Module – V Syllabus

Compressible Flows

Compressible Flows: Speed of sound, adiabatic and isentropic steady flow, Isentropic flow with area change stagnation and sonic properties, normal and oblique shocks, flow through nozzles.

Introduction to CFD: Necessity, limitations, philosophy behind CFD, applications

Table of Contents

1. Compressible flow

is defined as that flow in which the density of the fluid does not remain constant during flow. This means that the density changes from point to point in compressible flow. But in case of incompressible flow, the density of the fluid is assumed to be constant.

But in case of fluids such as

i. Flow of gases through orifices and nozzles
ii. Flow of gases in machines such as compressors and
iii. Projectiles and airplanes flying at high altitude with high velocities, the density of the fluid changes during the flow. The change in density of a fluid is accompanied by the changes in pressure and temperature and hence the thermodynamic behaviour of the fluids will have to be taken into account

2. Basic thermodynamic relations

The following are the basic thermodynamic relations useful for gases

1. Characteristic equation of state
2. Specific heat
3. Internal energy
4. Enthalpy

2.1. The characteristic equation of state

The characteristic equation of state for a perfect gas correlates the density of gas, absolute pressure and temperature

It is given by

$p\forall = RT$ (1)

Where, p= Absolute pressure in N/m^2

∀= Specific Volume of gas in m^3/kg

R= Gas constant in J/kgK

T= Absolute temperature in K

The specific volume ∀ is which is the reciprocal of density

$$\forall = \frac{1}{\rho}$$

Substituting this value of ∀ in above equation , we get

$$\frac{p}{\rho} = RT$$

Specific heat

Specific heat for a solid or liquid is defined as the amount of heat required to raise the temperature of unit mass to one degree

For gases there may be infinite number of specific heats but only the following two are defined

- Specific heat at constant volume, C_v
- Specific heat at constant pressure, C_p

The relationship between C_p and C_v is given by

$$C_p - C_v = R$$

And $\frac{C_p}{C_v} = \gamma$

Where γ is ratio of specific heat

Internal energy

Internal energy is defined as the amount of heat energy stored in the gas

For a perfect gas internal energy is a function of temperature only

Enthalpy

Enthalpy is the amount of heat content used or released in the system at constant pressure

It is expressed as the sum of internal energy and product of pressure and volume

H= U+ pV

3. Basic thermodynamic processes

The basic thermodynamic processes related to the gases are given as follows

1. Constant volume process or Isochoric process
2. Constant pressure process or Isobaric process
3. Constant temperature or Isothermal process
4. Adiabatic process

Constant volume process ($\forall = \boldsymbol{Constant}$)

In this process system changes its state at constant volume. For example , heating or cooling of a gas in an enclosed space

$$\frac{\boldsymbol{p_1 \forall_1}}{\boldsymbol{T_1}} = \frac{\boldsymbol{p_2 \forall_2}}{\boldsymbol{T_2}}$$

But $\forall_1 = \forall_2$ (as volume is constant)

$$\therefore \frac{p_1}{T_1} = \frac{p_2}{T_2}$$

In this system ,

Work done W=0

Heat added Q=$mC_v(T_2 - T_1)$

Constant pressure process (p=Constant)

In this process, system changes its state at constant pressure. For example, heating of a gas in a cylinder and piston moves against deadweight due to work by the gas and keeps the pressure constant

$$\frac{p_1\forall_1}{T_1} = \frac{p_2\forall_2}{T_2}$$

But $p_1 = p_2$ (as pressure is constant)

$$\therefore \frac{\forall_1}{T_1} = \frac{\forall_2}{T_2}$$

In this system ,

Work done W=$p(\forall_2 - \forall_1)$

Heat added Q=$mC_p(T_2 - T_1)$

Isothermal process (T= Constant or $p\forall = Constant$)

The process is called as isothermal when the expansion or compression takes place at constant temperature

For example during isothermal expansion, heat must be added continuously to keep the temperature constant and during isothermal compression heat must be removed continuously

In this process gas obeys Boyle's law

$p\forall = Constant$

$p_1\forall_1 = p_2\forall_2$

$\frac{\forall_2}{\forall_1} = \frac{p_1}{p_2}$

Also $\frac{p}{\rho}$ = Constant (as $\rho = \frac{1}{\forall}$)

Heat added Q= $p_1\forall_1 \ln\left(\frac{\forall_2}{\forall_1}\right) = p_1\forall_1 \ln\left(\frac{p_1}{p_2}\right)$

Adiabatic process

An adiabatic process is the process in which there is no heat transfer from system to surrounding and vice-versa

∴ Heat added Q=0

Work done W= $\frac{p_1\forall_1 - p_2\forall_2}{\gamma-1} = \frac{R(T_1-T_2)}{\gamma-1}$ (As $p_1\forall_1 = RT_1, p_2\forall_2 = RT_2$)

For this process

$p\forall^{\gamma} = C$

$\therefore p_1\forall_1^{\gamma} = p_2\forall_2^{\gamma}$

Also $\frac{p}{\rho^{\gamma}} = C$ $\left(since\ \forall = \frac{1}{\rho}\right)$

If the system is reversible, then this process is called as isentropic process

If the pressure and density are related in such a way that $\gamma \neq \frac{C_p}{C_v}$ but is equal to any positive value then the process is called as polytropic process

$\frac{p}{\rho^n} = Constant$ (where $n \neq \gamma$)

4. Basic equations of compressible flow

The basic equations of the compressible flows are

1. Continuity equation
2. Bernoulli's equation or energy equation
3. Momentum equation
4. Equation of state

4.1. Continuity equation

This is based on law of conservation of mass which states that matter cannot be created nor be destroyed. Or in other words, the matter or mass is constant.

For one-dimensional steady flow, the mass per second $\dot{m} = \rho AV$

Where $\rho = Mass\ density$, A = Area of cross section, V=Velocity

As mass or mass per second is constant according to law of conservation of mass. Hence

$\rho AV = constant$

Differentiating above equation, $d(\rho AV) = 0$

or $\rho d(AV) + AVd\rho = 0$

$\rho[AdV + VdA] + AVd\rho = 0$ Or $\rho AdV + \rho VdA + AVd\rho = 0$

Dividing by $\rho AV, we\ get$ $\frac{dV}{V} + \frac{dA}{A} + \frac{d\rho}{\rho} = 0$

Above equation is also known as continuity equation in differential form

4.2. Bernoulli's equation

The flow of fluid particle along a stream-line in the direction of S is considered. The resultant force on the fluid particle in the direction of S is equated to the mass of the fluid particle and its acceleration. As the flow of compressible fluid is steady,

$$\frac{dp}{\rho} + VdV + gdZ = 0$$

Integrating the above equation, we get

$$\int \frac{dp}{\rho} + \int VdV + \int gdZ = Constant$$

Or $\int \frac{dp}{\rho} + \frac{V^2}{2} + gZ$=Constant

In case of incompressible flow, the density ρ is constant and hence integration of $\int \frac{dp}{\rho}$ is equal to $\frac{p}{\rho}$. But in case of compressible flow, the density ρ is not constant. Hence ρ cannot be taken outside the integration sign. With the change of ρ, the pressure p also changes for compressible fluids. This change of ρ and p takes place depending upon the type of process during compressible flow.

