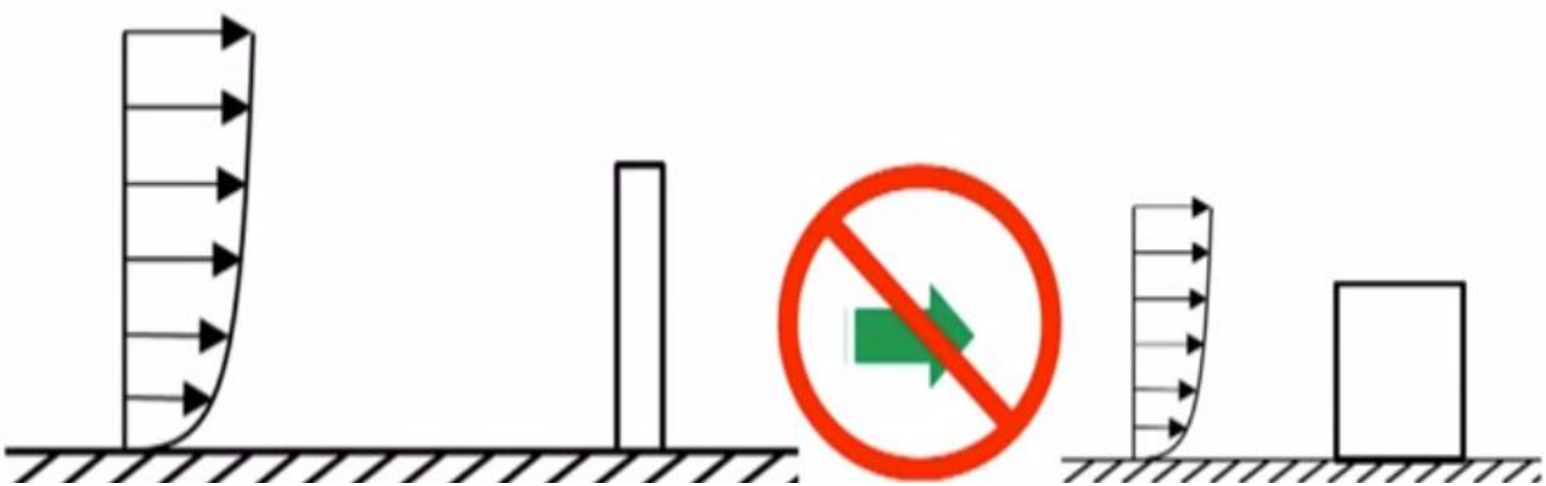
Similarity

Wind-tunnel testing in **building aerodynamics**: most often **scale models** are used to predict full-scale behavior.

Scale model and full-scale reality should be **"similar"** *(also called "similitude")*

Different **types** of similarity:

- **Geometric** similarity (concerning building dimensions, z_0, z_G, …)



Non- Dimensional Numbers

Dimensionless numbers

$$Re = \frac{VL}{\nu}$$

Reynolds number: Inertia force / viscous force

$$Ma = \frac{V}{c}$$

Mach number: Inertia force / elasticity force

$$Fr = \sqrt{\frac{V^2}{Lg}}$$

Froude number: Inertia force / gravity force

$$Gr = \frac{\beta g L^3 (T_s - T_{ref})}{\nu^2}$$

Grashof number: Buoyancy force / viscous force

(ν = kinematic viscosity, c = speed of sound in air, g = gravitational acceleration, β is thermal expansion coefficient of air, T_s = surface temperature, T_{ref} = reference temperature)

Importance of dimensionless numbers

- Froude number is only important for dynamic tests in which model as well as the aerodynamic are involved.
- For where the model is held **stationary**, Re and Ma are the significant similarity and flow quality parameters.
- If same R_e and M_a in model experiments as in full-scale application, then the model and the full-scale flows will be **dynamically similar.**
- This means: the **non-dimensional** functions of fluid velocity components, pressure coefficient, density viscosity, temperature, force coefficients **will be same** for the model and full-scale flows.

