

DC Generator : Principle of Operation, Constructional details, Induced emf expression, Types of generators, Relation between induced emf and terminal voltage, Simple numerical.

DC Motor : Principle of operation, back emf and its significance, Torque equation, Type of motors, characteristics and speed control (Armature & field) of DC Motors (Series and shunt only), Applications of DC Motors, Simple Numerical.

DC Machines

An electrical machine deals with the energy transfer either from mechanical to electrical or electrical to mechanical form.

DC ~~Motor~~ Generator

A dc generator is ~~an~~ machine which converts mechanical energy to electrical energy. The electrical energy is generated in the form of alternating voltage in the windings of the generator. This alternating voltage is converted to direct voltage by commutator.

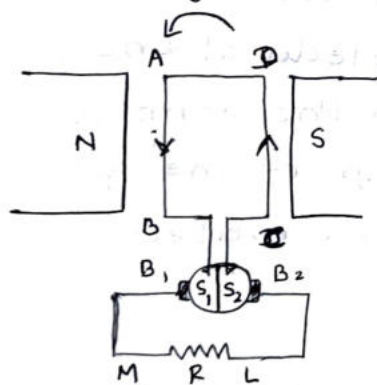
Working Principle of DC Generator

A dc generator works on the principle of Faraday's laws of Electromagnetic induction. According to this law, whenever the flux linked with a coil changes, an emf is induced in it. The magnitude of the induced emf is equal to the rate of change of flux linkage.

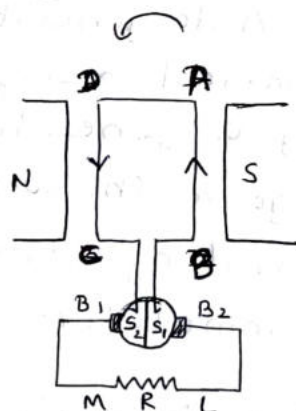
When the armature conductors are rotating in the magnetic field, an emf is induced in the armature conductor. The emf induced here is sinusoidally induced emf and is sinusoidal in

mature. This emf causes a current to flow, if the conductor circuit ~~is~~ is closed. In dc generator, this alternating emf is converted to direct voltage by commutator.

For simplicity consider a single turn rectangular copper coil ABCD, rotating in a stationary magnetic field. The ends of the coil are connected to the split rings. The split ring has two separate segments S_1 & S_2 separated by an insulating medium. Two brushes B_1 & B_2 just slide on the ring segments whose function is to collect the current induced in the coil. The function of the commutator (split rings) is to ~~make~~ convert the alternating current to dc.



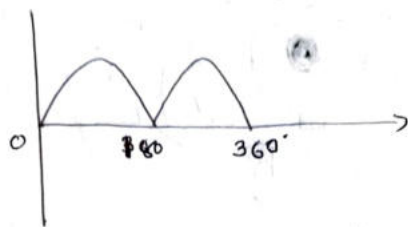
(i) $\theta \rightarrow 0^\circ - 180^\circ$



(ii) $\theta \rightarrow 180^\circ - 360^\circ$

When this coil ABCD starts rotating at a uniform speed in the magnetic field in anticlockwise direction, conductor AB rotates under the influence of north pole & conductor CD rotates under the influence of south pole. The direction of the current is shown in fig. ~~For dc generator~~ The direction of current can be obtained by Fleming's right hand rule.

The alternating emf induced is converted to dc by commutator action. The current flowing through the external resistance is same in both the cases. i.e. from $M \rightarrow L$. For rotation $0-180^\circ$ & $\theta=180^\circ-360^\circ$. Hence the split rings (commutator) converts the alternating current to unidirectional (direct) current.



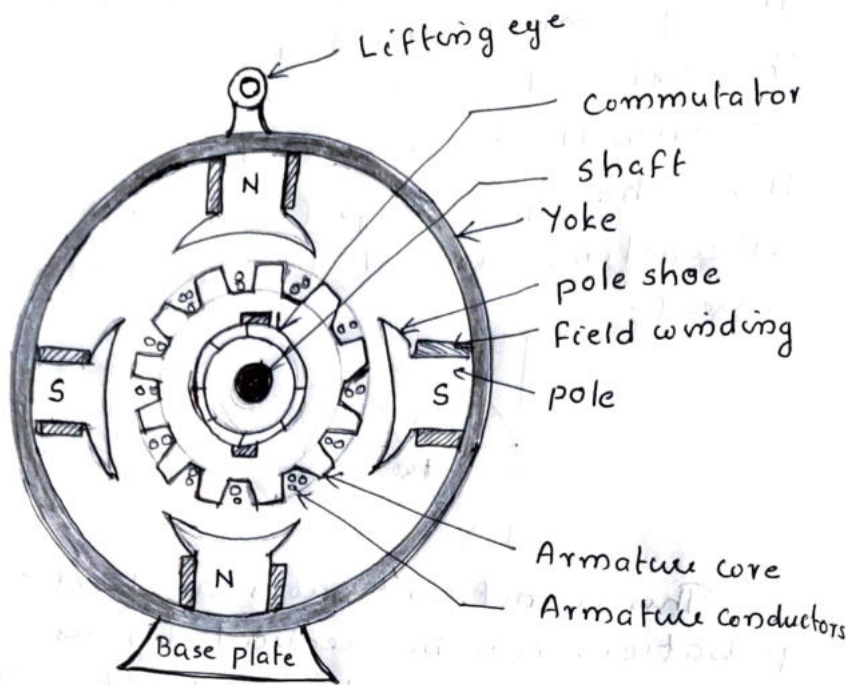
This is not continuous as direct current. The pulsations can be reduced by using many conductors on the armature. The resultant emf is thus nearly steady.

Constitutional Features of DC Generator.

The essential or main parts of a DC generator are as follows.

1. Field System
 - Yoke
 - pole cores & pole shoes
 - field coils.
2. Armature core and windings
3. Commutator
4. Brushes.

These parts of DC generator are explained below.



1. The Field System

The field system consists of the following parts (i) Yoke (ii) Pole core & Pole shoes (iii) Field coils

The field system creates a uniform magnetic field, within which the armature rotates. When direct current is passed through the field coil, the pole cores & shoes get magnetised and a magnetic field is produced.

(i) Yoke

It is cylindrical in shape to which even number of poles are bolted. The yoke is made up of cast iron and cast steel. The function of yoke is

- (i) to provide path for magnetic flux
 - (ii) to provide mechanical support to the poles and to serve as the outer cover of the generator.
- (ii) Pole core & pole shoes

The pole cores and pole shoes are fixed to the yoke, or magnetic frame by bolts. They are made of cast iron or cast steel.

Each pole is fitted ~~to the pole~~ with an extension known as pole shoes. The function of poles and pole shoes are

- (i) To support the field coil
- (ii) to spread out the flux in the air gap.
- (iii) Field coils (exciting coil)

Field coils are made of copper wires. It is provided on the poles. When direct current is passed through the field coils, pole cores and shoes get magnetised and a uniform magnetic field is produced.

2. Armature Core and Armature windings

The armature core is cylindrical in shape and is made up of silicon steel laminations. There are slots cut uniformly over the outer periphery of the armature core and armature windings are placed in these slots. The armature conductors are not only insulated from one another, but also from the armature slots.

3. Commutator

The commutator converts the alternating current induced in the armature conductors into unidirectional current in the external load circuit. The commutator is cylindrical in shape and consists of a number of copper segments, each segment insulated by a thin layer of mica.

4. Brushes

In small machines, the current collecting brushes are made of carbon & in big machines, they are made of copper carbon alloy or copper. They rest on the commutator segments. The function of the brushes is to collect the current from the armature windings & deliver the same to the external circuit.

Base Plate enables the generator to be placed conveniently on ground or any platform. The lifting eye is used to lift the generator to shift it from one place to another.

Armature Windings

The armature conductors are placed in the armature slots and are suitably connected.

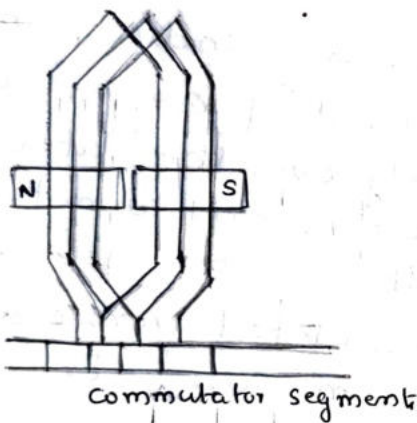
The number of armature conductors are connected in a specific manner to form armature windings.