4.2.1. Bernoulli's equation for isothermal process

For isothermal process, the relation between pressure p and density ρ is given by

$$\frac{p}{\rho} = Constant = C_1 \text{(say)}$$

$$\therefore \rho = \frac{p}{C_1}$$

Hence $\int \frac{dp}{\rho} = \int \frac{dp}{\frac{p}{C_1}} = \int \frac{C_1 dp}{p} = C_1 \int \frac{dp}{p}$ (as C_1 is constant)

$= C_1 lnp = \frac{p}{\rho} lnp$ (as $\frac{p}{\rho} = C_1$)

Substituting the value of $\int \frac{dp}{\rho}$ in Bernoulli's equation we get

$$\frac{p}{\rho} lnp + \frac{V^2}{2} + gZ = \text{Constant}$$

Dividing by 'g',

$$\frac{p}{\rho g} lnp + \frac{V^2}{2g} + Z = \text{Constant} \quad (1)$$

Equation (1) is the Bernoulli's equation for compressible flow undergoing isothermal process

For the two points 1 and 2, this equation is written as

$$\frac{p_1}{\rho_1 g} lnp_1 + \frac{V_1^2}{2g} + Z_1 = \frac{p_2}{\rho_2 g} lnp_2 + \frac{V_2^2}{2g} + Z_2$$

4.2.2. Bernoulli's equation for adiabatic process

For the adiabatic process the relation between pressure p and density ρ is given by

$$\frac{p}{\rho^{\gamma}} = constant = say\ C_2$$

$$\therefore \rho^{\gamma} = \frac{p}{C_2} \text{ or } \rho = \left(\frac{p}{C_2}\right)^{\frac{1}{\gamma}}$$

Hence $\int \frac{dp}{\rho} = \int \frac{dp}{\left(\frac{p}{C_2}\right)^{\frac{1}{\gamma}}} = \int \frac{C_2^{\frac{1}{\gamma}}}{p^{\frac{1}{\gamma}}} dp = C_2^{\frac{1}{\gamma}} \int \frac{1}{p^{\frac{1}{\gamma}}} dp$

$$= C_2^{\frac{1}{\gamma}} \int p^{-\frac{1}{\gamma}} dp = C_2^{\frac{1}{\gamma}} \left(\frac{p^{\left(-\frac{1}{\gamma}+1\right)}}{\left(-\frac{1}{\gamma}+1\right)} \right)$$

$$= \frac{C_2^{\frac{1}{\gamma}} p^{\left(\frac{\gamma-1}{\gamma}\right)}}{\left(\frac{\gamma-1}{\gamma}\right)} = \left(\frac{\gamma}{\gamma-1}\right) C_2^{\frac{1}{\gamma}} p^{\left(\frac{\gamma-1}{\gamma}\right)}$$

$= \left(\frac{\gamma}{\gamma-1}\right)\left(\frac{p}{\rho^{\gamma}}\right)^{\frac{1}{\gamma}} p^{\left(\frac{\gamma-1}{\gamma}\right)}$ (as $C_2 = \frac{p}{\rho^{\gamma}}$)

$= \left(\frac{\gamma}{\gamma-1}\right)\frac{p^{\frac{1}{\gamma}}}{\rho} p^{\left(\frac{\gamma-1}{\gamma}\right)} = \left(\frac{\gamma}{\gamma-1}\right)\frac{p^{\frac{1}{\gamma}+\frac{\gamma-1}{\gamma}}}{\rho} = \left(\frac{\gamma}{\gamma-1}\right)\frac{p}{\rho}$

Substituting the value of $\int \frac{dp}{\rho} = \left(\frac{\gamma}{\gamma-1}\right)\frac{p}{\rho}$ in Bernoulli's equation we get

$\left(\frac{\gamma}{\gamma-1}\right)\frac{p}{\rho} + \frac{V^2}{2} + gZ = \text{Constant}$

Dividing by g

$\left(\frac{\gamma}{\gamma-1}\right)\frac{p}{\rho g} + \frac{V^2}{2g} + Z = \text{Constant}$

Equation above is the Bernoulli's equation for compressible flow undergoing adiabatic process

For the two points 1 and 2, this equation is written as

$\left(\frac{\gamma}{\gamma-1}\right)\frac{p_1}{\rho_1 g} + \frac{V_1^2}{2g} + Z_1 = \left(\frac{\gamma}{\gamma-1}\right)\frac{p_2}{\rho_2 g} + \frac{V_2^2}{2g} + Z_2$

4.3. Momentum equations

The momentum per second of a flowing fluid (or momentum flux) is equal to the product of mass per second and the velocity of the flow. Mathematically, the momentum per second of a flowing fluid (compressible or incompressible) is

$= \rho AV \times V$, Where ρAV = Mass per second

The term ρAV is constant at every section of flow due to continuity equation. This means the momentum per second at any section is equal to the product of a constant quantity and the velocity. This also implies that momentum per second is independent of compressible effect. Hence the momentum equation for incompressible and compressible fluid is the same.

The momentum equation for compressible fluid for any direction may be expressed as

Net force in the direction of S =rate of change of momentum in the direction of S

= Mass per second× (change of velocity)

= $\rho AV(V_2 - V_1)$

Where, V_2 = Final velocity in the direction of S

V_1 = initial velocity in the direction of S

5. Velocity of sound or pressure wave in a fluid

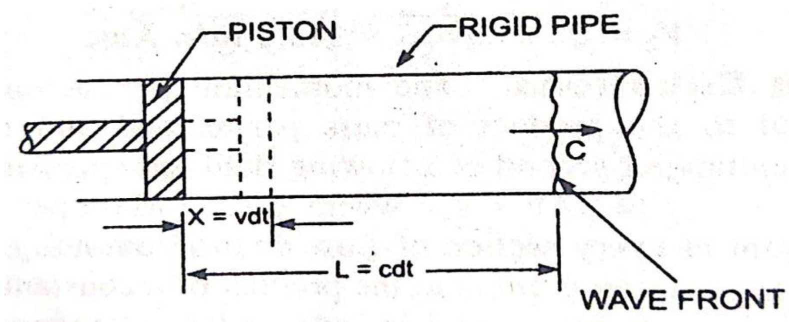
The disturbance in a solid, liquid or gas is transmitted from one point to the other. The velocity with which the disturbance is transmitted depends upon the distance between the molecules of the medium. In case of solids, the molecules are closely packed and hence the disturbance is transmitted instantaneously. In case of liquids and gases(or fluids) the molecules are relatively apart. The disturbance will be transmitted from one molecule to the next molecule. But in case of fluids, there is some distance between two adjacent molecules. Hence each molecule will have to travel at a certain distance before it can transmit the disturbance. Thus the velocity of disturbance in case of fluids will be less than the velocity of the disturbance in solids

The distance between the molecules is related with the density, which in turn depends upon pressure in case of fluids. Hence the velocity of disturbance depends upon the changes of pressure and density of the fluid

Expression for velocity of sound wave in a fluid

The disturbance creates the pressure waves in a fluid. These pressure waves travel with a velocity of sound waves in all directions. But for the sake of simplicity, one dimensional case will be considered

Fig below shows the model for one dimensional propagation of the pressure waves . It is a right long pipe of uniform cross sectional area, fitted with a piston. Let the pipe is filled with a compressible fluid, which is at rest initially. The piston is moved towards right and a disturbance is created in the fluid. This disturbance in the form of pressure wave, which travels in the fluid with a velocity of sound wave



Propagation of pressure wave.

Let A= Cross sectional area of the pipe

V= velocity of piston

p= pressure of the fluid in pipe before the movement of the piston

$\rho =$ density of fluid before the movement of the piston

dt = A Small interval of time with which piston is moved

C= velocity of pressure wave or sound wave travelling in the fluid

Distance travelled by the piston in time *dt*

= Velocity of piston$\times\ dt = V \times dt$ (V=$\frac{Distance}{Time}$ $\therefore$ Distance = V$\times$ $Time$)

Distance travelled by the pressure waves in time *dt*

= velocity of pressure wave$\times\ dt = C \times dt$

As the value of C will be very large, hence $C \times dt$ will be more than $V \times dt$. For the time interval dt, the pressure wave has travelled a distance L and piston has moved through x. thus in the length of the tube equal to $(L - x)$, the fluid will be compressed. Due to compression of the fluid, the pressure and density of the fluid will change

Let $p + dp =$ pressure after compression

$\rho + d\rho$ = Density after compression or the density of fluid in the length $(L - x)$

Now mass of fluid for a length L before compression

$$= \rho \times Volume\ of\ fluid\ upto\ length\ L$$

Density =$\frac{Mass}{Volume}$ ∴ Mass = Density× $Volume$

But Volume = Area × $Length$

=$\rho \times A \times L = \rho \times A \times C \times dt$ (as L= $C \times dt$) (1)

Mass of fluid after compression for length $(L - x)$

= Density after compression× $Area \times Length$

=$(\rho + d\rho) \times A \times (L - x)$

=$(\rho + d\rho) \times A \times (Cdt - Vdt)$(as L= $C \times d, x = Vdt$) (2)

From the continuity equation, we have

Mass of fluid before compression= mass of fluid after compression

$\rho \times A \times C \times dt = (\rho + d\rho) \times A \times (Cdt - Vdt)$ or

$\rho \times A \times C \times dt$=$(\rho + d\rho) A \times dt(C - V)$

Dividing by $A \times dt$

We get

$\rho \times C$=$(\rho + d\rho)(C - V)$=$\rho C - \rho V + Cd\rho - Vd\rho$

$\therefore\ Cd\rho = \rho C - \rho C + \rho V + Vd\rho = \rho V + Vd\rho$ (3)

But the velocity of the piston V is very small as compared to the velocity of the pressure wave C. Also the value of $d\rho$ is very small. Hence the term $Vd\rho$ will be very very small and can be neglected. Hence equation (3) becomes

$Cd\rho = \rho V$ (4)

Now when the piston is moved with a velocity V for time dt, the fluid which is at rest initially will move with a velocity equal to the velocity of the piston. Also the pressure of the fluid will increase from p to $p + dp$ due to the movement of the piston.