SIMILITUDE

- Similitude in a general sense is the indication of known relationship between two phenomena.
- In fluid Mechanics, this is usually the relation between a full scale model and a flow involving smaller but geometrically similar boundaries.
- The following similarities should be ensured between the model and the prototype

1. Geometric **2. Kinematic** **3. Dynamic**

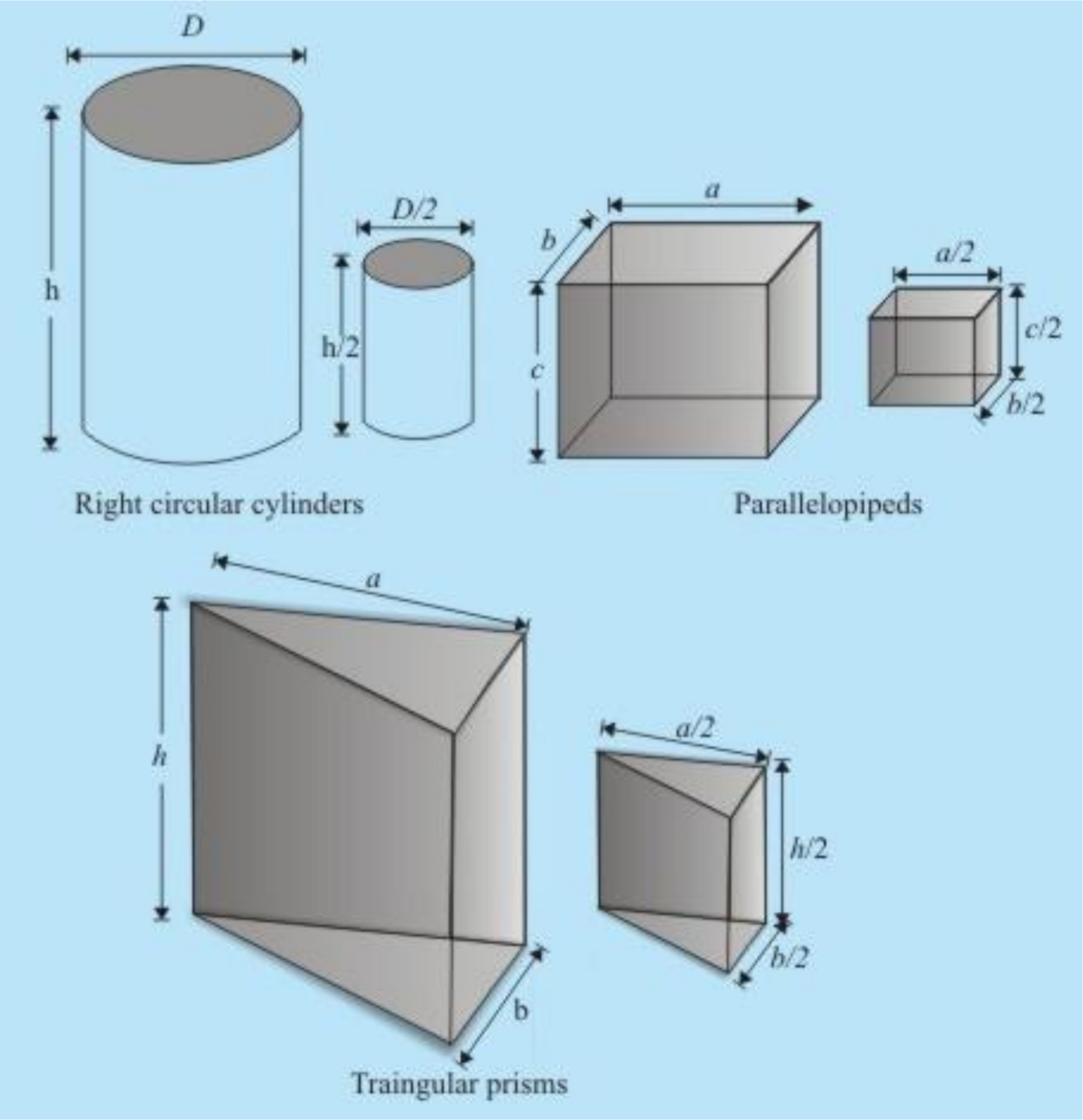
SIMILITUDE

- **Geometric similarity** – the model is the same shape as the application, usually scaled.
- **Kinematic similarity** – fluid flow of both the model and real application must undergo similar time rates of change motions. (fluid streamlines are similar)
- **Dynamic similarity** – ratios of all forces acting on corresponding fluid particles and boundary surfaces in the two systems are constant.