According to the way of connecting the conductors, there are two types of armature windings. (i) Lap winding.

(ii) Wave winding.

Comparison of Lap winding and Wave winding

Lap Winding

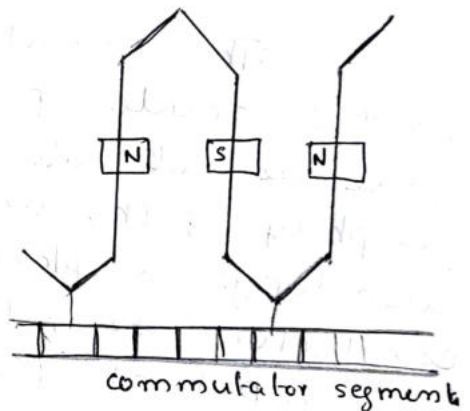


(2) This winding is called as lap winding because when the winding progresses, it appears as if one coil is lapping over the other.

(3) No. of parallel paths,
 $A = \text{No. of poles}$
(i.e. $A = P$)

4. Lap winding is used when a low voltage, high current supply is required.

Wave Winding



(2) This winding is called as wave winding because when the winding progresses, it appears like a wave.

(3) No. of parallel paths
 $A = 2$.

(4) Wave winding is used when a high voltage, low current supply is required.

EMF Equation of DC Generator.

Let $Z \rightarrow$ Total No. of armature conductors.

$\phi \rightarrow$ useful flux/pole in Webers

$N \rightarrow$ speed of the armature in rpm

$P \rightarrow$ No. of poles

$A \rightarrow$ No. of parallel paths.

The flux cut by the conductor in one revolution
 $= \phi P$

The time taken by the conductor to make one revolution $= \frac{60}{N}$ seconds.

emf induced in one conductor $= \frac{Nd\phi}{dt}$

$$= 1 \cdot \frac{P\phi}{60/N}$$

$$E_g = \frac{\phi P N}{60} \text{ volts}$$

emf induced/parallel path

$$= (\text{emf induced/conductor}) \times (\text{No. of conductor/parallel path})$$

$$E_g = \frac{\phi P N}{60} \times \frac{Z}{A}$$

$$\boxed{E_g = \frac{\phi P N Z}{60 A}} \quad \text{--- (1)}$$

Equation (1) is the emf equation of a DC generator. From the emf equation it is clear that the emf induced depends upon the flux and speed of the generator.

Problems

- 1) An 8 pole DC generator has 500 armature conductors & has useful flux per pole of 0.045 Wb. What will be emf generated if it is lap connected & runs at 1000 rpm. What must be the speed at which it is to be driven to produce the same emf if it is wave connected.

Solution

$$P = 8, Z = 500, \phi = 0.045 \text{ Wb}$$

$$N = 1000 \text{ rpm}$$

(i) Lap connected $\therefore A = P$

$$E_g = \frac{\phi Z N P}{60 A}$$

$$= \frac{0.045 \times 500 \times 1000 \times 8}{60 \times 8}$$

$$= \underline{541.67 \text{ V}}$$

$$E_g = 541.67 \text{ V}$$

(ii) Wave connected, $A = 2$

$$E_g = 541.67 \text{ V}$$

$$N = ?$$

$$E_g = \frac{\phi Z N P}{60 A}$$

$$N = \frac{E_g \times 60 \times A}{\phi Z P}$$

$$= \frac{541.67 \times 60 \times 2}{0.045 \times 500 \times 8} = 250 \text{ rpm}$$

$$N = 250 \text{ rpm.}$$

2. An 8 pole, lap connected armature has 40 slots with 12 conductors per slot, generates a voltage of 500V. Determine the speed at which it is running if the flux per pole is 50 mWb.

Soln

$P = 8$ Lap connected $\therefore A = P = 8$

slots = 40 conductors/slots = 12

$$\bullet \text{ No. of conductors} = \text{slots} \times \text{conductors/slot} \\ = 40 \times 12 = 480$$

$$\phi = 50 \text{ mWb} \\ = 50 \times 10^{-3} \text{ Wb}$$

$$E_g = 500 \text{ V}$$

$$N = ?$$

$$E_g = \frac{\phi Z P N}{60 A}$$

$$N = \frac{E_g \times 60 A}{\phi Z P} = \frac{500 \times 60 \times 8}{50 \times 10^{-3} \times 480 \times 8} \\ = \underline{\underline{1250 \text{ rpm}}}$$

3. A 4 pole generator with wave wound armature has 51 slots, each having 24 conductors. The flux per pole is 0.01 Wb . At what speed must the armature rotate to give an induced emf of 220 V . What will be the voltage developed if the winding is lap. & the armature rotates at the same speed.

Solution

$$P = 4$$

conductors per slot = 24

$$\text{slots} = 51$$

$$\phi = 0.01 \text{ Wb}$$

Total No. of conductors = No. of slots $\times$ Conductors/slot

$$Z = 51 \times 24 = \underline{\underline{1224}}$$

$$E_g = 220 \text{ V}$$

(i) Wave Connected $\therefore A = 2$

$$E_g = \frac{\phi Z P N}{60 A}$$

$$N = \frac{E_g \times 60 A}{\phi Z P} = \frac{220 \times 60 \times 2}{0.01 \times 1224 \times 4}$$

$$N = \underline{\underline{540 \text{ rpm}}}$$

(ii) Lap Connected $\therefore A = P = 4$

$$N = 540 \text{ rpm}$$

$$E_g = \frac{\phi Z P N}{60 A} = \frac{0.01 \times 1224 \times 4 \times 540}{60 \times 4} = \underline{\underline{110.16}}$$

$$E_g = \underline{\underline{110.16 \text{ V}}}$$

4. An 8 pole, lap connected armature has 960 ~~slots~~ conductors, flux of 40 mWb per pole and speed of 400 rpm. Calculate the emf generated. If the armature is wave connected, at what speed must it be driven to generate 400 V.

Solution

(i) $P = 8$ Lap connected $\therefore A = P = 8$

$$Z = 960$$

$$\phi = 40 \text{ mWb} = 40 \times 10^{-3} \text{ Wb}$$

$$N = 400 \text{ rpm}$$

$$E_g = \frac{\phi Z P N}{60 A}$$

$$= \frac{40 \times 10^{-3} \times 960 \times 8 \times 400}{60 \times 8}$$

$$= \underline{\underline{256 \text{ V}}}$$

(ii) Wave Connected $A = 2$

$$E_g = 400 \text{ V}$$

$$E_g = \frac{\phi Z P N}{60 A}$$

$$N = \frac{E_g \times 60 A}{\phi Z P}$$

$$= \frac{400 \times 60 \times 2}{40 \times 10^{-3} \times 960 \times 8}$$

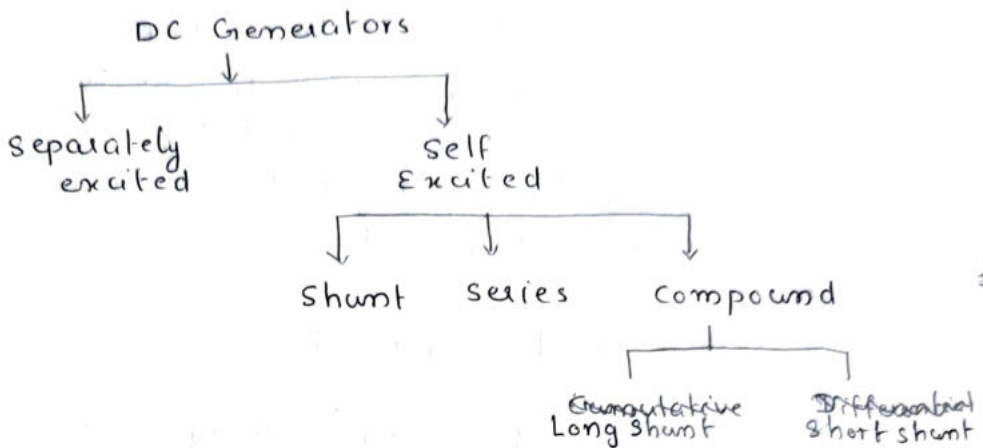
$$= \underline{\underline{156 \text{ rpm}}}$$

$$\boxed{N = 156 \text{ rpm}}$$

Types of DC Generators

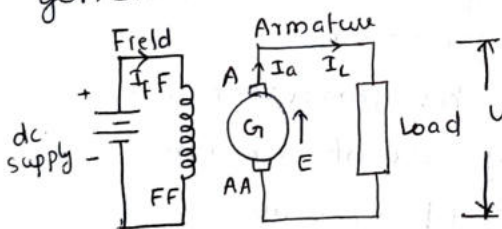
DC generators are classified based on how their fields are excited.

