

Normal forms:-

There are two types of normal forms

- i) CNF (Chomsky normal form)
- ii) GNF (Greibach normal form)

i) CNF (Chomsky normal form):-

Let  $G = (V, T, P, S)$  be a CFG. The grammar  $G$  is said to be in CNF if all the productions are of the form

$$A \rightarrow BC$$

$A \rightarrow a$ , where  $A, B, C$  are variables and ' $a$ ' is a terminal. To

get the CFG into CNF form the prerequisites are

- i) eliminate the useless symbols.
- ii) eliminate the  $\epsilon$ -productions
- iii) eliminate the unit productions.

ii) Elimination of useless symbols:-

'X' is a useful grammar for  $G = (V, T, P, S)$ , it should satisfy the following two conditions

- i) X should generate some terminal
- ii) X is reachable from start symbol

Eliminate useless symbols from the following

1.  $S \rightarrow aAa|aBC$

$A \rightarrow aS|b$

$B \rightarrow aBa|b$

$C \rightarrow abb|b$

$D \rightarrow aDa$

D is not generating a terminal. So, it is a useless symbol, eliminate D.

Eliminate D:-

$S \rightarrow aAa|aBC$

$A \rightarrow aS$

$B \rightarrow aBa|b$

$C \rightarrow abb$

2.  $S \rightarrow aA|bB$

$A \rightarrow aA|a$

$B \rightarrow bB$

~~$D \rightarrow ab|Ea$~~

$E \rightarrow ac|d$

D is not reachable from start symbol, so eliminate D

Eliminate D:-

$S \rightarrow aA|bB$

$A \rightarrow aA|a$

$B \rightarrow bB$

~~$E \rightarrow ac|d$~~

E is dependent on D and is not reachable from start symbol, so eliminate E

$S \rightarrow aA|bB$

$A \rightarrow aA|a$

$B \rightarrow bB$

B is not generating a terminal, so eliminate B

|                      |
|----------------------|
| $S \rightarrow aA$   |
| $A \rightarrow aA a$ |

3.  $S \rightarrow aA|a|Bb|c$

$A \rightarrow aB$

$B \rightarrow a|Aa$

$C \rightarrow cCD$

$D \rightarrow ddd$

C is not generating a terminal, so eliminate C

$S \rightarrow aA|a|Bb$

$A \rightarrow aB$

$B \rightarrow a|Aa$

~~$D \rightarrow ddd$~~

Eliminate D as D is dependent on C and is not reachable from start symbol

|                         |
|-------------------------|
| $S \rightarrow aA a Bb$ |
| $A \rightarrow aB$      |
| $B \rightarrow a Aa$    |

2) Eliminating  $\epsilon$ -productions:-

A production of the form  $A \rightarrow \epsilon$  is undesirable in CFG unless an empty string is derived from the start symbol (i.e., don't eliminate from start symbol)

Example:-  $S \rightarrow Aab$

$A \rightarrow aA | \epsilon$

Eliminate  $\epsilon$ -productions

|                                                    |
|----------------------------------------------------|
| $S \rightarrow Aab   ab$<br>$A \rightarrow aA   a$ |
|----------------------------------------------------|

Eliminate  $\epsilon$  productions from the following grammar

1.  $S \rightarrow AB$

$A \rightarrow aB | \epsilon$

$B \rightarrow bB | \epsilon$

Eliminate  $\epsilon$ -productions

|                                                                                           |
|-------------------------------------------------------------------------------------------|
| $S \rightarrow AB   A   B   \epsilon$<br>$A \rightarrow aB   a$<br>$B \rightarrow bB   b$ |
|-------------------------------------------------------------------------------------------|

2.  $S \rightarrow ABCa | bD$

$A \rightarrow BC | b$

$B \rightarrow b | \epsilon$

$C \rightarrow c | \epsilon$

$D \rightarrow d$

Eliminate  $\epsilon$ -productions

|                                                                                                                                                                 |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $S \rightarrow ABCa   bD   a   ABa   ACa   BCa   Ca   Aa   Ba$<br>$A \rightarrow BC   b   C   B$<br>$B \rightarrow b$<br>$C \rightarrow c$<br>$D \rightarrow d$ |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------|

