Module 2

Elastic properties of materials

Elasticity:

When the deforming forces applied to a body is removed the body tends to recover its original condition i.e. the body will recover its original shape or size. This property of material body to recover its original condition when deforming forces are removed is called elasticity.

Elastic Bodies and Plastic bodies:

The bodies which recover its original condition completely on the removal of deforming force are called perfectly elastic.

Ex. Steel, Quartz fibre, Phospor bronze, Rubber etc

The bodies which do not show any tendency to recover their original condition on the removal of deforming forces are called perfectly plastic body.

Ex. Clay, Wax, Putty.

Deforming Force:

Consider a body which is not free to move and is acted upon by external forces. Due to the action of external forces the body changes its shape or sizes changes and now body is said to be deformed. Thus the applied external force which cause deformation is called deforming force.

Restoring force:

When deforming force is applied to a body then molecules of body tend to displace from their equilibrium position. As a result of this a reaction force developed within the body which tries to bring the molecule to its equilibrium position. This reaction force which is developed in the body is called internal force or elastic force or restoring force.

Stress:

The restoring force per unit area set up inside the body is called stress. The restoring force is equal in magnitude but opposite that of the applied force. Therefore **stress is given by the ratio of the applied force to the area. Unit of stress is Nm^{-2}.**

Strain:

It is defined as t**he ratio of change in dimension of the body to its original dimension.**

It is not having any unit.

Tensile stress (Longitudinal stress):

The stress which brings about change in length of the object is called as Longitudinal stress. It is applied normal to the body.

Ex. Load suspended normally from the wire due to which the wire undergoes change in length.

If ‘F’ is the force applied and ‘a’ is the area of cross section.

Longitudinal stress = $\frac{F}{a}$

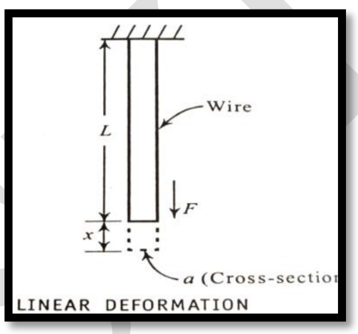
Longitudinal or Tensile strain:

If ‘x’ is the change in length produced due to the applied stress for an original length of ‘L’ then,

$$\textbf{Longitudinal} = \frac{change\ in\ length}{original\ length} = \frac{x}{L}$$

Tangential stress or Shear stress.

The stress which brings about change in shape is called as tangential stress. It is applied parallel to the surface.



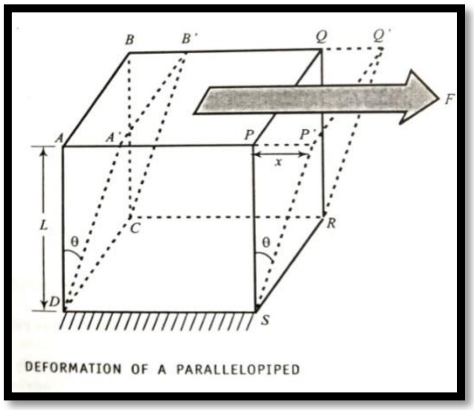
If ‘F’ is the force applied parallel to the surface and ‘a’ is the surface area.

Shear stress = $\frac{F}{a}$

Shear strain:

Shear strain is defined as the ratio of change in the shape of the object to original shape of the object.

If a force is applied tangentially to a free portion of the body whose other part is fixed then its layers slide one over the other; the body experiences a turning effect and changes its shape. This is called **shearing** and the angle through which the turning takes place is called **shearing angle (θ).**



DEFORMATION OF A PARALLELOPIPED

Within elastic limit it is measured by the ratio of relative displacement of one plane to its distance from fixed plane. It is also measured by the angle through which a line originally perpendicular to fixed plane is turned.

Shearing strain= $\theta = tan\ \theta = \frac{x}{L}$

<u>Compressive stress or volume stress :</u>

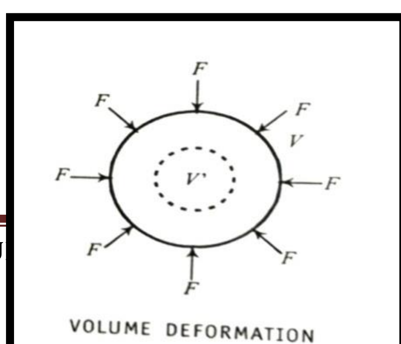
The compressive stress is restoring force developed within the body when body compressed under the action of deforming force. It brings about change in the volume of the object.

