text{or} \quad V_d = \mu_n E$$

where μ_n —mobility of charge carriers

In intrinsic semiconductor the movement of electrons in nearly empty conduction band is different from that of movement of holes in nearly filled valance band. Hence the conductivity and mobility of electrons and holes is different.

Conductivity of semiconducting materials:

Consider a semiconductor of area of cross section A , carrying a current I . Let V_d be the drift velocity of the charge carriers

Since in semiconductor both the holes and the electrons contribute to the conductivity of the semiconductor

Let us consider first the conductivity in a semiconductor is due to the flow of electrons

$$\text{Only } I = NAeV_d \quad \dots\dots\dots (1)$$

$N = N_e$ = charge concentration of electrons

The current density of electrons is given by $J_e = e N_e V_d \quad \dots\dots(2) \quad [J = I/A]$

Using $V_d = \mu_e E$ in the above equation

$$J_e = e N_e \mu_e E \quad \dots\dots(3)$$

But current density is given by $J_e = \sigma_e E \quad \dots\dots(4)$

Equating eqn (3) and (4)

$$\begin{aligned} \sigma_e E &= e N_e \mu_e E \\ \sigma_e &= e N_e \mu_e \quad \dots\dots\dots (5) \end{aligned}$$

eqn(5) represents the conductivity due to electrons in conduction band.

similarly the conductivity due holes in valance band is given by

$$\sigma_h = e N_h \mu_h \dots\dots\dots (6)$$

Total conductivity $\sigma = \sigma_e + \sigma_h = e(N_e \mu_e + N_h \mu_h)$

$$\sigma = e(N_e \mu_n + N_h \mu_h)$$

For intinsic semiconductors $n = p = n_i$

$$\text{Hence for semiconductors} \quad \sigma = n_i e (\mu_e + \mu_h) \quad \dots\dots\dots(7)$$

Concentration of electrons and holes in intrinsic semiconductors:

At room temperature in intrinsic semiconductors the conduction band consists of few electrons and valance band consists of equal number of holes due to thermal agitation and lattice vibrations.

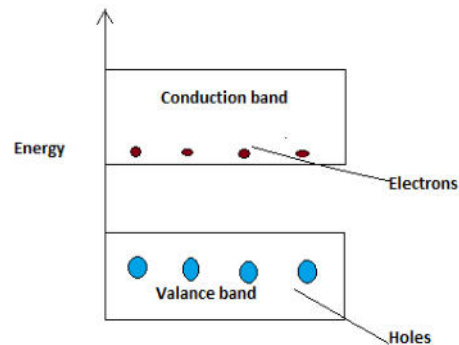


Fig. 8 Energy gap of semiconductor

The number of electrons or the concentration of electrons in conduction band is given by

$$N_e = \frac{4\sqrt{2}}{h^3} (\pi m_e * KT)^{3/2} e^{\frac{E_f - E_g}{KT}}$$

Similarly the number of holes or the concentration of holes in valance band is given by

$$N_h = \frac{4\sqrt{2}}{h^3} (\pi m_h * KT)^{3/2} e^{\frac{-E_f}{KT}} \quad E_f \text{----- Fermi level}$$

Law of mass action:

In a semiconducting material (intrinsic or extrinsic) the product of electron concentration and hole concentration is a constant.

$$\text{We know that } N_e = \frac{4\sqrt{2}}{h^3} (\pi m_e * KT)^{3/2} e^{\frac{E_f - E_g}{KT}} \quad \text{and } N_h = \frac{4\sqrt{2}}{h^3} (\pi m_h * KT)^{3/2} e^{\frac{-E_f}{KT}}$$

$$N_e N_h = \frac{32}{h^6} (\pi KT)^3 (m_e * m_h)^{3/2} e^{\frac{-E_g}{KT}}$$

The above equation shows that the concentration of charge carriers does not depends on (E_f) Fermi level.

Therefore $N_e N_h = \text{constant}$

For intrinsic semiconductors $N_e = N_h$

$N_e N_h = n_i^2 = \text{constant}$ where n_i is intrinsic charge carrier density.

Fermi level in intrinsic semiconductors:

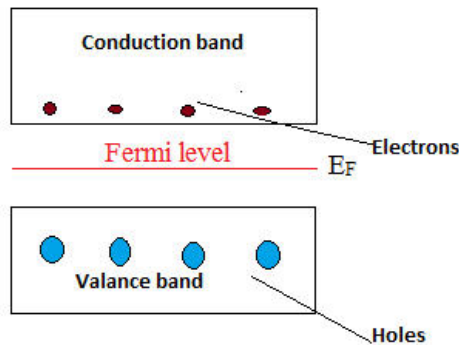


Fig. 9 Fermi level between Energy gap of semiconductor

At $T=0K$ all the energy level in valance band are completely empty. At high temperature the electron in top valance band undergoes excitation to the bottom of conduction. Therefore for such a distribution the average energy of the electrons taking part in conduction will be almost equal to $\frac{1}{2} E_g$.