1. Geometric

- The model should be an exact geometric replica of prototype.
- The ratio of corresponding lengths in the model and in the prototype must be same.
- If the scale ratio for the linear dimensions of the **model** and **prototype** is same, then the **model** is said to be **undistorted model**.
- A **distorted model** is said to be a **distorted model** only when it is not geometrically similar to prototype

Example figures



1. Geometric

- L_m = Length of Models
- H_m = Height of Models
- D_m = Diameter of Models
- A_m = Area of Models
- V_m = Volume of Models

And L_p, H_p, D_p, A_p , V_p corresponding values of Prototype

Then geometric similarity , we must have the following relation.

$$\text{Length,} \quad \frac{L_p}{L_m} = L_r$$

$$\text{Area,} \quad \frac{A_p}{A_m} = \frac{{L_p}^2}{{L_m}^2} = {L_r}^2$$

$$\text{Volume,} \quad \frac{V_p}{V_m} = \frac{{L_p}^3}{{L_m}^3} = {L_r}^3$$

2.Kinematic Similarity

Velocity or acceleration ratios are same and velocity/acceleration vector point in the same direction and the two flows are said to be kinematic ally similar

Velocity scale ratio, $V_r = \frac{V_p}{V_m}$

As time, T is dimensionally $\frac{L}{V}$. By that, time scale ratio, $\frac{T_p}{T_m} = T_r$ and $T_r = \frac{L_r}{V_r}$

While, for discharge and acceleration scale ratio

$$\text{Discharge scale ratio, } Q_r = \frac{Q_p}{Q_m} = \frac{{L_p}^3/T_p}{{L_m}^3/T_m} = \frac{{L_p}^3/{L_m}^3}{T_p/T_m} = \frac{{L_r}^3}{T_r}$$

$$\text{Acceleration scale ratio, } a_r = \frac{a_p}{a_m} = \frac{L_p/{T_p}^2}{L_m/{T_m}^2} = \frac{L_p/L_m}{{T_p}^2/{T_m}^2} = \frac{L_r}{{T_r}^2}$$

3. Dynamic Similarity

- Two systems have dynamic similarity if, in addition to dynamic similarity, corresponding forces are in the same ratio in both. The force scale ratio is

$$\frac{F_p}{F_m} = F_r$$

- Basically, if the geometric and kinematics similarities exist, it shows two system are dynamically similar. The ratios of these systems of all corresponding forces are the same. The respective forces includes:
    1. Gravity
    2. Viscosity
    3. Elasticity
    4. Surface tension
    5. Inertia

Advantages of Using Similarities

- Performances of object can be predicted.
- Economy and easy to build, where design of model can be done many times until reach a certain values.
- Non-functional structure also can be measured such as dam.

TYPES OF FORCES ENCOUNTERED IN FLUID PHENOMENON

Inertia Force, F_i: It is equal to product of mass and acceleration in the flowing fluid.

Viscous Force, F_v: It is equal to the product of shear stress due to viscosity and surface area of flow.

Gravity Force, F_g: It is equal to product of mass and acceleration due to gravity.

Pressure Force, F_p: it is equal to product of pressure intensity and cross-sectional area of flowing fluid.

Surface Tension Force, F_s: It is equal to product of surface tension and length of surface of flowing fluid.

Elastic Force, F_e: It is equal to product of elastic stress and area of flowing fluid.

DIMENSIONLESS NUMBERS

Dimensionless numbers are the numbers which are obtained by dividing the inertia force by viscous force or gravity force or pressure force or surface tension force or elastic force.

As this is ratio of once force to other, it will be a dimensionless number. These are also called non-dimensional parameters.

The following are most important dimensionless numbers.

- Reynold's Number
- Froude's Number
- Euler's Number
- Weber's Number
- Mach's Number

DIMENSIONLESS NUMBERS

Reynold's Number, Re: It is the ratio of inertia force to the viscous force of flowing fluid.

$$\mathrm{Re}=\frac{Fi}{Fv}=\frac{Mass.\dfrac{Velocity}{Time}}{Shear\ Stress.\ Area}=\frac{\rho\dfrac{Volume}{Time}.Velocity}{Shear\ Stress.\ Area}$$

$$=\frac{\rho Q.V}{\tau.A}=\frac{\rho AV.V}{\mu\dfrac{du}{dy}.A}=\frac{\rho AV.V}{\mu\dfrac{V}{L}.A}=\frac{\rho VL}{\mu}=\frac{VL}{\upsilon}$$

- **Froude's Number, Fe:** It is the ratio of inertia force to the gravity force of flowing fluid.

$$Fe=\sqrt{\frac{Fi}{Fg}}=\sqrt{\frac{Mass.\dfrac{Velocity}{Time}}{Mass.\ Gavitational\ Acceleraion}}=\sqrt{\frac{\rho\dfrac{Volume}{Time}.Velocity}{Mass.\ Gavitational\ Acceleraion}}$$

$$=\sqrt{\frac{\rho Q.V}{\rho Volume.g}}=\sqrt{\frac{\rho AV.V}{\rho AL.g}}=\sqrt{\frac{V^2}{gL}}=\frac{V}{\sqrt{gL}}$$