If F is the force applied uniformly and normally on a surface area ‘a’ then.

Compressive stress is = $\frac{F}{a}$

<u>Volume strain:</u>

If a uniform force is applied all over the surface of a body then the body undergoes a change in its volume (however the shape is retained in case of solid bodies). If *v* is the change in volume to an original volume *V* of the body then,



VOLUME DEFORMATION

Volume strain = $\dfrac{\text{change in volume}}{\text{original volume}} = \dfrac{v}{V}$

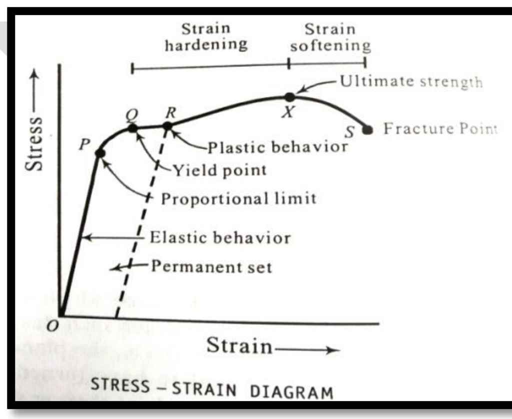
Hooke's law:

The fundamental law of elasticity was given by Robert Hook. It states that **"Stress produced in a body is directly proportional to the strain within the elastic limit".** Thus in such a case the ratio of stress to strain is a constant and it is called the modulus of elasticity or coefficient of elasticity. i.e., stress α strain,

or, $\dfrac{stress}{strain} = \text{a constant (E)}$

Stress – strain Graph when a wire is stressed

The relationship between stress and strain is studied by plotting a graph for various values of stress and the accompanying strain. This graph is obtained by plotting various values of stress and the accompanying strain of simple case of a bar or wire subjected to increasing tension. The graph obtained is in the general form as shown in the figure and is known as stress-strain diagram.



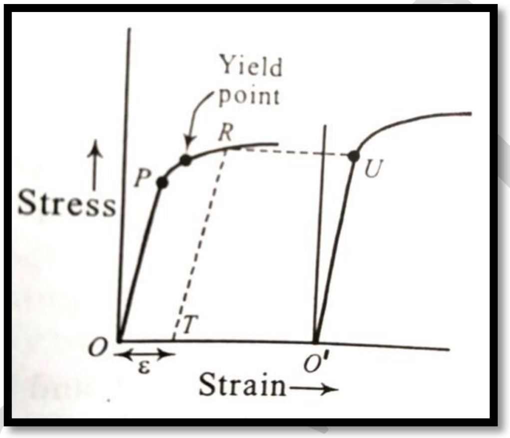
- In the graph the straight and slopping part OP of the curve shows that the strain produced is directly proportional to the stress or the Hook's law is obeyed perfectly up to P. In this region the material will recover its original condition of zero strain, on the removal of the stress. **The point 'P' is called as proportionality limit.**
- If the material is stressed between The region P and Q. the material will regain its original shape i.e it exhibits elasticity but it will not obey Hookes law. **The point Q is called as Yield point or elastic limit.**

- If the material is stressed between the region Q and S. it will not regain its original shape and size. it will not trace the original path instead it will trace the RT. i.e it has undergone a permanent deformation or plastic deformation.

  The point 'x' is called as the ultimate strength. It is the maximum stress that amaterial can withstand beyond which the material breaks.
- **The point 's' is called as the Breaking point.** The region 'QX' is called as **strain hardening.** The region XS is called as **Strain softening.**

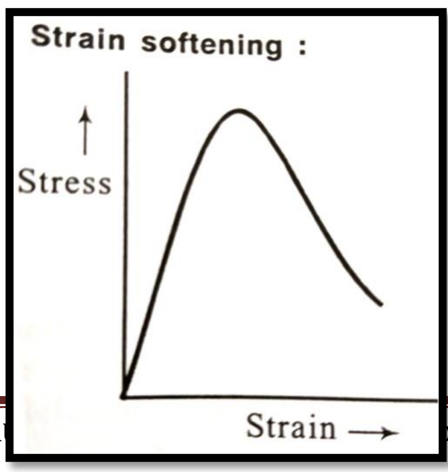
Strain Hardening and strain softening:

Certain materials that are plastically deformed earlier are stressed again, shows an **increased** yield point. This effect is called as **strain hardening.**



It is one of the process of Making a material harder by plastic deformation. It is also known as 'work hardening' or ' cold working'.