Based on field excitation methods, the dc generators are classified as follows.



separately Excited DC Generator

When the field windings of a dc generator is excited by an external, separate dc supply, then the generator is called as separately excited dc generator.



The field winding of this type of generator consists of large number of turns with thin wire. So the resistance of the field winding is high in order to limit the field current.

From the circuit, we can see that

Armature current = Load current

$$I_a = I_L$$

If

$V \rightarrow$ The terminal voltage

$E =$ emf induced

$I_a \rightarrow$ armature current

$R_a \rightarrow$ armature resistance

Then

'Voltage Equation is

$$E = V + I_a R_a + V_{\text{brush}}$$

where

$E \rightarrow$ Induced emf

$$E = \frac{\phi P Z N}{60 A}$$

$I_a R_a \rightarrow$ Armature resistance drop

$V_{\text{brush}} \rightarrow$ Brush contact drop at the commutator

Generally Brush contact drop will be negligible. It can be neglected.

Self Excited DC generator

When the field winding is supplied from the armature of the generator itself then it is called as self excited generator. Unless the generator produce some voltage, such generators cannot be excited.

This is possible by residual magnetism in case of self excited generators.

Practically, though the generator is not working without any current through the field winding the field poles possess some magnetic flux. This is called as residual flux and this property is called as residual magnetism.

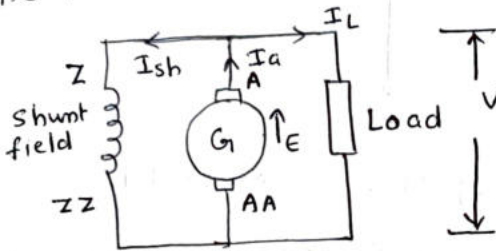
Thus when the generator is started, due to the residual flux, it develops a small emf which drives a small current through the field winding. This tends to increase the flux produced. This in turn increases the emf induced. This process continues till the generator develops the rated voltage across the armature.

The different type of Self excited DC generators are

- (i) DC Shunt generator
- (ii) DC series generator
- (iii) DC compound generator

DC Shunt Generator

In DC shunt generator, the field winding is connected in parallel with the armature winding. The load is connected across the armature terminal.



The field winding of shunt generator has large number of turns of thin wire. So, it has high resistance.

Let $R_{sh} \rightarrow$ Resistance of the ^{shunt} field winding

$I_a \rightarrow$ Armature current

$I_{sh} \rightarrow$ Field current

$I_L \rightarrow$ Load current

From the circuit

$$I_a = I_L + I_{sh}$$

The Field current I_{sh} can be written as

$$I_{sh} = \frac{V}{R_{sh}}$$

The Voltage equation can be written as

$$E = V + I_a R_a + V_{brush}$$

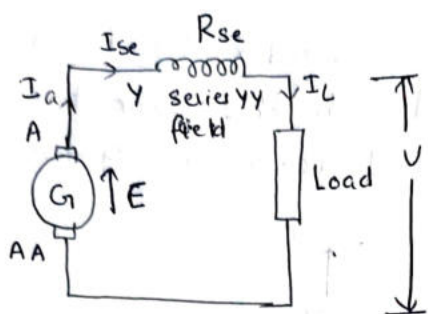
where E is induced emf $E = \frac{\phi Z N P}{60 A}$

$I_a R_a \rightarrow$ Armature resistance drop

$V_{brush} \rightarrow$ Brush contact drop.

DC Series Generator

When the field winding is connected in series with the armature winding, then that generator is called series generator.



The resistance of the series field winding is very small with less number of turns, and thick cross section of wire.

Let $R_{se} \rightarrow$ Resistance of the series field winding

$I_a \rightarrow$ Armature current

$I_{se} \rightarrow$ Field current

$I_L \rightarrow$ Load current.

From the circuit,

$$I_a = I_{se} = I_L$$

The voltage equation is

$$E = V + I_a R_a + I_{se} R_{se} + V_{brush}$$

where $E \rightarrow$ induced emf

$$E = \frac{\phi P Z N}{60 A}$$

$I_a R_a \rightarrow$ Armature resistance drop

$I_{se} R_{se} \rightarrow$ Series field resistance drop

$V_{brush} \rightarrow$ Brush contact drop.

DC Compound Generator

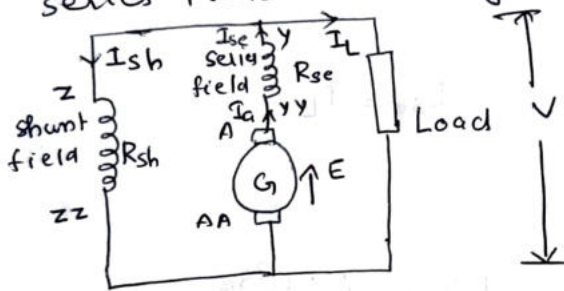
In this type, the part of field winding is connected in parallel with armature and a part in series with the armature.

Depending upon the connection of shunt & series field windings, compound generators are further classified into

1. Long shunt Compound Generator
2. Short shunt Compound Generators

Long Shunt Compound Generator

In this type, the shunt field winding is connected across the entire series combination of armature and series field winding.



From the circuit

$$I_a = I_{se}$$

$$I_a = I_{sh} + I_L$$

$$I_{sh} = \frac{V}{R_{sh}}$$

$R_{sh} \rightarrow$ Shunt field Resistance

$\therefore$ Voltage Equation

$$E = V + I_a R_a + I_{se} R_{se} + V_{brush}$$

where $E = \frac{\Phi P Z N}{60 A}$

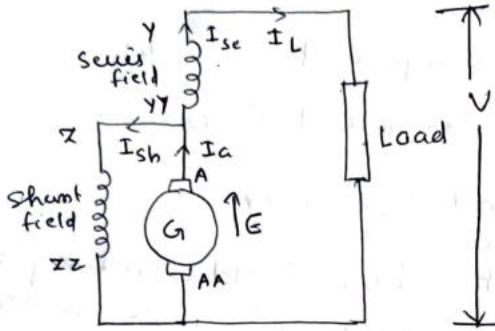
$I_a R_a \rightarrow$ Armature resistance drop

$I_{se} R_{se} \rightarrow$ Series field resistance drop

$V_{brush} \rightarrow$ Brush contact drop.

Short Shunt Compound Generator

In compound generator, if the shunt field winding is connected only across the armature excluding the series field winding, then it is called as short shunt compound generator.



From the circuit

$$I_a = I_{se} + I_{sh}$$

$$I_{se} = I_L$$

or

$$I_a = I_L + I_{sh}$$

$$I_{sh} = \frac{E - I_a R_a}{R_{sh}}$$

$$I_{sh} = \frac{V + I_L R_{se}}{R_{sh}}$$

Voltage Equation is

$$E = V + I_a R_a + I_{se} R_{se} + V_{brush}$$

As $I_{se} = I_L$, This can be written as

$$E = V + I_a R_a + I_L R_{se} + V_{brush}$$

$$E = \frac{\phi P Z N}{60 A}$$

$I_a R_a \rightarrow$ Armature resistance drop

$I_{se} R_{se} \rightarrow$ series field resistance drop

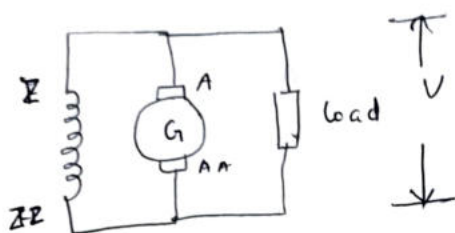
$V_{brush} \rightarrow$ brush contact drop.

Problems

1. The emf generated in the armature of a shunt generator is 625V, when delivering its full load current of 400A to the external circuit. The field current is 6A and armature resistance is 0.06Ω . What is the terminal voltage.

Solution

Shunt generator



$$E = 625V$$

$$I_L = 400A$$

$$I_{sh} = 6A \quad R_a = 0.06\Omega$$

$$I_a = I_L + I_{sh}$$

$$= 400 + 6 = 406A$$

Q Voltage Equation

$$E = V + I_a R_a + V_{brush}$$

V_{brush} is neglected.