DIMENSIONLESS NUMBERS

Eulers's Number, Eu: It is the ratio of inertia force to the pressure force of flowing fluid.

$$E_u = \sqrt{\frac{Fi}{Fp}} = \sqrt{\frac{Mass.\frac{Velocity}{Time}}{\Pr essure.\,Area}} = \sqrt{\frac{\rho\frac{Volume}{Time}.Velocity}{\Pr essure.\,Area}}$$

$$= \sqrt{\frac{\rho Q.V}{P.A}} = \sqrt{\frac{\rho AV.V}{P.A}} = \sqrt{\frac{V^2}{P/\rho}} = \frac{V}{\sqrt{P/\rho}}$$

- Weber's Number, We: It is the ratio of inertia force to the surface tension force of flowing fluid.

$$We = \sqrt{\frac{Fi}{Fg}} = \sqrt{\frac{Mass.\frac{Velocity}{Time}}{Surface\,Tension\,per.\,Length}} = \sqrt{\frac{\rho\frac{Volume}{Time}.Velocity}{Surface\,Tension\,per.\,Length}}$$

$$= \sqrt{\frac{\rho Q.V}{\sigma.L}} = \sqrt{\frac{\rho AV.V}{\sigma.L}} = \sqrt{\frac{\rho L^2V^2}{\sigma.L}} = \frac{V}{\sqrt{\frac{\sigma}{\rho L}}}$$

DIMENSIONLESS NUMBERS

Mach's Number, M: It is the ratio of inertia force to the elastic force of flowing fluid.

$$M = \sqrt{\frac{Fi}{Fe}} = \sqrt{\frac{Mass.\dfrac{Velocity}{Time}}{Elastic\ Stress.\ Area}} = \sqrt{\frac{\rho\dfrac{Volume}{Time}.Velocity}{Elastic\ Stress.\ Area}}$$

$$= \sqrt{\frac{\rho Q.V}{K.A}} = \sqrt{\frac{\rho AV.V}{K.A}} = \sqrt{\frac{\rho L^2 V^2}{KL^2}} = \frac{V}{\sqrt{K/\rho}} = \frac{V}{C}$$

$$Where: C = \sqrt{K/\rho}$$

Model laws

- The laws on which the models are designed for dynamic similarity are called model laws.
- The ratio of the corresponding force is acting at the corresponding points in the model and prototype should be equal For dynamic similarity.
- The following are the model law:-

1.Reynold’s model law

2.Froude model law

3.Euler model law

4.Weber model law

5.Mach model law

Reynolds Model Law

- Reynolds model law is the law in which models are based on Reynolds number.
- Models based on Reynolds number includes:-
  - Pipe flow
  - Resistance experienced by submarines, airplanes etc.
- Let,
  - Vm = velocity of fluid in model
  - Pm = density of fluid in model
  - Lm = Length dimension of the model
  - Um = viscosity of fluid in modeL

$$[R_e]_p = [R_e]_p \; or \; \frac{\rho_m V_m L_m}{\mu_m} = \frac{\rho_p V_p L_p}{\mu_p}$$

Froude Model law

- Froude model law is the law in which the models are based on froude Number which means to dynamic similarity between the models and prototype the froude number for both of them should be equal.
- Froude model law is applied in the following fluid flow problems:-
  1. Flow of jet from nozzle
  2. Where waves are likely to be formed on surface.

Let,

Vm = velocity of fluid in model

Lm = linear dimension

Gm = acceleration due to gravity at a place where model is tested.

$$\left(F_e\right)_{model} = \left(F_e\right)_{prototype} \quad or \quad \frac{V_m}{\sqrt{g_m L_m}} = \frac{V_p}{\sqrt{g_p L_p}}$$

Euler Model Law

- Euler's model law is the law in which The models are designed on euler's number which means for dynamic similarity between the model and prototype,the Euler number of model and prototype should be equle.
- Let,

  Vm = velocity of fluide in model

  Pm = pressure of fluid in model

  Pm = density of Lode in model

  Vp,Ppa,ppa = corresponding value in prototype,

$$\frac{V_m}{\sqrt{p_m / \rho_m}} = \frac{V_p}{\sqrt{p_p / \rho_p}}$$

Weber Model Law

- Weber model law is the law in which model are based on weber's number, which is the ratio of the square root of inertia force to surface tension force
- Let,