Certain materials like concrete or soil, are stressed their stress strain graph shows negative slope soon after the elastic region. The negative slope indicates the that there is softening effect of the material. This effect is called as **'strain softening'.**



Types of elasticity

Corresponding to the three types of strain, we have three types of elasticity

a) Linear Elasticity or Elasticity of length or Young's modulus

b) Elasticity of volume or Bulk modulus

c) Modulus of Rigidity (corresponding to shear strain)

a) Young's modulus (Y).

The ratio of longitudinal stress to linear strain within elastic limit is called the coefficient of direct elasticity or Young's modulus and is denoted by Y .

If F is the force applied normally, to a cross-sectional area a, then the stress is F/a. If L is original length and x is change in length due to the applied force, the strain is given by x/L.

$$\mathbf{Y} = \frac{\text{Normal stress}}{\text{Longitudinal strain}} = \frac{F/a}{x/L} = \frac{FL}{ax} \ \mathbf{N/m^2}$$

b) Elasticity of volume or Bulk modulus (K):

The ratio of normal stress or pressure to the volume strain without change in shape of the body within the elastic limits is called Bulk modulus.

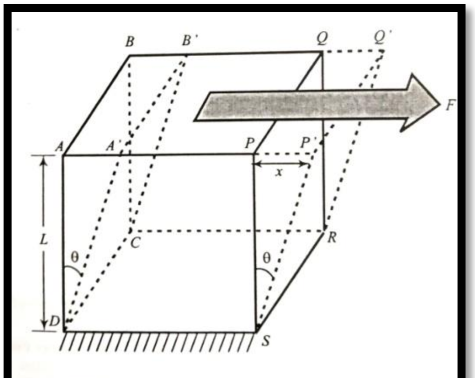
If 'F' is the force applied uniformly and normally on a surface area 'a' the stress or pressure is 'F/a' or P and if '*v*' is the change in volume produced in an original volume V, the strain is given by '*v*/V' and therefore

$$\mathbf{K} = \frac{\text{Normal stress}}{\text{Volume strain}} = \frac{F/a}{v/V} = \frac{FV}{av} = \frac{PV}{v} \ \mathbf{N/m^2}$$

Bulk modulus is referred to as incompressibility and hence its reciprocal is called **compressibility** (strain per unit stress).

c) Modulus of Rigidity (corresponding to shear strain)

In this case, while there is a change in the shape of the body, there is no change in its volume. It takes place by the movement of contiguous layers of the body, one over the other.



DEFORMATION OF A PARALLELOPIPED

Q^1 that is the planes of the two faces ABCD and PQRS can be said to have turned through an angle θ. This angle θ is called the angle of shear or shearing strain. Tangential stress is equal to the force F divided by area 'a' of the face APQB.

The rigidity modulus is defined as the ratio of the tangential stress to the shearing strain.

Hence **tangential stress** = $\frac{F}{a}$ $\theta = PP^1 / PS = \frac{x}{L}$

Rigidity modulus η = $\frac{\text{tangential stress}}{\text{shearing strain.}} = \frac{F/a}{\theta} = \frac{F/a}{x/L} = \frac{FL}{ax}$ **N/m²**

Longitudinal Strain co efficient (α):

If x/L is the Longitudinal strain produced due to Longintudinal stress T

The from Hookes law

$x/L \propto T$

$x/L = \alpha\, T$

Where α is called as Longitudinal strain co efficient.

Logitudinal strain co efficient (α) is defined as the Longitudinal strain produced per unit stress.

Lateral Strain co efficient (β):

When an elastic material subjected to tensile stress it produces longitudinal strain as well as lateral strain in the direction perpendicular to longitudinal strain in the material.

Lateral strain is defined as the ratio of change in the diameter to original diameter.

If 'd' is the change in the diameter and 'D' is the original diameter then

Lateral strain = d/D

If d/D is the lateral strain produced due to Longintudinal stress T

The from Hookes law

$d/D \propto T$

$d/D = \beta\, T$

Where β is called as Lateral strain co efficient.