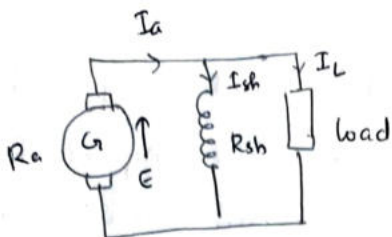
$$V = E - I_a R_a$$

$$= 625 - (406 \times 0.06)$$

$$= \underline{\underline{600.64V}}$$

2. A 30kW, 300V DC shunt generator has armature and field resistance of 0.05Ω & 100Ω respectively. Calculate the total power developed by the armature when it delivers full output power.

Shunt Generator.



$$I_L = \frac{P}{V} = \frac{30 \times 10^3}{300} = 100 \text{ A}$$

$$I_{sh} = \frac{V}{R_{sh}} = \frac{300}{100} = 3 \text{ A}$$

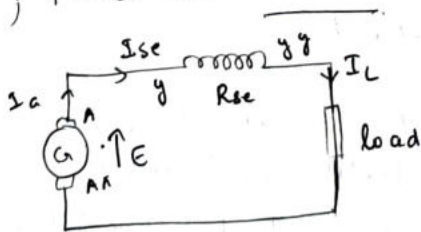
$$I_a = I_L + I_{sh} \\ = 100 + 3 = 103 \text{ A}$$

$$E = V + I_a R_a \\ = 300 + (103 \times 0.05) \\ = \underline{\underline{305.15 \text{ V}}}$$

Total Power developed by the armature

$$= E I_a \\ = 305.15 \times 103 \\ = \underline{\underline{31.4304 \text{ kW}}}$$

3. A dc series generator has armature resistance of 0.5Ω and series field resistance of 0.03Ω . It drives a load of 50 A . If the emf induced is 324 V and total brush drop is 2 V ; Find the terminal voltage



$$I_a = I_{se} = I_L = 50 \text{ A} \quad E = 324 \text{ V}$$

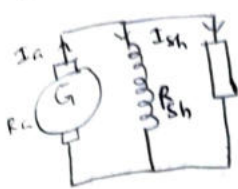
$$V_{brush} = 2 \text{ V}$$

$$E = V + I_a R_a + I_{se} R_{se} + V_{brush}$$

$$V = E - (I_a R_a + I_{se} R_{se} + V_{brush})$$

$$= 324 - (50 \times 0.5) - (50 \times 0.03) - 2 = \underline{\underline{295.5 \text{ V}}}$$

4. A 4 pole, 100V shunt generator with lap connected armature having field & armature resistance of 50Ω & 0.1Ω respectively, supplies a load of 60 lamps each lamp rated 100V, 40W. Calculate the armature current, the current per path & The generated emf. Allow a contact drop of 1V/brush.



$$\text{Total load} = 60 \times 40 = 2400W$$

$$\text{current through load } I_L = \frac{P}{V} = \frac{2400}{100} = 24A$$

$$I_{sh} = \frac{V}{R_{sh}} = \frac{100}{50} = 2A$$

$$(i) I_a = I_L + I_{sh} = 24 + 2 = 26A$$

$$(ii) \text{ Current/Path} = I_a / A = \frac{26}{4} = 6.5A \quad (\because \text{lap winding } A = P = 4)$$

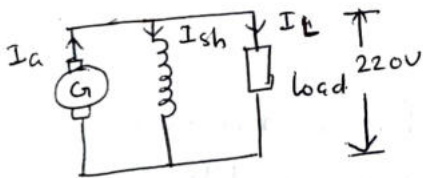
$$(iii) E_g = V + I_a R_a + BCD$$

$$= 100 + (26 \times 0.1) + 2 \times 1$$

$$V_{brush} = 2 \times 1 = 2V$$

$$E_g = \underline{\underline{104.6V}}$$

5. A 220V, 8.8kW, 1000 rpm dc shunt generator has armature resistance of 0.5Ω & field winding resistance of 110Ω . The brush drop is 1V/brush. Find the armature voltage at full load.



$$I_L = \frac{P}{V} = \frac{8.8 \times 10^3}{220} = 40A$$

$$I_{sh} = \frac{V}{R_{sh}} = \frac{220}{110} = 2A$$

$$I_a = I_L + I_{sh} = 40 + 2 = 42A$$

$$E_g = V + I_a R_a + V_{Brush}$$

$$= 220 + (42 \times 0.5) + 1 \times 2 = \underline{\underline{243V}}$$

DC Motor

A dc motor is a machine which converts electrical energy into mechanical energy. In construction, it is similar to DC generator. Hence a DC Motor can be used as a dc generator & vice versa.

Working Principle of a ^{DC} Motor

A DC Motor works on the principle that whenever a current carrying conductor is placed in a magnetic field, it experiences a force.

The magnitude of the force experienced by the conductor is given by

$$F = BIL$$

where $B \rightarrow$ flux density in Wb/m^2

$I \rightarrow$ current through the conductor in Ampere

$l \rightarrow$ Length of conductor in m

If the conductor is placed at an angle

$$F = BIL \sin \theta$$

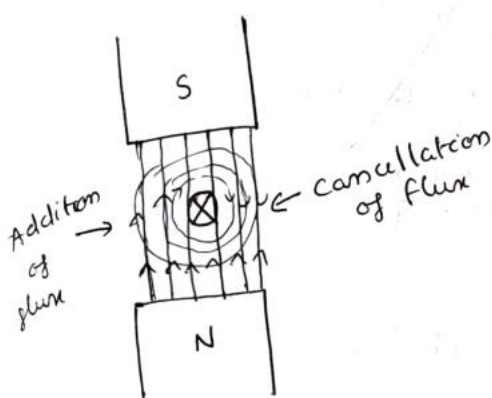
The direction of the force experienced by the conductor is given by Fleming's left hand rule.

When the thumb, forefinger and middle finger are held mutually perpendicular to each other, then the forefinger gives the direction of magnetic field; the middle finger gives the direction of the current & then the thumb represents the direction of the force experienced by the conductor.

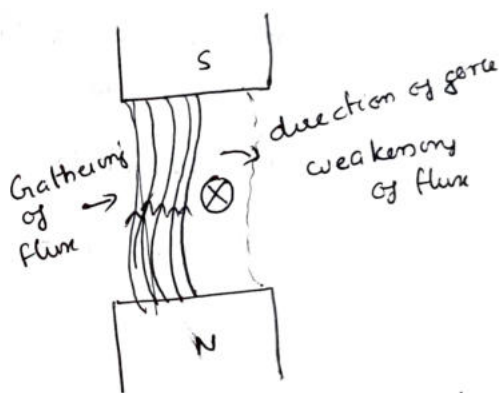
~~For~~ Consider a simple conductor placed in a magnetic field

Let the conductor be excited by a separate supply so that it carries current in a particular direction. Let the ^{direction of} current be away from the observer. ~~Thus~~ The current through the coil set up a magnetic field around the conductor. The direction of this magnetic field can be obtained by right hand thumb rule.

Now there are two flux present. (1) The flux produced by the main poles, (2) Flux produced by the current carrying conductor. These are shown in fig. Due to the interaction between these two fluxes a resultant field is set up as shown in fig(b). Because of this a force will be developed which rotates the coil in clockwise direction.



fig(a) - Interaction of two fluxes



fig(b) Force experienced by the conductor

Here, on one side of the conductor, ^{two} fluxes are in the same direction. There is a gathering of flux. On the right side of the conductor the two fluxes are in opposite direction & cancel each other & field will be weakened. Thus there will be a force developed and the conductor rotates in clockwise direction.

In actual machine, a large number of conductors are placed in the ~~influence~~ of armature. All the

conductors placed under the influence of North pole carry current in one direction & All conductors under the influence of South pole will be in other direction. Each conductor of the armature experiences a force which ~~causes~~ causes the armature to rotate.