Hence applying the impulse momentum equation, we get

Net force on the fluid = Rate of change of momentum

Or $(p + dp)A - p \times A = Mass\ per\ second$ (change of velocity of fluid)

Or $dp \times A = \frac{Total\ mass}{Time}[V - 0] = \frac{\rho AL}{dt}[V - 0]$

= $\frac{\rho ACdt}{dt}[V - 0]$ (as L=Cdt)

=$\rho AC[V - 0]$ =ρACV

Or $dp = \frac{\rho ACV}{A} = \rho CV$

C=$\frac{dp}{\rho V}$ (5)

Multiplying equation (4) and (5) we get

$$C^2 d\rho = \rho V \times \frac{dp}{\rho V} = dp$$

$$C^2 = \frac{dp}{d\rho}$$

$$\therefore C=\sqrt{\frac{dp}{d\rho}} \qquad (6)$$

Hence equation (6) gives the velocity of sound wave which is the square root of the ratio of change of pressure to the change of density of a fluid due to disturbance.

5.1.1. Velocity of sound in terms of bulk modulus

Bulk modulus K is defined as

$$K= \frac{Increase\ in\ pressure}{\frac{Decrease\ in\ volume}{Original\ volume}} = \frac{dp}{-\left(\frac{d\forall}{\forall}\right)}$$

Where $d\forall$ = decrease in volume, $\forall = Original\ volume$

Negative sign is taken as with the increase of pressure, volume decreases

Now we know mass of a fluid is constant. Hence

$\rho \times Volume = Constant$ (Since Mass = $\rho \times Volume$)

Or $\rho \times \forall = Constant$

Differentiating the above equation (ρ and $\forall$ *are variables*)

$\rho\ d\forall + \forall d\rho = 0\ \ or\ \rho\ d\forall = -\forall d\rho$

$$-\frac{d\forall}{\forall} = \frac{d\rho}{\rho}$$

Substituting the value of $\left(-\frac{d\forall}{\forall}\right)$ in equation (4), we get

K= $\frac{dp}{\left(\frac{d\rho}{\rho}\right)} = \rho\frac{dp}{d\rho}$ or $\frac{dp}{d\rho} = \frac{K}{\rho}$

From equation (6), the velocity of sound wave is

C= $\sqrt{\frac{dp}{d\rho}} = \sqrt{\frac{K}{\rho}}$ (7)

Equation (7) gives the velocity of sound wave in terms of bulk modulus and density. This equation is applicable for liquids and gases.

5.1.2. Velocity of sound for isothermal process

Isothermal process is given by the equation

$\frac{p}{\rho} = Constant$ or $p\rho^{-1}$= Constant

Differentiating the above equation (p and ρ both are variables)

$$p(-1)\rho^{-2}d\rho + \rho^{-1}dp = 0$$

Dividing by ρ^{-1}, we get $-p\rho^{-1}d\rho + dp = 0$

or $\frac{-p}{\rho}d\rho + dp = 0$

$dp = \frac{p}{\rho}d\rho$ or $\frac{dp}{d\rho} = \frac{p}{\rho} = RT$ (As $\frac{p}{\rho} = RT$)

Substituting the value of $\frac{dp}{d\rho}$ in C= $\sqrt{\frac{dp}{d\rho}}$

C=$\sqrt{RT}$

5.1.3. Velocity of sound for adiabatic process

Adiabatic process is given by $\frac{p}{\rho^{\gamma}} = Constant$

Or $p\rho^{-\gamma}$ = Constant

Differentiating the above equation, we get

$p(-\gamma)\rho^{-\gamma-1}d\rho + \rho^{-\gamma}dp = 0$

Dividing by $\rho^{-\gamma}$, we get $-p\gamma\rho^{-1}d\rho + dp = 0$

Or $dp = \frac{p\gamma}{\rho}d\rho$

$\frac{dp}{d\rho} = \frac{p}{\rho}\gamma = RT\gamma \quad (As \quad \frac{p}{\rho} = RT \ )$

$=\gamma RT$

Substituting the value of $\frac{dp}{d\rho}$ in C=$\sqrt{\frac{dp}{d\rho}}$

C=$\sqrt{\gamma RT}$

NOTE:

1. For the propagation of the minor disturbances through air, the process is assumed to be adiabatic. The velocity of the disturbances (pressure waves) through air is very high and hence there is no time for any appreciable heat transfer
2. Isothermal process is considered for the calculation of the velocity of the sound waves (or pressure waves) only when it is given in the numerical

problem that process is isothermal. If no process is mentioned, it is assumed to be adiabatic

6. Mach number

Mach number is defined as the square root of the ratio of the inertia force of a flowing fluid to the elastic force

$$\therefore \text{Mach number} = M = \sqrt{\frac{Inertia\ force}{Elastic\ force}} = \sqrt{\frac{\rho A V^2}{KA}}$$

$$= \sqrt{\frac{V^2}{\frac{K}{\rho}}} = \frac{V}{\sqrt{\frac{K}{\rho}}} = \frac{V}{C}\left[Since\ \sqrt{\frac{K}{\rho}} = C\right]$$

Thus Mach number = M

$$= \frac{Velocity\ of\ fluid\ or\ body\ moving\ in\ fluid}{Velocity\ of\ sound\ in\ the\ fluid} = \frac{V}{C}$$

For the compressible fluid flow, Mach number is an important non-dimensional parameter. On the basis of the Mach number, the flow is defined as

1. Subsonic flow
2. Sonic flow
3. Supersonic flow

Subsonic flow

A flow is said to be sub-sonic flow if the Mach number is less than 1.0 (or $M < 1$) which means the velocity of flow is less than the velocity of sound wave (or $V < C$)

Sonic flow

A flow is said to be sonic flow if the Mach number (M) is equal to 1.0. this means that when the velocity of flow V is equal to the velocity of sound C, the flow is said to be sonic flow

Super-sonic flow

A flow is said to be super-sonic flow if the Mach number is greater than 1.0 (or M> 1). This means that when the velocity of flow V is greater than the velocity of sound wave, the flow is said to be super-sonic flow.

7. Propagation of pressure waves (or disturbances) in a compressible fluid

Whenever any disturbance is produced in a compressible fluid, the disturbance is propagated in all direction with a velocity of sound(i.e, equal to C). The nature of propagation of the disturbance depends upon the Mach number. Let us consider a small projectile moving from left to right in a straight line in a stationary fluid. Due to the movement of the projectile, the disturbances will be created in the fluid. This disturbance will be moving in all directions with a velocity C

Hence Let V= Velocity of the projectile

C= velocity of pressure wave or disturbance for different Mach numbers

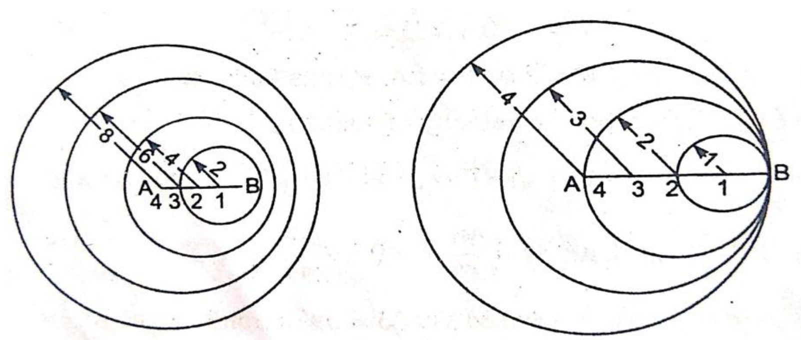
Let us find the nature of propagation of the disturbance for different Mach numbers