Lateral strain co efficient (β) is defined as the Lateral strain produced per unit stress.

Poisson's Ratio (σ)

It is commonly observed fact that when we stretch string or wire it becomes longer but thinner. That is an increase in length is always accompanied by a decrease in its cross section. In other words a linear or a tangential strain produced in a wire is accompanied by a transverse or lateral strain of opposite kind in a direction right angle to the direction of applied force. This change which occurs in a direction perpendicular to the direction along which the deforming force is acting is called **lateral change**.

***Within elastic limits of a body, the ratio of lateral strain (β) to the longitudinal strain (α) is a constant and is called Poisson's ratio* (σ).**

If a deforming force acting on a wire of length 'L' produces a change in length 'x' accompanied by a change in diameter of 'd' in it which has a original diameter of 'D'

$$\therefore \textbf{ Poisson's ratio, } \sigma = \frac{\textit{Lateral strain}}{\textit{Longitudinal strain}} = \frac{Ld}{xD}$$

The lateral strain coefficient $\beta = \frac{d}{D}$ and Longitudinal strain coeficient $\alpha = \frac{x}{L}$,

$$\therefore \textbf{ Poisson's ratio, } \sigma = \frac{Ld}{xD} = \frac{\beta}{\alpha}$$

Poisson's ratio is a dimensionless quantity.

Limiting value of Poisson's ratio:

W.K.T.

$$Y = 2\eta\,(1 + \sigma), \text{ and } Y = 3K\,(1-2\sigma)$$

$$\therefore\ 2\eta\,(1 + \sigma) = 3K\,(1\text{-}2\sigma)$$

where k & η are essentially positive quantities.

(i) If σ be positive quantity, then the right hand side and left hand side expression must be positive. i.e. $(1\text{-}2\sigma) > 0$

$$\text{Or } 2\sigma < 1$$

$$\therefore \sigma < 0.5$$

(ii) If σ is A negative quantity, then the right hand side and left hand side expression must be positive. i.e. $(1 + \sigma) > 0$

$$\text{Or } \sigma > -1$$

Thus the limiting value of Poisson's ratio is $-1 > \sigma < 0.5$

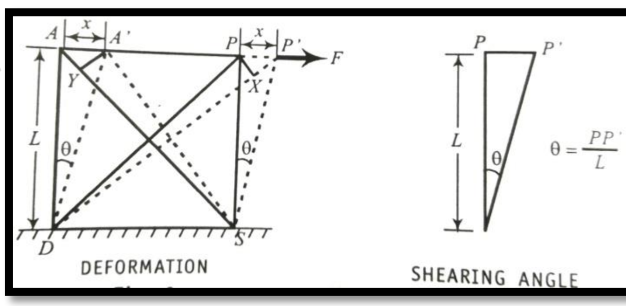
If σ = 0.5, then Bulk modulus is infinite. It means that, the substance is perfectly incompressible. Actually there is no substance which perfectly incompressible.

If σ = -1, the rigidity modulus is infinite. It means that, if a body is extended linearly, then it should also extend laterally. No substance exhibits such phenomenon.

In actual practice, the value of σ varies from 0.2 to 0.4 **(Practical limit)**

Relation between Y, n and σ

Consider a cube with each of its sides of length L under the action of tangential stress T. let tangential force F be applied to its upper face. It causes the plane of the faces perpendicular to the applied force F turn through an angle θ. as a result diagonal AC undergoes contraction and diagonal DB^1undergoes elongation of equal amount.



Now, shearing strain occurring along AP can be treated as equivalent to a longitudinal strain, along DP' and an equal lateral strain along the diagonal A'S i.e., perpendicular to DP. Let α and β be the longitudinal strain co efficient and lateral strain co efficients

Since 'T' is the applied stress, therefore extension produced for the length DP due to tensile stress produces the longitudinal and lateral strain given by

Longitudinal Strain = T. DP.α

Lateral strain = T.DP.β.