3.  $S \rightarrow ABC$

$A \rightarrow BC | a$

$B \rightarrow bAC | \epsilon$

$C \rightarrow cAB | \epsilon$

Eliminate  $\epsilon$ -productions

$$\begin{aligned} S &\rightarrow ABC|A|B|C|AB|AC|BC|\epsilon \\ A &\rightarrow BC|a|B|C \\ B &\rightarrow bAC|bA|bC|b \\ C &\rightarrow CAB|C|A|B|CA|CB|AB \end{aligned}$$

4.  $S \rightarrow BAAB$

$A \rightarrow OAa|aA|\epsilon$

$B \rightarrow AB|IB|\epsilon$

Eliminate  $\epsilon$ -productions

$$\begin{aligned} S &\rightarrow BAAB|AAB|BAB|BAA|AA|BB|A|B|\epsilon|BA|AB \\ A &\rightarrow OAa|aA|Oa|aO \\ B &\rightarrow AB|IB|A|B|I \end{aligned}$$

3> Eliminating unit productions:-

Consider the production  $A \rightarrow B$ , the LHS of the production and RHS of the productions contain only one variable, such productions are called unit productions.

Eliminate unit productions from the following grammar

1.  $A \rightarrow B$

$B \rightarrow aB|b$

$$\begin{aligned} A &\rightarrow aB|b \\ B &\rightarrow aB|b \end{aligned}$$

2.  $S \rightarrow AB$

$A \rightarrow a$

$B \rightarrow C|b$

$C \rightarrow D$

$D \rightarrow E|bc$

$E \rightarrow d|Ab$

$$\begin{aligned} S &\rightarrow AB \\ A &\rightarrow a \\ B &\rightarrow d|Ab|bc|b \\ C &\rightarrow d|Ab|bc \\ D &\rightarrow d|Ab|bc \\ E &\rightarrow d|Ab \end{aligned}$$

$$\begin{aligned}
 3. \quad S &\rightarrow A0|B \\
 B &\rightarrow A|11 \\
 A &\rightarrow 0|12|B
 \end{aligned}$$

$$\begin{aligned}
 S &\rightarrow A0|0|12|11 \\
 B &\rightarrow 0|12|11 \\
 A &\rightarrow 0|12|11
 \end{aligned}$$

Eliminate useless symbol and unit production

$$\begin{aligned}
 1. \quad S &\rightarrow Aa|B|Ca \\
 B &\rightarrow aB|b \\
 C &\rightarrow Db|D \\
 D &\rightarrow E|d \\
 E &\rightarrow ab
 \end{aligned}$$

$$ddd|aac|dd|ac$$

Eliminate A as A is not generating terminal, it is useless symbol

$$\begin{aligned}
 S &\rightarrow B|Ca \\
 B &\rightarrow aB|b \\
 C &\rightarrow Db|D \\
 D &\rightarrow E|d \\
 E &\rightarrow ab
 \end{aligned}$$

Eliminate unit productions

$$\begin{aligned}
 S &\rightarrow aB|b|Ca \\
 B &\rightarrow aB|b \\
 C &\rightarrow Db|ab|d \\
 D &\rightarrow ab|d \\
 E &\rightarrow ab
 \end{aligned}$$

$$\begin{aligned}
 2. \quad S &\rightarrow aAa|bBb|e \\
 A &\rightarrow C|a \\
 B &\rightarrow C|b \\
 C &\rightarrow CDE|E \\
 D &\rightarrow A|B|ab
 \end{aligned}$$

Eliminate E as E is not generating terminal. It is useless symbol

$$S \rightarrow aAa|bBb|\epsilon$$

$$A \rightarrow c|a$$

$$B \rightarrow c|b$$

$$C \rightarrow \epsilon$$

$$D \rightarrow A|B|ab$$

Eliminate D as it is not reachable from start state

$$S \rightarrow aAa|bBb|\epsilon$$

$$A \rightarrow c|a$$

$$B \rightarrow c|b$$

$$C \rightarrow \epsilon$$

Eliminate  $\epsilon$  productions

$$S \rightarrow aAa|bBb|\epsilon|aa|bb$$

$$A \rightarrow a$$

$$B \rightarrow b$$

Convert

$$1. S \rightarrow AB|a$$

$$A \rightarrow aab$$

$$B \rightarrow Ac$$

No useless,  $\epsilon$ -production and unit productions.