For an intrinsic semiconductor, the number of holes/unit volume in valance band is equal to number of electrons/unit volume in conduction band.

$$\text{i.e.} \quad N_e = N_h$$

$$\text{we know that } N_e = \frac{4\sqrt{2}}{h^3} (\pi m_e^* K T)^{3/2} e^{\frac{E_f - E_g}{K T}} \quad \text{and} \quad N_h = \frac{4\sqrt{2}}{h^3} (\pi m_h^* K T)^{3/2} e^{\frac{-E_f}{K T}}$$

$$\frac{4\sqrt{2}}{h^3} (\pi m_e^* K T)^{3/2} e^{\frac{E_f - E_g}{K T}} = \frac{4\sqrt{2}}{h^3} (\pi m_h^* K T)^{3/2} e^{\frac{-E_f}{K T}}$$

$$(m_e^*)^{3/2} e^{\frac{E_f - E_g}{K T}} = (m_h^*)^{3/2} e^{\frac{-E_f}{K T}}$$

$$(m_e^*/m_h^*)^{3/2} = e^{\frac{-2E_f + E_g}{K T}}$$

Take natural log on both sides

$$\frac{3}{2} \ln (m_e^*/m_h^*) = \frac{-2E_f + E_g}{K T}$$

$$2E_f = E_g - \frac{3KT}{2} \ln (m_e^*/m_h^*)$$

$$E_f = \frac{E_g}{2} - \frac{3KT}{4} \ln (m_e^*/m_h^*)$$

For all practical considerations $m_e^* = m_h^*$

$$E_f = \frac{E_g}{2}$$

Thus Fermi level is in the middle of the band gap for an intrinsic semiconductor.

Alternative method

For intrinsic semiconductors the concentration of electrons in conduction band is equal to the concentration of holes in valance band.

$$\text{i.e } n = p$$

$$\text{But } n = N_c \exp (E_f - E_c / KT) \text{ and } p = N_v \exp (E_v - E_f / KT)$$

$$N_c \exp (E_f - E_c / KT) = N_v \exp (E_v - E_f / KT)$$

$$(N_c / N_v) = \exp (E_v - E_f / KT) \exp - (E_f - E_c / KT)$$

$$(N_c / N_v) = \exp \{ (E_v - E_f - E_f + E_c) / KT \}$$

$$\ln (N_c / N_v) = (E_v + E_c - 2E_f) / KT$$

$$(E_v + E_c - 2E_f) = KT \ln (N_c / N_v)$$

$$2E_f = E_v + E_c - KT \ln (N_c / N_v)$$

$$E_f = (E_v + E_c) / 2 - KT / 2 \ln (N_c / N_v)$$

Substituting the value of N_c and N_v in the above equation

$$E_f = (E_v + E_c) / 2 - KT / 2 \ln (m_e^* / m_h^*)^{3/2}$$

$$E_f = (E_v + E_c) / 2 - 3KT / 4 \ln (m_e^* / m_h^*)$$

For intrinsic semiconductor $m_e^* = m_h^*$, therefore $\ln (m_e^* / m_h^*) = 0$

$$E_f = (E_v + E_c) / 2 \text{ where } (E_v + E_c) = E_g$$

$$E_f = E_g / 2$$

Hence Fermi level lies at the center of valance band and conduction band for intrinsic semiconductors or Fermi level lies at the center of forbidden energy gap.

Hall Effect:

The Hall effect is a phenomenon observed when a current-carrying conductor or semiconductor is placed in a magnetic field perpendicular to the direction of current. It results in the development of a transverse voltage across the material.

Importance of Hall effect

It helps to determine

- 1) The type of semiconductor
- 2) The majority charge carrier concentration
- 3) The mean drift velocity of the charge carriers

Hall Effect in semiconductors**Experimental Setup**

- A rectangular sample of a semiconductor (such as n-type or p-type) is placed in a magnetic field B .
- A current I is passed through the sample along the x-axis.
- The magnetic field B is applied along the z-axis, i.e., perpendicular to the direction of current.
- As a result, a Hall voltage V_H develops along the y-axis (transverse direction).