1st case: When M< 1

- When Mach number is less than 1.0, the flow is called Sub-sonic flow. For M< 1 means $\frac{V}{C} < 1$ or $V < C$.
- To find the nature of propagation for this case, let V=1 unit and C = 2 unit, so that $\frac{V}{C} = \frac{1}{2}$ which is less than 1.0.
- Let the projectile is at A and is moving towards right. Let in 4 seconds the projectile reaches to the position B.
- At A, the point 4 is also marked. The position of the projectile after 1 sec, 2 sec, 3 sec and 4 sec along the lines are shown by points 3, 2 ,1 and B respectively.
- The projectile moves from A to B in 4 seconds and hence the distance AB $=4 \times V = 4 \times 1 = 4$ units. The disturbance created at A in 4 seconds will move a distance = 4C=$4 \times 2 = 8$ units in all directions. Hence taking A as centre and

radius equal to 8 units, a circle is drawn. This circle gives the position of disturbance after 4 seconds

- When the projectile is at point 3, it will reach B in 3 seconds and distance 3B= $3\times V = 3\times 1 = 3$ units. But the disturbance created at point 3 in 3 seconds will move a distance having a radius $=3\times C = 3\times 2 = 6\, units$
- Similarly at point 2, the disturbance will have a radius $= 2\times C = 4\, units$ and at point 1, the disturbance will have a radius $=1\times C = 1\times 2 = 2 units$. This is shown in fig a
- As in this case $V < C$, the pressure wave is always ahead of the projectile and point B is inside the sphere of radius 8 units



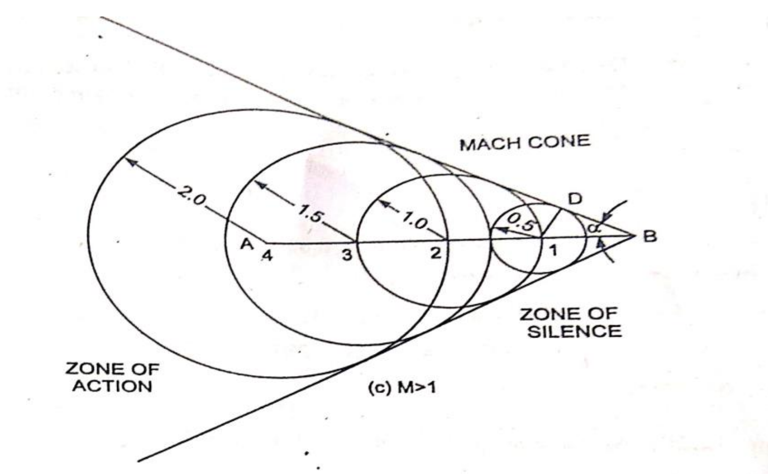
(a) M<1 (b) M=1

2nd case: When M=1

- When M=1, the flow is known as sonic flow.
- In this case, the disturbance always travels with the projectile as shown in fig b.
- Let V=1 unit and C=1 unit, so that M=$\frac{V}{C} = 1.0$.
- Let the projectile moves from A to B in 4 seconds. The disturbance created at A in 4 seconds will move a distance having radius $=4\times C = 4\times 1 = 4\, units$ in all directions.
- The projectile from point 3 will move to position B in 3 seconds. The disturbance created at point 3, will move a distance having radius $=3\times C = 3\times 1 = 3\, units$ in all directions in 3 seconds.
- Similarly at the point 2 and point 1, the disturbance created at these points will move a distance having radius 2 and 1 in all directions respectively.

3rd case: When $M > 1$

- When $M > 1$, the floe is known as supersonic flow. Let V=1 units and C=0.5 units so that M=$\frac{V}{C} = \frac{1}{0.5} =$ 2.0, which is greater than unity.
- Let the projectile moves from A to B in 4 seconds. The distance travelled by the projectile in 4 seconds =$4 \times V = 4 \times 1 = 4\,units$. Hence take AB =4 units. The disturbance created at A will move in all directions and in 4 seconds, the radius of disturbance will be equal to $4 \times C = 4 \times 0.5 = 2\,units$. Hence taking A as centre draw a circle with radius equal to 2 units.
- After one second from A, the projectile will be at point 3 and distance A3=$V \times 1 = 1 \times 1 = 1\,unit$. The projectile from point 3 will reach point B in 3 seconds. Hence the disturbance created at point 3 will move in all directions and in 3 seconds, the radius of disturbance from point 3 will be equal to $3 \times C = 3 \times 0.5 = 1.5\,units$.
- Similarly the radius of disturbance at point 2 and 1 will be $2 \times C = 2 \times 0.5 = 1.0\,unit$ and $1 \times C = 1 \times 0.5 = 0.5\,unit$ respectively as shown in fig c.
- In this case the sphere of propagation of disturbance always lags behind the projectile.
- If we draw a tangent to the different circles which represent the propagated spherical waves on both sides, we shall get a cone with vertex at B. This cone is known as **Mach cone**



Propagation of disturbance for different Mach numbers.

Mach angle

This is defined as the half of the angle of the Mach cone. In fig c, angle α is known as Mach angle. In the $\Delta 1BD$ of fig c, the distance 1B= Velocity of projectile=V, the distance 1D= Velocity of sound wave=C. Hence we have

$$sin\alpha = \frac{1D}{1B} = \frac{C}{V} = \frac{1}{\frac{V}{C}} = \frac{1}{M}$$

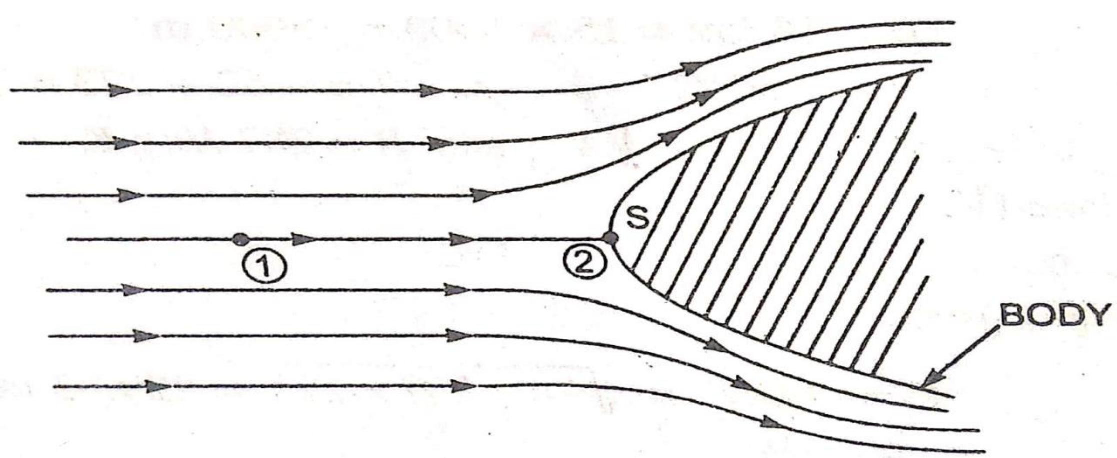
Zone of action

When M> 1, the effect of the disturbance is felt only in the region inside the Mach cone. This region is called the zone of action

Zone of silence

When M> 1, there is no effect of disturbance in the region outside the Mach cone. This region which is outside the Mach cone is called Zone of silence

Stagnation properties



Stagnation properties.

When a fluid is flowing past an immersed body,and at a point on the body if the resultant velocity becomes zero, the values of pressure, temperature and density at that point are called stagnation properties. The point is called the stagnation point. The values of pressure, density and temperature are called stagnation pressure, stagnation density and stagnation temperature respectively. They are denoted by p_s,ρ_s and T_s respectively.