  Vm = velocity of fluid in model

  M= surface tensile force in model

  Lm = length of surface in model

  Lp = Corresponding values of fluid in prototype

$$\frac{V_m}{\sqrt{\sigma_m / \rho_m L_m}} = \frac{V_p}{\sqrt{\sigma_p / \rho_p L_p}}$$

Mach model law

- Mach model law is the law in which models are designed on Mach number which is the ratio of the square root of inertia force to elastic force of a fluid.
- Let,

  Vm = velocity of fluid in model

  Km = Elastic stress for model

  Pm = density of fluid in model

  Vp,Kp and Ppa =Corresponding valued for prototype.

$$\frac{V_m}{\sqrt{K_m / \rho_m}} = \frac{V_m}{\sqrt{K_p / \rho_p}}$$

Types of Models

- The hydraulic models basically two types as,

1. Undistorted models
2. Distorted models

1.Undistorted model:

- The this model is geometrical is similar to its prototype.
- The scale ratio for corresponding linear dimension of the model and its prototype are same.
- The behaviour of the prototype can be easily predicted from the result of these type of model.

Advantages of undistorted model

1. The basic condition of perfect geometric similarity is satisfied.
2. Predication of model is relatively easy.
3. Results obtained from the model tests can be transferred to directly to the prototype.

Limitations of undistorted models

1. The small vertical dimension of model can not measured accurately.
2. The cost of model may increases due to long horizontal dimension to obtain geometric similarity.

Numerical Problems

2. A geometrically Similar model of an air duct is built to 1/25 scale and tested with water which is 50 times more viscous and 800 times denser than air. When tested under dynamically similar conditions, the pressure drop is 2 bar in the model. Find the corresponding pressure drop in the full scale prototype.

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Module 3: Dimensional Analysis and Similarity

Dimensional Homogeneity – Method of repeating variables- Buckingham Pi Theorem- types of similarity and similitude. Dimensionless numbers. Model laws

Numerical Problems on Similitude

(Model laws)

EXAMPLE 3.5 The pressure drop per unit length in a 6 mm diameter gasoline fuel line is to be determined from a laboratory test using the same tube but with water as the fluid. The pressure drop at a gasoline velocity of 1 m/s is of interest. (a) Determine the water velocity required. (b) At the properly scaled velocity from part (a), the pressure drop per unit length was found to be 65 Pa/m. What is the predicted pressure drop per unit length for the gasoline line? For gasoline, the density is 680 kg/m^3 and the viscosity $\mu = 3.1 \times 10^{-4}\ \mathrm{N \cdot s/m^2}$; for water the density is 1000 kg/m^3 and the viscosity $\mu = 1.12 \times 10^{-3}\ \mathrm{N \cdot s/m^2}$.

Hint: For dynamics similarity between gasoline and water flows, their Reynolds number should be equal,

The pressure drop through the water tube per unit length is given by

$$\Delta p_{\mathrm{w}} = f \frac{L}{D} \frac{1}{2} \rho V^2$$

EXAMPLE 3.4 It is required to calculate the drag experienced by a small submarine hull when it is moving far below the surface of water. A 1/10 scale model is to be tested. What dimensionless groups should be matched between the model and the prototype and why? If the drag on the prototype at 1 knot is desired, at what speed should the model be moved to give the drag to be experienced by the prototype? Would this result be true if the prototype were to move close to the water surface? Explain.

Hint: When the submarine travels far below the water surface, the viscous forces predominate and, therefore, the Reynolds number must be matched for dynamic similarity.

HW

EXAMPLE 3.6 A scaled model of an automobile vehicle of width 2.44 m and frontal area 7.8 m^2 was tested in a wind tunnel to find the aerodynamic drag that would act on the vehicle when it plies at 100 km/h and to find the power required to overcome this drag. The scale of the model was 16:1. The test revealed that the drag coefficient on the model was 0.46 and it becomes independent of Reynolds number for Reynolds numbers greater than 10^5. Estimate the drag force and the power required for the actual vehicle, if the test-section air flow conditions were identical to sea level atmospheric conditions.

HINT:

But the density and viscosity are invariants. Thus,

$$(VL)_{\mathrm{m}} = (VL)_{\mathrm{p}}$$

$$D_{\mathrm{p}} = C_D \, \frac{1}{2} \, \rho V_{\mathrm{p}}^2 \, A_{\mathrm{p}}$$

The power required to overcome this drag is

$$\text{Power} = D_{\mathrm{p}} V_{\mathrm{p}}$$