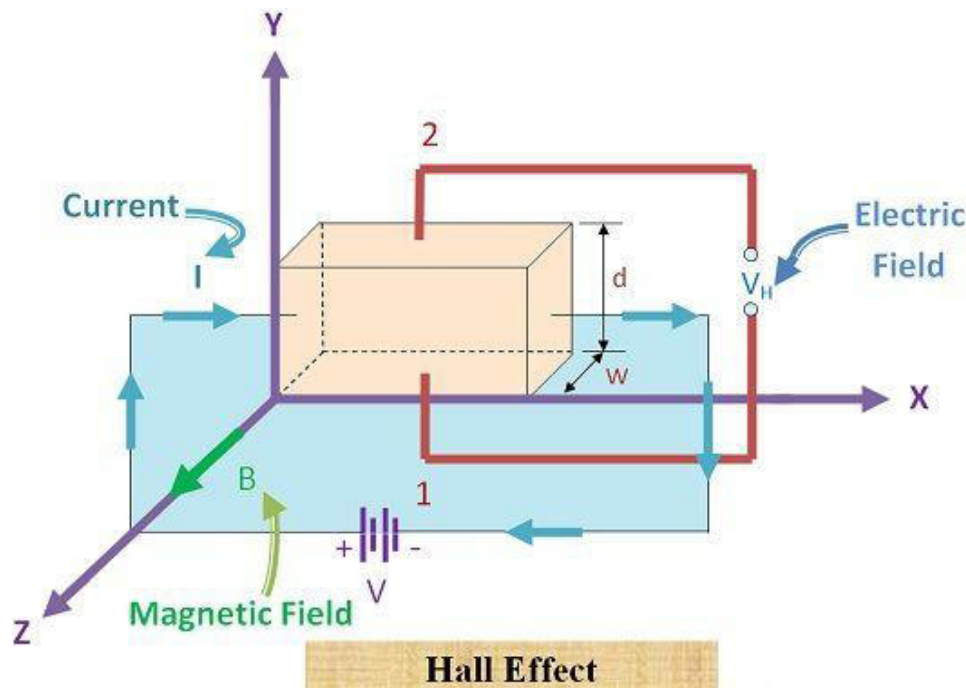


Fig. 10 Circuit for Hall effect

Principle

When charge carriers (electrons or holes) move through a magnetic field, they experience a Lorentz force given by:

$$F = q(v \times B)$$

This force deflects the moving charges to one side of the conductor, producing a potential difference (Hall voltage) across the sample.

Derivation

Let us consider a rectangular plate of p-type semiconducting material carrying a current I is kept in magnetic field B at right angles to I . 'w' is width and 'd' is the thickness of the semiconducting material.

Let V velocity of electrons along x-direction. electrons are the current carriers in metals. The magnetic field applied along the y-direction, then electrons experience a force F_L

$$F_L = BeV$$

along z-direction (Fleming's left hand rule) called Lorentz force.

Before the application of the magnetic field holes move parallel to F & F^1 on the application of the magnetic field the holes experience a downward force

$$F_L = BeV_d$$

Due to this force, Holes are deflected towards the face F and pile up there, due to this corresponding equivalent negative charge is left on the rear face F^1 these opposite charges produce a transverse Electric field E_H known as **Hall field** and hence **Hall potential difference** will be produced, whose direction is F to F^1 due to this holes experience an electric force additional to Lorentz force due to Hall field E_H .

$$F_H = e E_H$$

At equilibrium $F_L = F_H$ (1)

$$BeV_d = e E_H$$

$$E_H = BV_d \text{(2)}$$

Since $E_H = V_H/d$ where d is the thickness of the slab.

$$V_H = E_H d$$

$$V_H = BV_d d \text{(3)}$$

$$I = neAV_d$$

$$V_d = \frac{I}{neA}$$

$$V_H = \frac{BdI}{neA}$$

$$V_H = \frac{BI}{(ne)w} \quad V_H = R_H \frac{BI}{w}$$

Hall coefficient (R_H) in terms of Hall voltage:

Hall coefficient is defined as Hall effect per unit current current density per unit magnetic induction

$$R_H = E_H / (JB)$$

$$E_H = BV_d$$

$$\text{Current density } J = neV_d$$

$$R_H = (BV_d) / (neV_d B)$$

$$R_H = 1 / (ne)$$

Where $R_H = (1/ne)$ called Hall coefficient

$n \rightarrow$ electron concentration $e \rightarrow$ magnitude of the charge carriers.

Since electrons are the charge carriers in metals, Hall coefficient is negative for metals.