7.1. Expression for stagnation pressure(p_s)

Consider a compressible fluid flowing past an immersed body under frictionless adiabatic conditions as in fig below. Consider two points 1 and 2 on a stream line as shown in fig

Let p_1 =Pressure of compressible fluid at point 1

V_1 = Velocity of fluid at 1 and

ρ_1 = Density of fluid at 1

p_2, V_2 , ρ_2 = Corresponding values of pressure , velocity and density at point 2

Applying Bernoulli's equation for adiabatic flow at point 1 and 2

$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p_1}{\rho_1 g}+\frac{V_1^2}{2g}+Z_1=\left(\frac{\gamma}{\gamma-1}\right)\frac{p_2}{\rho_2 g}+\frac{V_2^2}{2g}+Z_2$$

But $Z_1 = Z_2$

$$\therefore \left(\frac{\gamma}{\gamma-1}\right)\frac{p_1}{\rho_1 g}+\frac{V_1^2}{2g}=\left(\frac{\gamma}{\gamma-1}\right)\frac{p_2}{\rho_2 g}+\frac{V_2^2}{2g}$$

$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p_1}{\rho_1}+\frac{V_1^2}{2}=\left(\frac{\gamma}{\gamma-1}\right)\frac{p_2}{\rho_2}+\frac{V_2^2}{2} \quad \text{(Cancelling } \frac{1}{g}\text{)}$$

Point 2 is a stagnation point. Hence velocity will become zero at stagnation point and pressure, density will be denoted by p_s and ρ_s

V_2 =0, $p_2 = p_s$ and $\rho_2 = \rho_s$

Substituting these values in the above Bernoulli's equation

$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p_1}{\rho_1}+\frac{V_1^2}{2}=\left(\frac{\gamma}{\gamma-1}\right)\frac{p_s}{\rho_s}+0$$

Or $\left(\frac{\gamma}{\gamma-1}\right)\left[\frac{p_1}{\rho_1}-\frac{p_s}{\rho_s}\right]=-\frac{V_1^2}{2}$

Or $\left(\frac{\gamma}{\gamma-1}\right)\frac{p_1}{\rho_1}\left[1-\frac{p_s}{\rho_s}\times\frac{\rho_1}{p_1}\right]=-\frac{V_1^2}{2}$

Or $\left(\frac{\gamma}{\gamma-1}\right)\frac{p_1}{\rho_1}\left[1-\frac{p_s}{p_1}\times\frac{\rho_1}{\rho_s}\right]=-\frac{V_1^2}{2}$ (1)

But for adiabatic process, we have

$$\frac{p}{\rho^\gamma}=Constant$$

or $\frac{p_1}{\rho_1^\gamma}=\frac{p_s}{\rho_s^\gamma}$

or $\frac{p_1}{p_s}=\frac{\rho_1^\gamma}{\rho_s^\gamma}$

or $\left(\frac{\rho_1}{\rho_s}\right)=\left(\frac{p_1}{p_s}\right)^{\frac{1}{\gamma}}$ (2)

Substituting the value of $\frac{\rho_1}{\rho_s}$ in equation (1), we get

$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p_1}{\rho_1}\left[1-\frac{p_s}{p_1}\times\left(\frac{p_1}{p_s}\right)^{\frac{1}{\gamma}}\right]=-\frac{V_1^2}{2}$$

Or $\left(\frac{\gamma}{\gamma-1}\right)\frac{p_1}{\rho_1}\left[1-\frac{p_s}{p_1}\times\left(\frac{p_s}{p_1}\right)^{-\frac{1}{\gamma}}\right]=-\frac{V_1^2}{2}$

Or $\left(\frac{\gamma}{\gamma-1}\right)\frac{p_1}{\rho_1}\left[1-\left(\frac{p_s}{p_1}\right)^{1-\frac{1}{\gamma}}\right]=-\frac{V_1^2}{2}$

Or $\left[1-\left(\frac{p_s}{p_1}\right)^{1-\frac{1}{\gamma}}\right] = -\frac{V_1^2}{2}\times\left(\frac{\gamma-1}{\gamma}\right)\frac{\rho_1}{p_1}$

Or $1+\frac{V_1^2}{2}\left(\frac{\gamma-1}{\gamma}\right)\frac{\rho_1}{p_1} = \left(\frac{p_s}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$ (3)

Now for adiabatic process, the velocity of sound is given by

$C = \sqrt{\gamma RT} = \sqrt{\gamma\frac{p}{\rho}}$ (Since $\frac{p}{\rho} = RT$)

For the point 1, $C_1 = \sqrt{\gamma\frac{p_1}{\rho_1}}$ Or $C_1^2 = \gamma\frac{p_1}{\rho_1}$

Substituting the value of $C_1^2 = \gamma\frac{p_1}{\rho_1}$ in equation (3)

$$1+\frac{V_1^2}{2}\left(\frac{\gamma-1}{\gamma}\right)\frac{1}{\frac{p_1}{\rho_1}} = \left(\frac{p_s}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$1+\frac{V_1^2}{2}(\gamma-1)\frac{1}{C_1^2} = \left(\frac{p_s}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

or $1+\frac{V_1^2}{2C_1^2}(\gamma-1) = \left(\frac{p_s}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$

Or $1+\frac{M_1^2}{2}(\gamma-1) = \left(\frac{p_s}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$ (As $\frac{V_1^2}{C_1^2} = M_1^2$)

Or $\left(\frac{p_s}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left[1+\frac{(\gamma-1)}{2}M_1^2\right]$

Or $\frac{p_s}{p_1} = \left[1+\frac{(\gamma-1)}{2}M_1^2\right]^{\frac{\gamma}{\gamma-1}}$ (4)

$$\therefore p_s = p_1\left[1 + \frac{(\gamma-1)}{2} M_1^2\right]^{\frac{\gamma}{\gamma-1}}$$

Above equation gives the value of stagnation pressure

7.2. Expression for stagnation density(ρ_s)

We have $\left(\frac{\rho_1}{\rho_s}\right) = \left(\frac{p_1}{p_s}\right)^{\frac{1}{\gamma}}$ from equation (2)

Taking reciprocal

$$\left(\frac{\rho_s}{\rho_1}\right) = \left(\frac{p_s}{p_1}\right)^{\frac{1}{\gamma}}$$

$$\rho_s = \rho_1\left(\frac{p_s}{p_1}\right)^{\frac{1}{\gamma}}$$

Substituting the value of $\frac{p_s}{p_1}$ from equation (4)

$$\rho_s = \rho_1\left(\left[1 + \frac{(\gamma-1)}{2} M_1^2\right]^{\frac{\gamma}{\gamma-1}}\right)^{\frac{1}{\gamma}}$$

Or $\rho_s = \rho_1\left[1 + \frac{(\gamma-1)}{2} M_1^2\right]^{\frac{1}{\gamma-1}}$

7.3. Expression for stagnation temperature

Equation of state is given by $\frac{p}{\rho} = RT$

For the stagnation point, we have equation of state as $\frac{p_s}{\rho_s} = RT_s$

$$T_s = \frac{1}{R}\frac{p_s}{\rho_s}$$

Substituting the values of p_s and ρ_s, we get

$$T_s = \frac{1}{R}\frac{p_1\left[1+\frac{(\gamma-1)}{2}M_1^2\right]^{\frac{\gamma}{\gamma-1}}}{\rho_1\left[1+\frac{(\gamma-1)}{2}M_1^2\right]^{\frac{1}{\gamma-1}}}$$

$$= \frac{1}{R}\frac{p_1}{\rho_1}\left[1+\frac{(\gamma-1)}{2}M_1^2\right]^{\frac{\gamma}{\gamma-1}-\frac{1}{\gamma-1}}$$

$$T_s = \frac{1}{R}\frac{p_1}{\rho_1}\left[1+\frac{(\gamma-1)}{2}M_1^2\right]^{\frac{\gamma-1}{\gamma-1}}$$

$$= \frac{p_1}{\rho_1 R}\left[1+\frac{(\gamma-1)}{2}M_1^2\right]$$

$$T_s = \frac{RT_1}{R}\left[1+\frac{(\gamma-1)}{2}M_1^2\right] \quad \left(\text{As} \quad \frac{p_1}{\rho_1} = RT_1\right)$$

$$T_s = T_1\left[1+\frac{(\gamma-1)}{2}M_1^2\right]$$

8. Area velocity relationship for compressible flow

The area velocity relationship for incompressible fluid is given by the continuity equation as

$$A \times V = Constant$$

From the above equation, it is clear that with the increase of area, velocity decreases. But in case of compressible fluid, the continuity equation is given by,

$$\rho \times A \times V = Constant \ (1)$$

From this relation, it is clear that with the change of area, both the velocity and density are affected. Hence to find the relation between area and velocity for compressible fluid we proceed as given below

Differentiating equation (1), we get

$\rho d(AV) + AVd\rho = 0$

$\rho[AdV + VdA]$+AV$d\rho$ = 0

$\rho AdV + \rho VdA +$ AV$d\rho$ = 0

Dividing by ρAV, we get $\frac{dV}{V} + \frac{dA}{A} + \frac{d\rho}{\rho} = 0$ (2)

The Euler's equation for compressible fluid is given by equation

$$\frac{dp}{\rho} + VdV + gdz = 0$$

Neglecting the Z term, the above equation is written as $\frac{dp}{\rho} + VdV = 0$

This equation can also be written as $\frac{dp}{\rho} \times \frac{d\rho}{d\rho} + VdV$ =0 (Dividing and multiplying by $d\rho$)

Or $\frac{dp}{d\rho} \times \frac{d\rho}{\rho} + VdV$ =0

But $\frac{dp}{d\rho} = C^2$

Hence the above equation becomes

$C^2 \times \frac{d\rho}{\rho} + VdV$ =0

Or $C^2 \times \frac{d\rho}{\rho} = -VdV$

Or $\frac{d\rho}{\rho} = \frac{-VdV}{C^2}$

Substituting the value of $\frac{d\rho}{\rho}$ in equation (2), we get

$$\frac{dV}{V} + \frac{dA}{A} - \frac{VdV}{C^2} = 0 \text{ or } \frac{dA}{A} = \frac{VdV}{C^2} - \frac{dV}{V} = \frac{dV}{V}\left[\frac{V^2}{C^2} - 1\right]$$

$$\therefore \frac{dA}{A} = \frac{dV}{V}[M^2 - 1]$$

Above equation gives the relationship between change of area with change of velocity for different Mach numbers.