$\therefore$ Total extension = Longitudinal strain along DP + Lateral strain perpendicular to DP

$\therefore$ P'X = DP.T.(α+β),

P'X=PP' cos (PP'X)

but ∠ PP'X= ∠AP'D = ∠ APD = 45º

$\therefore\ P'X = PP'\cos 45^\circ = \dfrac{PP'}{\sqrt{2}}$

Let PP'= x

$P'X = \dfrac{x}{\sqrt{2}}$

And DP = $\sqrt{2}\,L$

$\therefore$ P'X=($\sqrt{2}\,L$).T(α+β),

Rearranging the terms

$$\therefore (\sqrt{2}L)\,T(\alpha+\beta) = \frac{x}{\sqrt{2}},$$

$$\text{Or, } \frac{1}{2}\frac{1}{(\alpha+\beta)} = \frac{TL}{x} = \frac{T}{(x/L)} = \frac{T}{\theta} = \eta$$

$$\therefore \eta = \frac{1}{2(\alpha+\beta)},$$

$$= \frac{1}{2\alpha(1+\beta/\alpha)},$$

$$\text{Or, } \eta = \frac{1/\alpha}{2(1+\sigma)}, \quad (\because \sigma = \beta/\alpha),$$

$$\eta = \frac{Y}{2(1+\sigma)}$$

$$or\ \ Y = 2\eta(1+\sigma)$$

Relation between Y and α

Consider a cube of unit side subjected to unit tension along one side. Let α be the elongation per unit length per unit tension along the direction of the force. Therefore,

$$\text{Stress} = \frac{\text{Force}}{\text{Area}} = 1$$

$$\text{Similarly, linear strain} = \frac{\text{change in length}}{\text{original length}} = \frac{\alpha}{1} = \alpha$$

$$\therefore \quad \mathbf{Y} = \frac{\text{Normal stress}}{\text{Longitudinal strain}} = \frac{1}{\alpha}$$

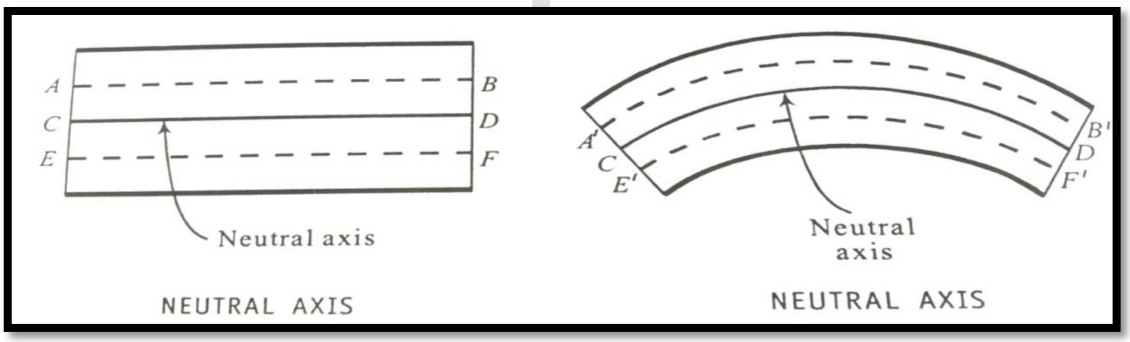
Mention the Relation between K, Y and σ

$Y=3K(1-2\sigma)$

BENDING OF BEAMS

A homogenous body of uniform cross section whose length is large compared to its other dimensions is called a beam.

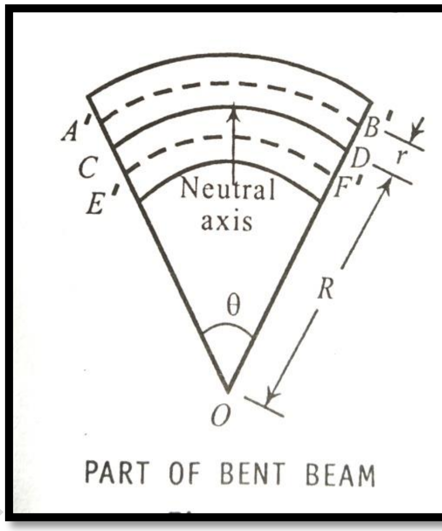
Neutral Surface: It is that layer of a uniform beam which does not undergo any change in its dimensions, when the beam is subjected to bending within its elastic limit.



Neutral axis: It is a longitudinal line along which neutral surface is intercepted by any longitudinal plane considered in the plane of bending.

Bending moment of a beam:

Consider a uniform beam whose one end is fixed at M. If now a load is attached to the beam, the beam bends. The successive layers are now strained. A layer like AB which is above the neutral surface will be elongated to A' B' and the one like EF below neutral surface will be contracted to E'F'. CD is neutral surface which does not change



The shape of each layers of the beam can be imagined to form part of concentric circles of varying radii. Let R be the radius of the circle to which the neutral surface forms a part.