Back emf (E_b)

In a dc motor, when the armature rotates, the conductor cut the magnetic flux and therefore emf is induced in the armature. The direction of this induced emf is such that it opposes the applied voltage ~~in~~ according to Lenz's law. Hence it is called as back emf.

$$\text{Back emf, } E_b = \frac{\phi Z N P}{60 A}$$

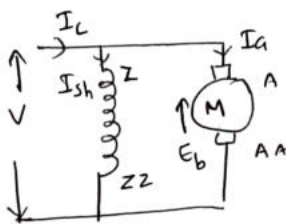
Torque Equation of a Motor

Back emf

In a dc motor, when the armature rotates, the conductor cut the magnetic flux and therefore emf is induced in the armature. The direction of this emf is such that it opposes the applied voltage according to Lenz's law. Hence it is called as back emf.

$$\text{Back emf, } E_b = \frac{\phi Z N P}{60 A}$$

Considering a dc shunt motor

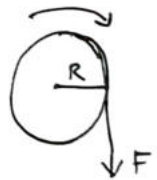


$$\text{Here } V = E_b + I_a R_a$$

Torque Equation of DC Motor

The turning or twisting force about an axis is called as torque.

Consider a wheel of radius R meters. ~~acted~~
upon



Let T_a be the torque produced by the armature of a motor running at N rpm

The angular speed is $\omega = \frac{2\pi N}{60}$ rad/sec

Work done in one revolution,

$$W = F \times \text{distance travelled in one revolution}$$

$$W = F \cdot 2\pi R$$

Power developed

$$P = \frac{\text{Work done}}{\text{Time}} = \frac{F \cdot 2\pi R}{\text{Time for 1 revolution}}$$

$$= \frac{F \cdot 2\pi R}{60/N}$$

$$= F \times R \quad \frac{2\pi N}{60}$$

$$P = \frac{2\pi N T_a}{60} \quad \text{--- (1)}$$

The Power developed in the armature of a DC Motor is also equal to

$$P = E_b I_a \quad \text{--- (2)}$$

Equating (1) & (2)

$$\frac{2\pi N T_a}{60} = E_b I_a$$

$$\text{WKT } E_b = \frac{\phi Z N P}{60 A}$$

$$\frac{2\pi N T_a}{60} = \frac{\phi Z N P}{60 A} \cdot I_a$$

$$T_a = \frac{1}{2\pi} \phi Z I_a \frac{P}{A}$$

$$T_a = 0.159 \phi Z I_a \left(\frac{P}{A} \right) \text{ N.m}$$

Shaft or Useful Torque.

The torque developed in the armature is not ^{fully} available to the shaft due to iron loss and frictional losses. The torque which is available as useful torque to the shaft is known as shaft torque.

~~Q~~ If $T_a \rightarrow$ Armature or gross torque,

$T_L \rightarrow$ Torque loss due to iron & friction losses.

Then

$$T_{\text{shaft}} = T_a - T_L$$

$$T_{\text{shaft}} = \frac{\text{o/p of the motor}}{\frac{2\pi N}{60}} \text{ N.m}$$

Significance of back emf

Due to the presence of back emf, the dc motor becomes a regulating machine. i.e. Motor adjusts itself to draw armature current just enough to draw satisfy the load demand. This can be explained by considering the following equations.

$$E_b = \frac{\phi Z P N}{60 A} \quad \text{--- (1)} \quad \therefore E_b \propto N$$

$$V = E_b + I_a R_a \quad \therefore I_a = \frac{V - E_b}{R_a} \quad \text{--- (2)}$$

$$T_a = 0.159 \phi Z I_a (P/A) \quad \text{Nm} \quad \text{--- (3)}$$

When the load of a DC Motor is suddenly increased, it tries to slow down. i.e. the speed of the motor, N reduces. Because of this the back emf also decreases (eq. 1). When back emf decreases, the armature current increases (eq. 2). i.e. motor draws more current. Due to the increase in armature current, the armature torque increases (eq. 3).

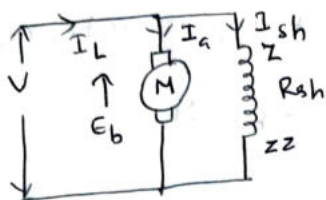
Similarly when the load of a DC Motor is decreased, the speed increases. As speed increases, the back emf E_b also increases (eq. 1). When back emf increases, the armature current decreases (eq. 2). As the armature current decreases the armature torque also reduces (eq. 3).

Thus back emf regulates the armature current so as to meet the load requirement.

Types of DC Motor

DC Shunt Motor

In shunt motor, the field winding is connected in parallel with the armature.



$$I_L = I_a + I_{sh}$$

$$I_{sh} = \frac{V}{R_{sh}}$$

$$V = E_b + I_a R_a + V_{brush} + ARD$$

$E_b \rightarrow$ Terminal voltage

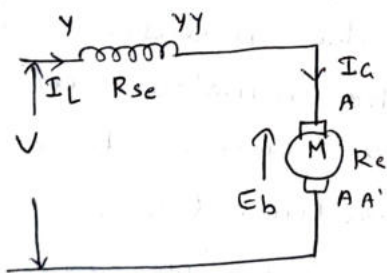
$I_a R_a \rightarrow$ Armature resistance drop

$V_{brush} \rightarrow$ Brush contact drop

$ARD \rightarrow$ armature reactance drop

DC series Motor

In a dc series motor, field winding is connected in series with the armature.



$$\text{Here } I_L = I_a = I_{se}$$

$$V = E_b + I_a R_a + I_{se} R_{se} + V_{brush} + ARD$$

$$V = E_b + I_a (R_a + R_{se}) + V_{brush} + ARD$$

$E_b =$ back emf

$I_a R_a \rightarrow$ armature resistance drop

$I_a R_{se} \rightarrow$ series field winding drop

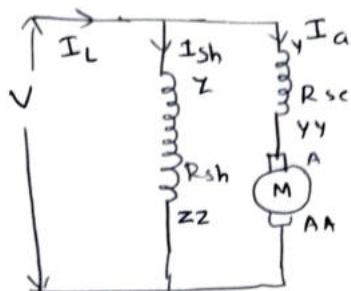
$V_{brush} \rightarrow$ brush contact drop

$ARD \rightarrow$ Armature reactance drop

Compound Motor.

A compound motor has both series and shunt field winding.

Long Shunt Compound Motor



$$\text{Here } I_L = I_{sh} + I_a$$

$$I_{sh} = \frac{V}{R_{sh}}$$

$$V = E_b + I_a(R_a + I_{se}) + V_{brush} + ARD$$

$$E_b = \text{back emf} \quad E_b = \frac{\phi Z N P}{60 A}$$

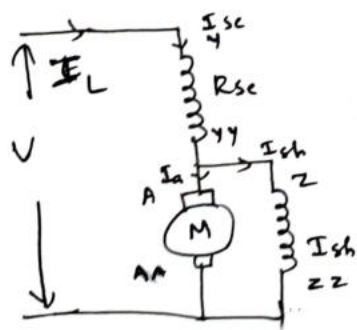
$I_a R_a \rightarrow$ armature resistance drop.

$I_{se} R_{se} \rightarrow$ series field winding drop.

$V_{brush} \rightarrow$ Brush contact drop

$ARD \rightarrow$ Armature reactance drop.

Short Shunt Compound Generator.



$$I_L = I_{se}$$

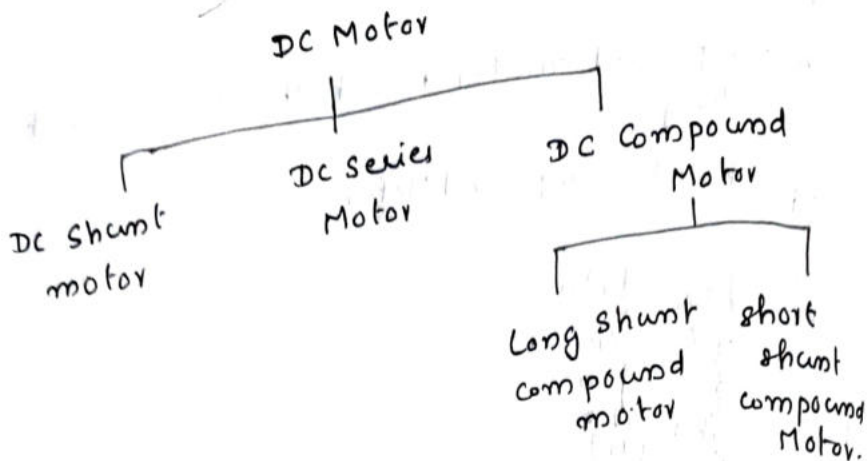
$$I_L = I_a + I_{sh}$$

$$I_{sh} = \frac{V - I_L R_{se}}{R_{sh}}$$

$$V = E_b + I_a R_a + I_{se} R_{se} + V_{brush} + ARD$$

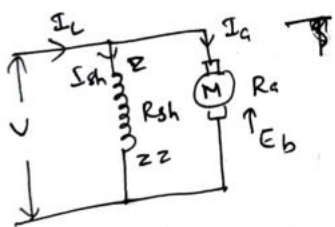
Cumulative Compound Winding : If the flux produced by the series winding is in same direction as that of shunt field winding.