The following are the important conclusions

i. For M< 1, the flow is sub-sonic and the right hand side of the equation is negative as $[M^2 - 1]$ is negative for values of M< 1. Hence $\frac{dA}{A} > 0$, $\frac{dV}{V} < 0$. This means that with the increase of area, the velocity decreases and vice versa

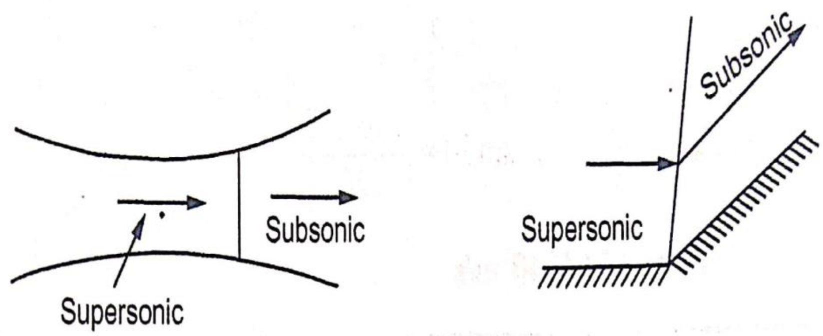
ii. For M> 1, the flow is super-sonic. The value of $[M^2 - 1]$ will be positive and hence right hand side of equation will be positive. Hence $\frac{dA}{A} > 0$, $\frac{dV}{V} > 0$. This means that with the increase of are, velocity also increases

iii. For M=1, the flow is called sonic flow. The value of $[M^2 - 1]$ is zero. Hence right hand side of the equation will be zero. Hence $\frac{dA}{A} = 0$. This means area is constant

9. Normal and Oblique shocks

- For compressible flows with little or small flow turning, the flow process is reversible and the entropy is constant
- The change in flow properties is given by the isentropic relations. But when an object moves faster than the speed of sound, and there is an abrupt decrease in the flow area, the flow process is irreversible and the entropy increases
- Shock waves are generated in very small regions in the gas where the gas properties change by large amount

- Across a shock wave, the static pressure, temperature, and gas density increases almost instantaneously. The mach number and speed of flow decrease across a shock wave
- If the upstream Mach number is in the low supersonic regime, the specific heat ratio of air remains a constant value(1.4) and air is said to be calorifically perfect. But under low hypersonic flow conditions or high total temperature conditions, the specific heat ratio changes and air is then said to be calorifically imperfect
- Because a shock wave does no work and there is no heat addition, the total enthalpy and the total temperature are constant through the shock. But because the flow is non-isentropic, the total pressure downstream of the shock is always less than the total pressure upstream of the shock; there is a loss of total pressure associated with a shock wave
- If the shock wave is perpendicular to the flow direction it is called a normal shock
- Depending upon the shape of the object and the speed of the floe , the shock wave may be inclined to the flow direction. When a shock wave is inclined to the flow direction it is called an oblique shock



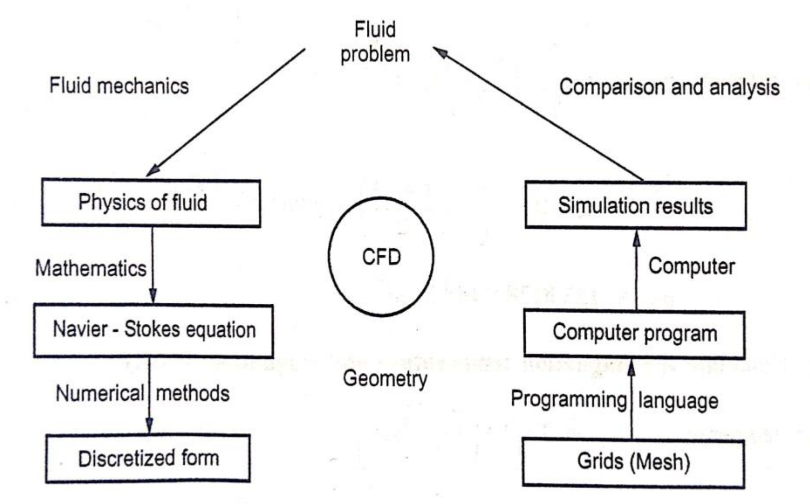
(a) Normal shock (b) Oblique shock

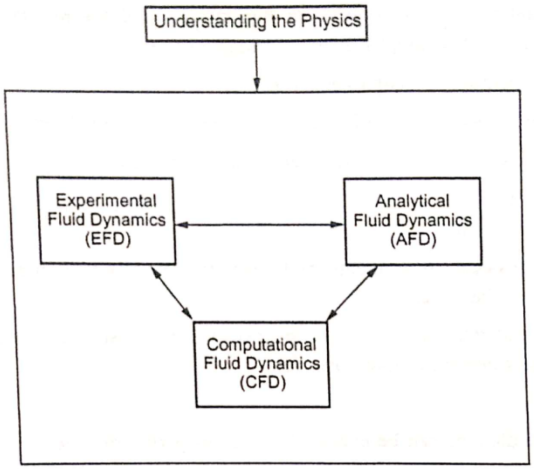
- Oblique shocks are generated by the nose and the leading edge of the wing and tail of a supersonic aircraft
- Oblique shocks are also generated at the trailing edges of the aircraft as the flow is brought back to free stream conditions. Oblique shocks also occur downstream of a nozzle if the expanded pressure is different from free steam conditions
- In high speed inlets, oblique shocks are used to compress the air going into the engine. The air pressure is increased without using any rotating machinery

- Similar to a normal shock wave, the oblique shock wave consists of a very thin region across which nearly discontinuous changes in the thermodynamic properties of a gas occur. While the upstream and downstream flow directions are unchanged across a normal shock, they are different for flow across an oblique shock wave

Computational Fluid Dynamics (CFD)

- Computational fluid dynamics, usually abbreviated as CFD, is a branch of fluid mechanics which uses numerical methods and algorithms to solve and analyze problems that involve fluid flows
- In this technique, computers are used to perform the calculations required to simulate the interaction of liquids and gases with surfaces defined by boundary conditions
- With high speed supercomputers, better solutions can be achieved. Ongoing research yields software that improves the accuracy and speed of complex simulation scenarios like transonic or turbulent flows
- In case of engineering problems the equations become more difficult. For example, problems involving chemical reactions, heat transfer, multiphase flows etc. In all these problems, CFD method is an effective tool to get solutions
- The fundamental basis of almost all CFD problems is the Navier-Stokes equations which define any single-phase fluid flow. These equations can be simplified by removing terms describing viscosity to yield the Eulers equations





Fluid dynamics is the science of fluid motion. Fluid flow is commonly studied in one of three ways

1. Experimental fluid dynamics
2. Analytical fluid dynamics
3. Computational fluid dynamics

In computational fluid dynamics approach the following procedure is used

i) During preprocessing

- The geometry of the problem is defined
- The volume occupied by the fluid is divided into discrete cells(mesh). The mesh may be uniform or non uniform
- The physical modelling is defined-for example, the equations of motions, enthalpy, radiation etc
- Boundary conditions are defined. This involves specifying the fluid behaviour and properties at the boundaries of the problem. For transient problems, the initial conditions are also defined

ii) The simulation is started and the equations are solved iteratively as a steady state or transient

iii) Finally a postprocessor is used for the analysis and the results are interpreted in the visual forms like graphs, images, animations etc