$\therefore$CD=Rθ

where 'θ' is the common angle subtended by the layers at common center O of the circles. The layer AB has been elongated to A^1B^1.

$\therefore$ Change in length = A' B' -AB

But AB=CD=Rθ

If the successive layers are separated by a distance r then,

$$A'B' = (R+r)\theta$$

$\therefore$Change in length=(R+r)θ-Rθ = rθ

But original length = AB=Rθ

$$\therefore \text{Linear strain} = \frac{r\theta}{R\theta} = \frac{r}{R}$$

Youngs Modulus Y= Longitudinal stress/linear strain

Longitudinal stress = Y× Linear strain

$$= Y \times \frac{r}{R}$$

$$\text{But stress} = \frac{F}{a}$$

Where F is the force acting on the beam and a is the area of the layer AB.

$$\frac{F}{a} = \frac{Yr}{R} = \frac{Yar}{R}$$

Moment of this force about the neutral axis=F x its distance from neutral axis.

$$= F \times r = \frac{Yar^2}{R}$$

Moment of force acting on the entire beam = $\Sigma \frac{Yar^2}{R}$

$$= \frac{Y}{R} \sum ar^2$$

The moment of inertia of a body about a given axis is given by Σmr^2, where Σm is the mass of the body. Similarly Σar^2 is called the geometric moment of Inertia Ig.

Ig=Σar^2 =Ak^2 , where A is the area of cross section of the beam and k is the radius of gyration about the neutral axis.

$$\text{Moment of force} = \frac{Y}{R} Ig$$

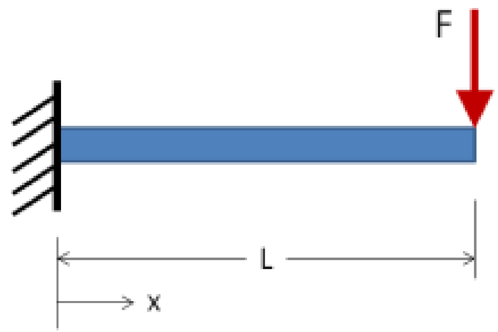
1) Bending moment of rectangular beam = $\frac{Y}{R} \frac{bd^3}{12}$

2) Bending moment of circular beam = $\frac{Y}{R} \frac{\pi r^4}{4}$

Cantilever and I Section Girders:

Cantilever:

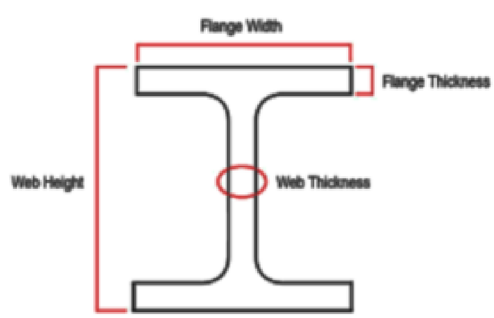
The cantilever is a beam fixed at one end and free at the opposite end. Structure : Typically it is firmly attached to a flat vertical surface like wall and extends from it outwards. It distributes the load back to support that force against a flash and shear stress.



Applications : Cantilever construction allows overhanging structures without additional supports and bracing. A cantilever beam allows the creation of a bay window, some bridges, balconies, truss, or slab.

I Section Girders:

I-shaped girders are made like the I-beam section and are composed of couple of load bearing flanges and web. Structure and Purpose The top and bottom flanges of the "I" shape provide horizontal support and resistance against bending, while the vertical web connects the two flanges and provides resistance against shearing forces. The "I" shape of steel girders and rails provides structural efficiency, material efficiency, and cost-effectiveness. Uses They are used in building of pulls, fly overs etc. They are also used in construction of public shades and public places.