Differentially Compound Winding : If the ~~shunt~~ series field flux opposes the shunt field flux, then it is differentially compound motor.



Problems on DC Motor

1. A 200V, 4 Pole, lap wound dc ^{shunt} motor has 800 conductors on its armature. The resistance of the armature winding is 0.5Ω & that of shunt field winding is 200Ω . The motor takes 21A. & flux/pole is 30mWb. Find the speed & gross Torque developed in the motor.



$$V = 200V$$

$$P = 4$$

$$\phi = 30\text{mWb} = 30 \times 10^{-3}\text{Wb}$$

Lap wound.

$$\therefore A = P = 4$$

$$I_L = 21A \quad R_a = 0.5 \quad R_{sh} = 200\Omega$$

$$I_{sh} = \frac{V}{R_{sh}} = \frac{200}{200} = 1A$$

$$I_L = I_a + I_{sh}$$

$$\therefore I_a = I_L - I_{sh}$$

$$= 21 - 1 = \underline{\underline{20A}}$$

$$(V_{brush} = 0)$$

$$V = E_b + I_a R_a$$

$$E_b = V - I_a R_a$$

$$E_b = 200 - (20 \times 0.5)$$

$$= \underline{\underline{190V}}$$

(i) Speed $N = ?$

$$E_b = \frac{\phi Z N P}{60 A}$$

$$N = \frac{E_b \times 60 A}{\phi Z P}$$

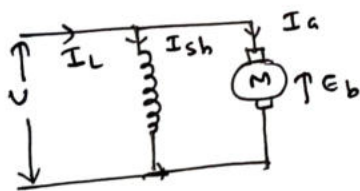
$$= \frac{190 \times 60 \times 4}{30 \times 10^{-3} \times 800 \times 4} = \underline{\underline{475 \text{ rpm}}}$$

(ii) Torque. $T_a = 0.159 \phi Z I_a \left(\frac{P}{A} \right)$

$$T_a = 0.159 \times 30 \times 10^{-3} \times 800 \times 20 \times \left(\frac{4}{4} \right)$$

$$T_a = \underline{\underline{76.32 \text{ Nm}}}$$

2. A 4 pole, 500V, DC shunt motor has 720 wave wound conductors on its armature. The full load armature current is 60A & the flux per pole is 0.03 Wb. The armature resistance is 0.2Ω & the contact drop is 1V per brush. Calculate the full load speed of the motor.



$$P = 4$$

Wave wound, $\therefore A = 2$

$$Z = 720 \quad I_a = 60 \text{ A}$$

$$\phi = 0.03 \text{ Wb} \quad R_a = 0.2 \Omega$$

$$V_{\text{brush}} = 1 \text{ V/brush}$$

$$= 1 \times 2 = 2 \text{ V}$$

$$V = E_b + I_a R_a + V_{\text{brush}}$$

$$E_b = V - I_a R_a - V_{\text{brush}}$$

$$= 500 - (0.2 \times 60) - 2$$

$$E_b = \underline{\underline{486 \text{ V}}}$$

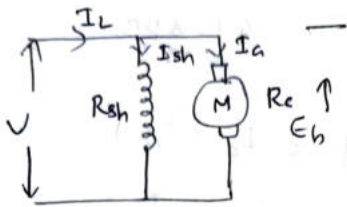
$$E_b = \frac{\phi Z P N}{60 A}$$

$$N = \frac{E_b \times 60 A}{\phi Z P} = \frac{486 \times 60 \times 2}{0.03 \times 720 \times 4}$$

$$N = \underline{\underline{675 \text{ rpm}}}$$

3. A 4 pole shunt motor takes 22.5A from 250V supply. $R_a = 0.5\Omega$, $R_{sh} = 125\Omega$. The armature is wave wound with 300 conductors. If the flux per pole is 0.02Wb . Calculate (i) speed (ii) Torque developed (iii) Power developed.

Soln



$$V = 250\text{V}$$

$$R_a = 0.5\Omega$$

$$R_{sh} = 125\Omega$$

Wave wound, $A = 2$

$$Z = 300$$

$$\text{Flux/pole } \phi = 0.02\text{Wb} \quad I_L = 22.5\text{A}$$

$$I_{sh} = \frac{V}{R_{sh}} = \frac{250}{125} = 2\text{A}$$

$$I_L = I_a + I_{sh}$$

$$I_a = I_L - I_{sh} = 22.5 - 2 = \underline{20.5\text{A}}$$

$$V = E_b + I_a R_a + V_{\text{brush}}^0$$

$$V = E_b + I_a R_a$$

$$E_b = V - I_a R_a$$

$$= 250 - (0.5 \times 20.5)$$

$$= \underline{239.75\text{V}}$$

$$E_b = \frac{\phi Z N P}{60 A}$$

$$\therefore \text{(i) Speed, } N = \frac{E_b \times 60 A}{\phi Z P}$$

$$= \frac{239.75 \times 60 \times 2}{0.02 \times 300 \times 4} = \underline{1199 \text{ rpm}}$$

$$\text{(ii) } T_a = 0.159 \phi Z I_a \left(\frac{P}{A}\right)$$

$$= 0.159 \times 0.02 \times 300 \times 20.5 \times (4/2)$$

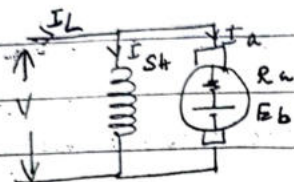
$$T_a = \underline{39.12\text{Nm}}$$

$$\text{(iii) } P = E_b I_a = 239.75 \times 20.5 = \underline{4.915\text{kW}}$$

- 4) A 4 pole, 500V, DC shunt motor has 720 wave connected conductors on its armature. The full load armature current is 60A & the flux/pole is 0.03 wb. The armature resistance is 0.2 Ω & the contact drop is 1V, per brush. Calculate the full load speed of the motor.

Solution:

$P=4, V=500V, Z=720, A=2, I_a=60A, \phi=0.03 \text{ wb}$
 $R_a=0.2 \Omega, BCD=1V/\text{brush}$



$$I_a = 60A$$

$$V = E_b + I_a R_a + BCD$$

$$500 = E_b + 60 \times 0.2 + 2$$

$$E_b = 500 - 60 \times 0.2 - 2 = 486V$$

$$E_b = 486V$$

$$E_b = \frac{\phi Z N P}{60 A} \Rightarrow N = \frac{E_b \times 60 \times A}{\phi Z P} = \frac{486 \times 60 \times 2}{0.03 \times 720 \times 4} = 675$$

$$N = 675 \text{ rpm}$$

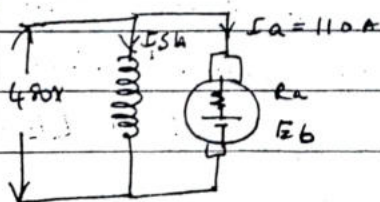
- 5) A DC shunt motor takes an armature current of 110A at 480V. The armature resistance is 0.2 Ω .

The machine has 6 poles & armature is lap connected with 864 conductors. The flux/pole is 0.05 wb. Calculate

- (i) speed (ii) the torque developed by the armature.

Solution:

JUN-14 [1064-25]



$$R_a = 0.2 \Omega, P=6, A=P=6, Z=864$$

$$\phi = 0.05 \text{ wb}$$

$$E_b = V - I_a R_a - BCD$$

$$= 480 - 110 \times 0.2 - 0$$

$$E_b = 458V$$

$$(i) E_b = \frac{\phi Z N P}{60 A} \Rightarrow N = \frac{E_b \times 60 \times A}{\phi Z P} = \frac{458 \times 60 \times 6}{0.05 \times 864 \times 6} = 636 \text{ rpm}$$

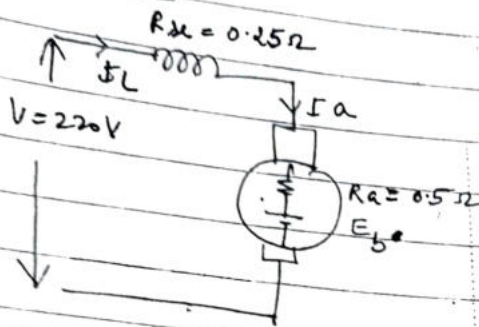
$$N = 636 \text{ rpm}$$

$$(ii) T_a = \frac{60}{2\pi} \phi Z I_a P / A = \frac{60}{2\pi} \times 0.05 \times 864 \times 110 \times 6 / 6 = 755.56 \text{ Nm}$$

- ⑥ A 220V series motor is taking a current of 40A. Resistance of armature is 0.5Ω , resistance of series field is 0.25Ω . Calculate (i) Voltage at armature (ii) Back emf (iii) Power wasted in armature (iv) Power wasted in series field.