The following methods are commonly used

i. Finite volume method

ii. Finite element method

iii. Finite difference method
iv. Spectral element method
v. Boundary element method

Advantages of CFD

- Relatively low cost
    - Using physical experiments and tests to get essential engineering data for design can be expensive
    - CFD simulations are relatively inexpensive, and costs are likely to decrease as computers becomes more powerful
- Speed
    - CFD simulations can be executed in a short period of time
    - Quick turnaround means engineering data can be introduced early in the design process
- Ability to simulate real conditions
    - Many flow and heat transfer processes can not be (easily) tested, ex hypersonic flow
    - CFD provides the ability to theoretically simulate any physical condition
- Comprehensive information
    - CFD allows the analyst to examine a large number of locations in the region of interest, and yields a comprehensive set of flow parameters for examination

Applications of CFD

- Design of vehicles to improve aerodynamic characteristics
- It is widely used in design of systems like pipes, nozzles, turbomachinery, aerodynamic objects etc
- It is used in medical field to cure diseases
- It can be used effectively for simulation in various fields like flows of gases and liquids, heat and mass transfer, bodies in motion, chemical reactions etc

Limitations of CFD

- It requires fair amount of knowledge and experience with both the CFD and the phenomenon under investigation
- The cost of tools or software required for CFD is very high

- The solution of CFD problem is not fully reliable
- To solve the CFD problem, it requires large number of input data
- The accuracy of the CFD solution is only as good as the initial/boundary conditions provided for the numerical model

10. Theory question bank

1. Define the following terms
   i. Subsonic flow ii. Sonic flow iii. Super-sonic flow iv. Mach number***(JUNE/JULY 2019)***
2. Derive an expression for velocity of sound in terms of bulk modulus***(JUNE/JULY 2019)***
3. Explain the necessity of CFD. Mention its applications and limitations***(JUNE/JULY 2019)***
4. What are normal and oblique shocks. Explain ***(JUNE/JULY 2019)***
5. Define CFD. Mention the applications of CFD***(DEC 2018/JAN 2019)***
6. Define stagnation properties. Obtain an expression for stagnation pressure of a compressible fluid in terms of mach number and pressure***(DEC 2018/JAN 2019)***
7. What are normal and oblique shocks***(DEC 2018/JAN 2019)***
8. Show that velocity of propagation of elastic wave in an adiabatic medium is given by $C=\sqrt{\gamma RT}$ starting from fundamentals***(DEC 2018/JAN 2019)***
9. Derive an expression for velocity of sound in a fluid***(JUNE/JULY 2018)***
10. Explain the meaning of CFD and its applications***(JUNE/JULY 2018)***
11. Define the following terms i. Mach number ii. Mach cone iii. Zone of action iv. Sonic flow v. supersonic flow ***(JUNE/JULY 2018)***
12. Define i. Mach line ii. Mach angle iii. Subsonic and supersonic flow***(DEC 2017/JAN 2018)***
13. Write short essay on the engineering application of CFD, bringing the advantages and the limitations***(DEC 2017/JAN 2018)***
14. Define the following terms and write the relevant equations for the same
    i. Stagnation temperature
    ii. Stagnation pressure ***(DEC 2017/JAN 2018)***

11. Problems Question Bank

1. Find the velocity of bullet fired in standard air if Mach angle is 30°, Take R=287.14J/kgK and $\gamma = 1.4$ for air and temperature of air is 15℃ ***(JUNE/JULY 2019)***
2. An air plane is flying at an altitude of 15 km where the temperature is -50 ℃. The speed of plane corresponds to Mach number 1.6. Assume $\gamma = 1.4$ and R=287 J/kgK for air. Find the speed of plane and Mach angle ***(JUNE/JULY 2018)***
3. A projectile travels in air of pressure 15 N/cm^2 at 10℃, at speed of 1500 km/hr. find the Mach number and Mach angle. Assume $\gamma = 1.4$ and R=287 J/kgK***(DEC 2018)***
4. An aeroplane is flying at 950 km/hr through still air having an absolute pressure of 80 KN/m^2 and temperature -7℃. Calculate stagnation pressure, stagnation temperature and stagnation density, on the stagnation point on the nose of the plane. Take $\gamma = 1.4$ and R=287 J/kgK for air ***(DEC 2018)***
5. Calculate the velocity and Mach number of a supersonic aircraft flying at an altitude of 1000 m where temperature is 280 K, sound of the aircraft is heard 2.15 seconds after the passage of air craft on the head of an observer. Take $\gamma = 1.41$ and R=287 J/kgK for air***(JUNE/JULY 2017, DEC2017/JAN 2018)***

Assignment Questions

1. A projectile travels at speed of 1500 km/hour at 20℃ temperature and 0.1 MPa air pressure. Calculate the Mach number and Mach angle. Take $\gamma = 1.4$ for air and R=287 J/kgK***(JUNE/JULY 2016, DEC 2018/JAN 2019)***
2. Calculate the Mach number at a point on a jet propelled air craft which is flying at 1100 km/hr at sea level where air temperature is 20℃. Assume $\gamma =$ 1.4 and R=287 J/kgK
3. A gas is flowing through a horizontal pipe having cross sectional area 45 cm^2 and pressure 50 N/cm^2 (absolute) and temperature 12℃. At another section the cross section is reduced to 25 cm^2 and pressure 40 N/cm^2 (absolute). Assuming isothermal process, find the velocities at these sections if the mass rate of flow of gas is 0.52 kg/s. Take gas constant R= 293 Nm/kgK
4. Find the sonic velocity for the following fluids
   i. Crude oil of sp.gr 0.8 and bulk modulus 153036 N/cm^2
   ii. Mercury having a bulk modulus of 2648700 N/cm^2

12. Problems Question Bank solution

1. **Find the velocity of bullet fired in standard air if Mach angle is 30°, Take R=287.14J/kgK and $\gamma = 1.4$ for air and temperature of air is 15℃ *(JUNE/JULY 2019)***

   Solution:

   T= 15℃ $= 288\ K$

   Velocity of sound wave is C=$\sqrt{\gamma RT}$=$\sqrt{1.4 \times 287.14 \times 288}$ = 340.257m/s

   Mach angle is given by

   $$sin\alpha = \frac{C}{V}$$

   $$\therefore\ sin30 = \frac{340.257}{V}$$

   $V = 680.5140$ m/s

2. **An air plane is flying at an altitude of 15 km where the temperature is -50 ℃. The speed of plane corresponds to Mach number 1.6. Assume $\gamma = 1.4$ and R=287 J/kgK for air. Find the speed of plane and Mach angle *(JUNE/JULY 2018)***

   Solution:

   h=15 km= 15×10^3m, T= -50 ℃= 223 K, M=1.6, $\gamma = 1.4$, R=287 J/kgK

   Velocity of sound wave is C=$\sqrt{\gamma RT}$=$\sqrt{1.4 \times 287 \times 223}$ = 299.3349 m/s

   Mach number is given by

   M=$\frac{V}{C} = \frac{V}{299.3349} = 1.6$

   V= 478.9358 m/s

   Velocity of plane, V= 478.9358 m/s

   Mach angle is given by

   $sin\alpha = \frac{1}{M} = \frac{1}{1.6}$

   $\alpha = 38.6821°$

3. **A projectile travels in air of pressure 15 N/cm² at 10℃, at speed of 1500 km/hr. find the Mach number and Mach angle. Assume $\gamma = 1.4$ and R=287 J/kgK*(DEC 2018)***

   Solution:

   T=10℃ $= 283\ K$,V= 1500 km/hr = 416.667 m/s ,$\gamma = 1.4$, R=287 J/kgK

Velocity of sound wave is C=$\sqrt{\gamma RT}$=$\sqrt{1.4 \times 287 \times 283}$ = 337.208 m/s

Mach number is given by

M=$\frac{V}{C} = \frac{416.667}{337.208} = 1.235$

Mach angle is given by

$sin\alpha = \frac{1}{M} = \frac{1}{1.235}$

$\alpha = 54.068°$

4. **An aeroplane is flying at 950 km/hr through still air having an absolute pressure of 80 KN/m² and temperature -7℃. Calculate stagnation pressure, stagnation temperature and stagnation density, on the stagnation point on the nose of the plane. Take $\gamma = 1.4$ and R=287 J/kgK for air *(DEC 2018)***
   Solution:

V= 950 km/hr=263.8888 m/s, $p_1 = 80\times 10^3$ N/m² T= -7℃=266 K, $\gamma = 1.4$,

R=287 J/kgK

Velocity of sound wave is C=$\sqrt{\gamma RT}$=$\sqrt{1.4 \times 287 \times 266}$ = 326.9232 m/s

Mach number is given by

M=$\frac{V}{C} = \frac{263.8888}{326.9232} = 0.8071$

Stagnation pressure

$$p_s = p_1\left[1 + \frac{(\gamma-1)}{2}M_1^2\right]^{\frac{\gamma}{\gamma-1}}$$

$$= 80\times 10^3\left[1 + \frac{(1.4-1)}{2}(0.8071)^2\right]^{\frac{1.4}{1.4-1}}$$

$= 122.8128 \times 10^3$ N/m²

Stagnation temperature

$$T_s = T_1\left[1 + \frac{(\gamma-1)}{2}M_1^2\right]$$

$$= 266\left(1 + \frac{(1.4-1)}{2}(0.8071)^2\right)$$

= 300.655 K

Stagnation density

$$\rho_s = \rho\left[1 + \frac{(\gamma - 1)}{2} M_1^2\right]^{\frac{1}{\gamma-1}}$$

$ideal\ gas\ equation\ p = \rho RT$

$\therefore 80 \times 10^3 = \rho \times 288 \times 266$

$\rho = 1.0479\ kg/m^3$

$$\rho_s = \rho\left[1 + \frac{(\gamma-1)}{2} M_1^2\right]^{\frac{1}{\gamma-1}}$$

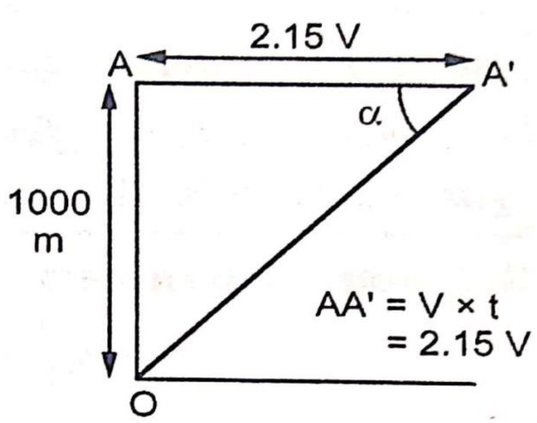
$$= 1.0479\left[1 + \frac{(1.4-1)}{2}(0.8071)^2\right]^{\frac{1}{1.4-1}}$$

= 1.9122 kg/m^3

5. **Calculate the velocity and Mach number of a supersonic aircraft flying at an altitude of 1000 m where temperature is 280 K, sound of the aircraft is heard 2.15 seconds after the passage of air craft on the head of an observer. Take $\gamma = 1.41$ and R=287 J/kgK for air*(JUNE/JULY 2017, DEC2017/JAN 2018)***
Solution:

H= 1000 m, T=280 K, t= 2.15 sec, γ = 1.41, R=287 J/kgK

Velocity of sound wave is C=$\sqrt{\gamma RT}$=$\sqrt{1.41 \times 287 \times 280}$ = 336.6119 m/s



In the given fig, O denotes the position of observer and A denotes position of air craft just above the observer

The aircraft reaches position A' after 2.15 seconds. Here α denotes Mach angle and OA' denotes the wave front

From fig

$tan\alpha = \frac{1000}{2.15V} = \frac{465.116}{V}$ (1)

Mach number is given by

M=$\frac{V}{C} = \frac{1}{sin\alpha}$

V= $\frac{C}{sin\alpha} = \frac{336.6119}{sin\alpha}$ (2)

Substituting (2) in (1)

$$tan\alpha = \frac{465.116}{\frac{336.6119}{sin\alpha}}$$

$$\frac{sin\alpha}{cos\alpha} = 1.3817\, sin\alpha$$

$$cos\alpha = \frac{1}{1.3817} = 0.7237$$

$\alpha = 43.6391°$

M=$\frac{V}{C} = \frac{1}{sin\alpha}$

$$\frac{V}{465.116} = \frac{1}{sin 43.6391}$$

V= 487.763 m/s

Assignment Questions Solution:

1. **A projectile travels at speed of 1500 km/hour at 20℃ temperature and 0.1 MPa air pressure. Calculate the Mach number and Mach angle. Take $\gamma = 1.4$ for air and R=287 J/kgK*(JUNE/JULY 2016, DEC 2018/JAN 2019)***
Solution:

T= 20℃ = 293 K V= 1500 km/hr = 416.6667m/s

Velocity of sound wave is C=$\sqrt{\gamma RT}$=$\sqrt{1.4 \times 287 \times 293}$ = 343.1142 m/s

Mach number is given by

M=$\frac{V}{C} = \frac{416.6667}{343.1142} = 1.2143$

Mach angle is given by

$sin\alpha = \frac{C}{V} = \frac{1}{M} = \frac{1}{1.2143}$

$\alpha = 55.436°$

2. **Calculate the Mach number at a point on a jet propelled air craft which is flying at 1100 km/hr at sea level where air temperature is 20℃. Assume $\gamma = 1.4$ and R=287 J/kgK**
   Solution:

   V= 1100 km/hr= 305.555 m/s, T=20℃ $= 293\ K, \gamma = 1.4$, R=287 J/kgK

   Velocity of sound wave is C=$\sqrt{\gamma RT}$=$\sqrt{1.4 \times 287 \times 293}$ = 343.114 m/s

   Mach number is given by

   M=$\frac{V}{C} = \frac{305.555}{343.114} = 0.8905$

3. **A gas is flowing through a horizontal pipe having cross sectional area 45 cm^2 and pressure 50 N/cm^2 (absolute) and temperature 12℃. At another section the cross section is reduced to 25 cm^2 and pressure 40 N/cm^2 (absolute). Assuming isothermal process, find the velocities at these sections if the mass rate of flow of gas is 0.52 kg/s. Take gas constant R= 293 Nm/kgK**
   Solution:

   A_1= 45 cm^2= 45$\times 10^{-4}$ m^2, p_1 = 50 N/cm^2=50 $\times 10^4$ N/m^2, T_1=12℃ = 285 K

   A_2= 25 cm^2= 25$\times 10^{-4}$ m^2, p_2 = 40 N/cm^2=40 $\times 10^4$ N/m^2, R= 292 Nm/kgK, $\dot{m}$ =0.52 kg/s

   Ideal gas equation

   For section 1

   p_1=$\rho_1 RT$

   $50 \times 10^4 = \rho_1 \times 292 \times 285$

   $\rho_1 = 6.0081\ kg/m^3$

   For isothermal process

   $\frac{p_1}{\rho_1} = \frac{p_2}{\rho_2}$

   $\therefore \frac{50 \times 10^4}{6.0081} = \frac{40 \times 10^4}{\rho_2}$

   $\rho_2 = 4.8064\ kg/m^3$

Mass flow rate $\dot{m} = \rho_1 A_1 V_1 = \rho_2 A_2 V_2$

$\dot{m} = \rho_1 A_1 V_1$

$0.52 = 6.0081 \times 45 \times 10^{-4} \times V_1$

$V_1 = 19.2332\ m/s$

$\dot{m} = \rho_2 A_2 V_2$

$0.52 = 4.8064 \times 25 \times 10^{-4} \times V_2$

$V_2 = 43.2756\ m/s$

4. **Find the sonic velocity for the following fluids**

i. Crude oil of sp.gr 0.8 and bulk modulus 153036 N/cm²

ii. Mercury having a bulk modulus of 2648700 N/cm²

Solution:

i. For crude oil sp gr =0.8

$\therefore$ Density $\rho = 0.8 \times 1000 = 800\ kg/m^3$

Bulk modulus K =153036 N/cm² =153036× 10^4 N/m²

For sonic velocity C= $\sqrt{\frac{K}{\rho}} = \sqrt{\frac{153036\times 10^4}{800}}$ =1383 m/s

ii. For mercury sp gr=13.6

$\therefore$ Density $\rho = 13.6 \times 1000 = 13600\ kg/m^3$

Bulk modulus K = 2648700N/cm² =2648700 × 10^4 N/m²

For sonic velocity C= $\sqrt{\frac{K}{\rho}} = \sqrt{\frac{2648700\times 10^4}{13600}}$ =1395.55 m/s