Types of Elastic Materials :

- **Linear Elastic Materials:** On loading exhibits a linear graph between of stress vs strain passing through the origin. Eg. Metals.
- **Non-Linear Elastic Material:** On loading Exhibits non linear graph of stress vs strain passing through the origin. Eg. Large strain rubber or Plastic.
- **Cauchy-elastic material:** In this material the stress at each point is determined only by the current state of deformation with respect to an arbitrary reference configuration.
- **Hyper-elastic materials:** They are conservative models that are derived from a strain energy density function.
  Ex: Bridge and Train pads, tyres.
- **Hypo-elastic Materials:** Hypo-elasticity implies that stress is not derivable from an energy potential.
- **Elastomers:** Rubbery material composed of long chain-like molecules (polymers) that are capable of recovering their original shape after being stretched to great extents. eg. Polyurethane.
  Some Examples of Elastic materials are Thermo Plastic Elastomer, Silicone Rubber Elastin, Poly Amide (Nylon), Natural Gum, Wool, Lycra

Elastic Fatigue and Fracture:

Consider an Elastic material which is subjected to number of cycles of loading and unloading. Thus, the material experiences repeated strain and relief. During the course the material experiences " Elastic tiredness" or "Elastic Fatigue". A material experiencing fatigue develops cracks which grow over cycles and the materials undergoes fracture and hence called fatigue failure.

Thus, the material (part) fails during the operation. Hence fatigue is an important concern in the design of some mechanical instruments and devices like Engine Pistons, Suspension springs etc.

Fracture is classified into two types.

• Brittle fracture

• Ductile fracture

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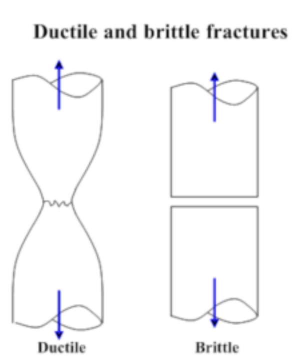

Brittle Fracture:

Brittle fracture means fracture of material without or with very small plastic deformation before fracture. Eg. Rock, concrete, and cast iron. Such materials are called brittle materials.

Ductile Fracture:

Ductile fracture is a type of failure seen in readily deformable (malleable) materials and is characterized by extensive plastic deformation or necking that occurs before the material finally cracks or breaks apart. Eg. Aluminium, Copper. Such materials are called ductile materials.

Stress Concentration:

A stress concentration is a point in a part where the stress is significantly greater than its surrounding area. Stress concentrations occur as a result of irregularities in the geometry or within the material of a component structure types of Elastic Materials that cause an interruption of the stress flow. These interruptions typically arise from discontinuities such as holes, grooves, notches and fillets. Stress concentrations may also be caused by accidental damage such as nicks and scratches.

Concentration Factor:

A stress concentration factor (Kt) is a dimensionless factor that is used to quantify how concentrated the stress is in a mechanical part. It is defined as the ratio of the highest stress in the part compared to a reference stress (Nominal Stress).

$$K_t = \frac{S_{max}}{S_{ref}}$$

Factors Affecting Fatigue:

Surface Effect:

There are always uneven machining marks on the machinated surface. These marks are equivalent to tiny gaps, which cause stress concentration on the surface of the material, thus reducing the fatigue strength of the material.

Design Effect:

Any notch or geometrical discontinuity can act as a stress raiser and fatigue crack initiation site; these design features include grooves, holes, keyways, threads, and so on. The sharper the discontinuity (i.e., the smaller the radius of curvature),the more severe the stress concentration.

Environmental Effects:

Two types of environment-assisted fatigue are thermal fatigue and corrosion fatigue. Thermal Fatigue Thermal fatigue is normally induced at elevated temperatures by fluctuating thermal stresses. Corrosion Fatigue Failure that occurs by the simultaneous action of a cyclic stress and chemical attack is termed corrosion fatigue. Corrosive environments have a deleterious influence and produce shorter fatigue lives.

Important questions:

1. State Hooke's Law and Explain Stress Vs Strain curve with the help of a neat sketch.
2. Discuss Strain Hardening and Softening.
3. Define Elastic Modulus and Explain the three types of Elastic Moduli.
4. Explain Longitudinal and Lateral Strain coefficients and hence define Poisson's ration.
5. Derive the relation between y, n and σ.
6. Discuss the limiting values of Poisson's Ratio.
7. Define Beam and explain the classification of beams.
8. Define Bending Moment, Restoring Moment, Neutral Surface, Neutral Axis.
9. Derive an expression for the Bending Moment of a beam.
10. Describe Cantilever and I section Girder with the held of neat sketches.
11. Discuss the types of elastic material.
12. Elucidate the stress concentration and concentration factor.
13. Discuss the Surface, Design and Environment factors affecting Fatigue.