Solution:

Jun-13 [10 Marks]



$$I_L = I_a = 40A$$

$$R_{se} = 0.25\Omega \quad R_a = 0.5\Omega$$

$$\text{Voltage at brush} = V - I_a R_{se} = 220 - 40 \times 0.25$$

$$(i) \quad \boxed{V_{\text{at brush}} = 210V}$$

$$(ii) \quad E_b = V - I_a R_{se} - I_a R_a - BCD$$

$$= 220 - 40 \times 0.25 - 40 \times 0.5 = 0$$

$$\boxed{E_b = 190V}$$

$$(iii) \quad \text{Power wasted in armature} = I_a^2 R_a = 40^2 \times 0.5 = 800 \text{ watts}$$

$$(iv) \quad \text{Power wasted in series field} = I_a^2 R_{se} = 40^2 \times 0.25 = 400 \text{ watts}$$

- ⑦ A series motor runs at 600 rpm when taking a current of 110A from 230V supply. The useful flux/pole for 110A is 24 mwb & that for 50A is 16 mwb. The armature resistance & series field resistance are 0.12Ω and 0.03Ω respectively. Calculate the speed when the current has fallen to 50A.

Solution:

Jun-16 [14 Marks]

$$N_1 = 600 \text{ rpm}, I_{a1} = 110A, \phi_1 = 24 \text{ mwb}$$

$$N_2 = ?, I_{a2} = 50A, \phi_2 = 16 \text{ mwb}$$

$$E_{b1} \propto \phi_1 N_1 \quad E_{b2} \propto \phi_2 N_2$$

$$\frac{E_{b1}}{E_{b2}} = \frac{\phi_1 N_1}{\phi_2 N_2}$$

$$E_{b1} = V - I_{a1} (R_a + R_{se})$$

$$= 230 - 110 \times (0.12 + 0.03)$$

$$E_{b1} = 213.5 \text{ V}$$

$$E_{b2} = V - I_{a2} (R_a + R_{se})$$

$$= 230 - 50 (0.12 + 0.03)$$

$$E_{b2} = 222.5 \text{ V}$$

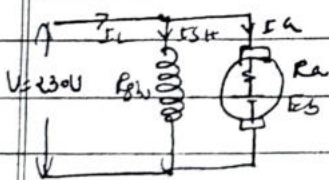
$$\frac{E_{b1}}{E_{b2}} = \frac{213.5}{222.5} = \frac{\phi_1 N_1}{\phi_2 N_2}$$

$$\Rightarrow 0.959 = \frac{24 \times 10^{-3} \times 600}{16 \times 10^{-3} \times N_2}$$

$$\Rightarrow N_2 = 938 \text{ rpm}$$

- ⑧ A 230V DC shunt motor takes no load current of 3A & runs at 1100 rpm. If the full load current is 41A, find the speed on full load. Assume armature resistance 0.25Ω & shunt field resistance 230Ω.

Dec-15 (14 & 15)

Solution:

$$I_{L0} = 3 \text{ A}, \quad N_0 = 1100 \text{ rpm}$$

$$I_{L1} = 41 \text{ A}, \quad N_1 = ?$$

$$R_a = 0.25 \Omega \quad R_{sh} = 230 \Omega$$

$$I_{sh} = V / R_{sh} = \frac{230}{230} = 1 \text{ A}$$

$$I_{a0} = I_{L0} - I_{sh} = 3 - 1 = 2 \text{ A} \quad I_{a1} = 41 - 1 = 40 \text{ A}$$

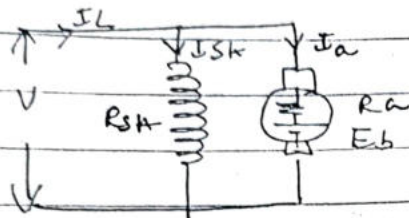
$$E_{b0} = V - I_{a0} R_a = 230 - 2 \times 0.25 = 229.5 \text{ V}$$

$$E_{b1} = V - I_{a1} R_a = 230 - 40 \times 0.25 = 220 \text{ V}$$

$$\boxed{N \propto E_b} \Rightarrow \frac{N_0}{N_1} = \frac{E_{b0}}{E_{b1}} \Rightarrow \frac{1100}{N_1} = \frac{229.5}{220} \Rightarrow \boxed{N_1 = 1054 \text{ rpm}}$$

- Q A 440V DC shunt motor takes an armature current of 20A & runs at 500rpm. The armature resistance is 0.6Ω . If the flux is reduced by 30% & the torque is increased 40%, calculate the new value of armature current & speed.

DEC-15 [14645]

Solution:

$$V = 440V, I_{a1} = 20A, N_1 = 500\text{rpm}$$

$$\phi_2 = 0.7\phi_1, T_2 = 1.4T_1$$

$$I_{a2} = ?, N_2 = ?$$

$$E_{b1} = V - I_{a1}R_a = 440 - 20 \times 0.6 = 428V$$

$$T \propto \phi I_a$$

$$\frac{T_1}{T_2} = \frac{\phi_1 I_{a1}}{\phi_2 I_{a2}} \Rightarrow \frac{T_1}{1.4T_1} = \frac{\phi_1 \times 20}{0.7\phi_1 \times I_{a2}}$$

Applications of D.C. Motors:

(7)

Shunt motor: It is used where almost a constant speed is required such as lathes, centrifugal pumps, fans & grinders etc.

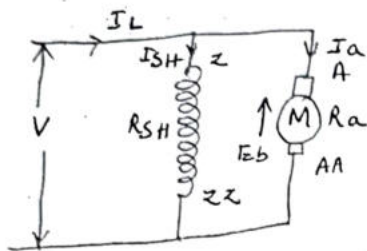
Series motor: It is used where high starting Torque is required such as cranes, lifts, electric trains etc.

Compound motor: Differential compound motors are rarely used because of their poor torque characteristic. However, cumulative-compound motors are used where a fairly constant speed is required with irregular loads such as rolling machines, lifts, printing machines etc.

Characteristics of D.C. motors:

- (i) Speed / armature current characteristics or speed characteristics
- (ii) Torque / Armature current characteristics or electrical characteristics
- (iii) Speed / Torque characteristics or mechanical characteristics

Characteristics of D.C. Shunt motor:



(i) $N - I_a$ characteristics:

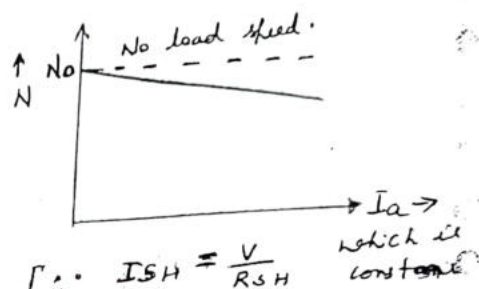
$$E_b = \frac{\phi Z N P}{60 A} \Rightarrow N \propto E_b / \phi$$

But for shunt motor ϕ is constant $\left[\because I_{SH} = \frac{V}{R_{SH}} \text{ which is constant} \right]$

Hence $N \propto E_b \propto V - I_a R_a$

$\therefore$ As I_a increases, $I_a R_a$ drop increases & hence speed decreases.

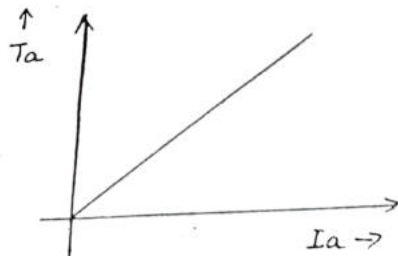
Since the speed remains almost constant with increase in load current, it may be considered as a constant speed motor.