$$S \rightarrow AB|a$$

$$A \rightarrow A_0A_0B_0$$

$$B \rightarrow AC_0$$

$$A_0 \rightarrow a$$

$$B_0 \rightarrow b$$

$$C_0 \rightarrow c$$

$$S \rightarrow AB|a$$

$$A \rightarrow A_0B_0$$

$$B \rightarrow AC_0$$

$$A_0 \rightarrow a$$

$$B_0 \rightarrow b$$

$$C_0 \rightarrow c$$

$$A_1 \rightarrow A_0A_0$$

$$\begin{aligned} S &\rightarrow AB_1 | 0 \\ A &\rightarrow 00A | 11 \\ B &\rightarrow 1A1 \end{aligned}$$

No useless,  $\epsilon$  and unit productions

$$\begin{aligned} S &\rightarrow ABB_0 | 0 \\ A &\rightarrow A_0A_0A | B_0B_0 \\ B &\rightarrow B_0AB_0 \\ A_0 &\rightarrow 0 \\ B_0 &\rightarrow 1 \end{aligned}$$

$$\begin{aligned} S &\rightarrow AB_1 | 0 \\ A &\rightarrow A_1A | B_0B_0 \\ B &\rightarrow B_0B_0 \\ A_0 &\rightarrow 0 \\ B_0 &\rightarrow 1 \\ A_1 &\rightarrow A_0A_0 \\ B_1 &\rightarrow BB_0 \\ B_2 &\rightarrow B_0A \end{aligned}$$

$$\begin{aligned} 3. S &\rightarrow aSb | ab | Aa \\ A &\rightarrow aab \end{aligned}$$

No useless,  $\epsilon$  and unit productions

$$\begin{aligned} S &\rightarrow A_0SB_0 | A_0B_0 | AA_0 \\ A &\rightarrow A_0A_0B_0 \\ A_0 &\rightarrow a \\ B_0 &\rightarrow b \end{aligned}$$

$$\begin{aligned} S &\rightarrow A_1B_0 | A_0B_0 | AA_0 \\ A &\rightarrow A_2B_0 \\ A_0 &\rightarrow a \\ B_0 &\rightarrow b \\ A_1 &\rightarrow A_0S \\ A_2 &\rightarrow A_0A_0 \end{aligned}$$

$$4. S \rightarrow aB/bA$$

$$A \rightarrow a/aS/bAA$$

$$B \rightarrow b/aS/aBB$$

No useless,  $\epsilon$  and unit productions

$$S \rightarrow A_0B/B_0A$$

$$A \rightarrow a/A_0S/B_0AA$$

$$B \rightarrow b/A_0S/A_0BB$$

$$A_0 \rightarrow a$$

$$B_0 \rightarrow b$$

$$S \rightarrow A_0B/B_0A$$

$$A \rightarrow a/A_0S/B_1A$$

$$B \rightarrow b/A_0S/A_1B$$

$$A_0 \rightarrow a$$

$$B_0 \rightarrow b$$

$$B_1 \rightarrow B_0A$$

$$A_1 \rightarrow A_0B$$

$$5. S \rightarrow AaA/CA/BaB$$

$$A \rightarrow aaBa/cDa/aa/DC$$

$$B \rightarrow bB/bAB/bb/aS$$

$$C \rightarrow Ca/bC/D$$

$$D \rightarrow bD/\epsilon$$

Eliminate  $\epsilon$ -productions:-

$$S \rightarrow AaA/CA/BaB$$

$$A \rightarrow aaBa/cDa/aa/DC/Ca/Da/c/D$$

$$B \rightarrow bB/bAB/bb/aS$$

$$C \rightarrow Ca/bC/a/b$$

$$D \rightarrow bD/b$$

Eliminating unit productions:-

$$S \rightarrow AaA \mid CA \mid BaB$$

$$A \rightarrow aaBa \mid CDa \mid aa \mid DC \mid Ca \mid bc \mid a/b \mid Da \mid bD \mid b$$

$$B \rightarrow bB \mid bAB \mid bb \mid aS$$

$$C \rightarrow Ca \mid bc \mid a/b$$

$$D \rightarrow bD \mid b$$