(ii) $T_a - I_a$ characteristics:

$$T_a = 0.159 \phi Z I_a (P/A)$$

$\therefore T_a \propto I_a \rightarrow (i)$

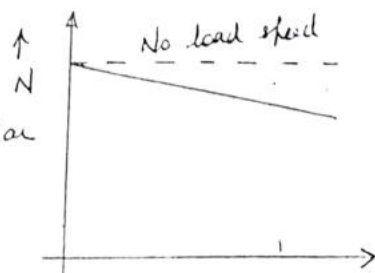


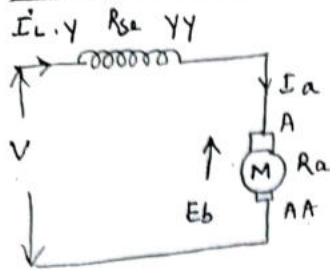
(iii) $N - T_a$ characteristics:

from equation (i) $T_a \propto I_a$

Therefore N / T_a characteristics will be similar

to N / I_a characteristics



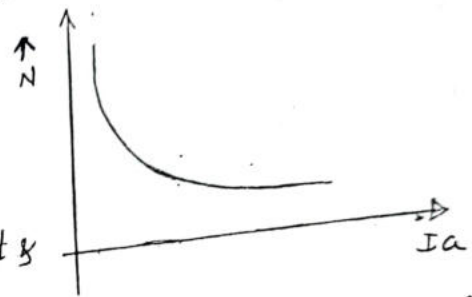
Series motor:N - Ia characteristics: // neglect

$$N \propto \frac{E_b}{\phi} \propto \frac{V - I_a(R_a + R_{se})}{\phi}$$

But in case of series motor, ϕ is not constant. As the load current increases, the flux also increases.

$$\therefore \phi \propto I_a \quad [\text{upto saturation point}]$$

$$N \propto \frac{V}{\phi} \propto \frac{1}{I_a}$$



But after saturation point, the ϕ is constant & curve is a st. line. Hence D.C.

series motor is a variable speed motor. At no load, I_a is very small & hence the speed will be dangerously high $\therefore N \propto 1/I_a$. Hence a D.C. series motor should never be started without load.

Ta - Ia characteristics:

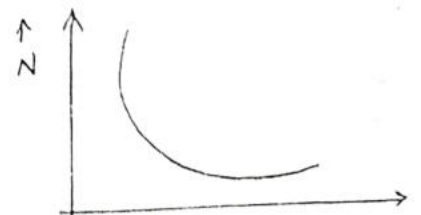
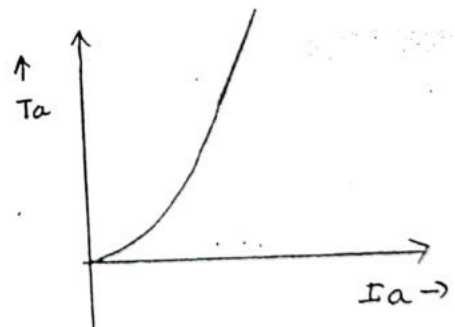
$$T_a \propto \phi I_a$$

$$\text{But } \phi \propto I_a \propto I_a$$

$$\text{Hence } T_a \propto I_a^2$$

But after saturation ϕ remains constant. $\therefore T_a \propto I_a$. Up to the saturation point the curve is a parabola & after saturation the curve is a st. line.

$\therefore$ In case of D.C. series motor the starting is very high as $T_a \propto I_a^2$.

N - Ta characteristics:

$$N \propto \frac{1}{I_a} \quad \& \quad T_a \propto I_a^2$$

Speed Control of DC Motor.

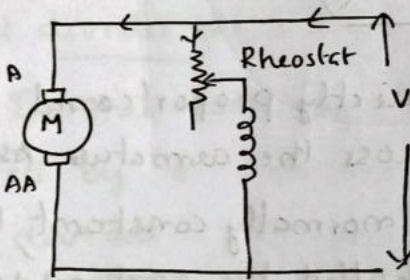
$$N \propto \frac{E_b}{\phi} \propto \frac{V - I_a R_a}{\phi}$$

Speed can be controlled by varying

- (1) Flux per pole $\rightarrow$ Flux control
- (2) Armature Resistance $\rightarrow$ Rheostatic Control
- (3) Applied voltage, $V \rightarrow$ Voltage control.

Speed Control of DC Shunt Motor

(i) Flux control



$$N \propto \frac{1}{\phi}$$

By decreasing ϕ , N can be increased

The flux in a dc shunt motor can be changed by changing I_{sh} with the help of a shunt field rheostat.

The speed control above rated speed is possible

Advantages

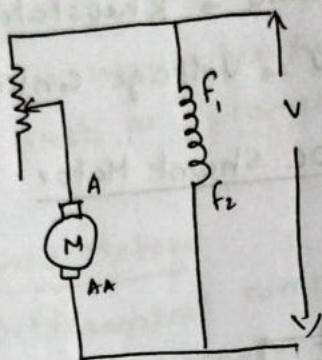
- (1) It provides relatively smooth & easy control
- (2) Speed control above rated speed is possible
- (3) As its field current is small, its size of the rheostat required is small

Disadvantages

1. The speed control below normal speed is not possible as flux can be increased only upto its rated speed.

(2) As flux reduces speed increases. But high speed ~~increases~~ affects the commutation, making motor unstable.

(ii) Armature Voltage Control or Rheostatic Control



The speed is directly proportional to the voltage applied across the armature. As the supply voltage is normally constant, the voltage can be controlled by ~~cont~~ adding a variable resistance in series with the armature.

When extra resistance is added in the armature circuit, I_a remains same.

$$I_a R_a \uparrow$$

$$V - I_a R_a \downarrow \text{ and } N \downarrow$$

By varying this extra resistance various speeds below rated speed can be obtained.

So constant load torque, the speed is directly proportional to the voltage across the armature.

Advantage

- (1) Easy & smooth control below normal is possible.

Disadvantages

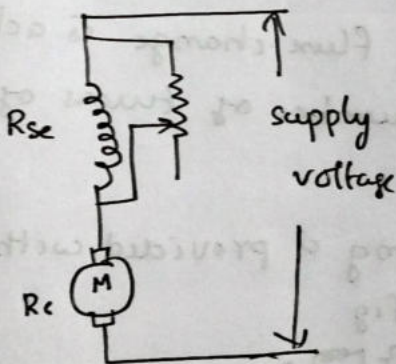
- (1) As the entire armature current passes through the external resistance, there are power losses.
- (2) Speed above rated speed is not possible
- (3) Due to large power loss the method is expensive & less efficient
- (4) Needs expensive heat-dissipation arrangements

Speed Control of DC series motor.

(1) Flux Control.

(i) Field divider method.

In this method, the series field winding is shunted by a variable resistance (R_x) known as field divider.



Due to parallel path of R_x , any amount of current can be diverted through divider.

Hence current can be adjusted in the field winding.

Due to which flux gets controlled & speed gets controlled.

Speed of the motor can be controlled above rated speed.

Armature Diverter Control

A diverter across the armature can be used for giving speeds lower than the normal speed.

For a constant given constant load torque,

Torque $T_a \propto \phi I_a$

I_a is reduced using diverter.

T_a will also reduce.

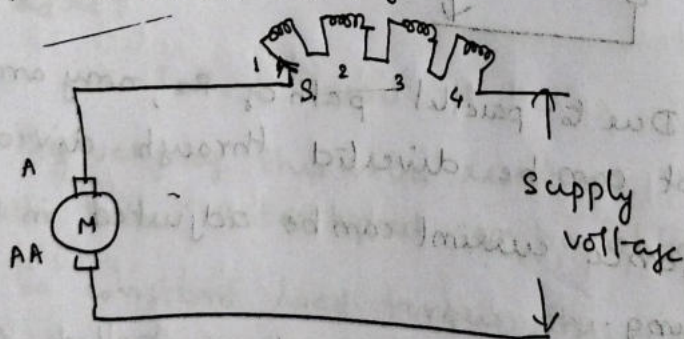
Then ϕ must increase

As flux ϕ increases, speed of the motor reduces. ($N \propto \frac{E_b}{\phi}$)

Tapped field Method.

In this method flux change is achieved by changing the number of turns of field winding.

The field winding is provided with the taps as shown in fig



The selector switch S is provided to select the number of turns (taps) as per requirement.

When the switch S is in position 1, the entire field winding is in the circuit and

motor runs with normal speed.

As the switch is moved from position 1 to 2 & onwards the number of turns of field winding in the circuit decrease, increasing the speed of the motor above rated speed.