Module 2

AC Fundamentals :

Equation of AC Voltage and Current, waveform, time period, frequency, amplitude, phase, phase difference, average value, RMS Value, form factor, peak factor (only definitions)

Voltage and Current Relationship with Phasor diagram in R, L & C circuits. Concept of impedance. Analysis R-L, R-C, R-L-C Series circuits. Active Power, Reactive Power and apparent Power. Concept of Power Factor (simple Numerical)

Three Phase Circuits

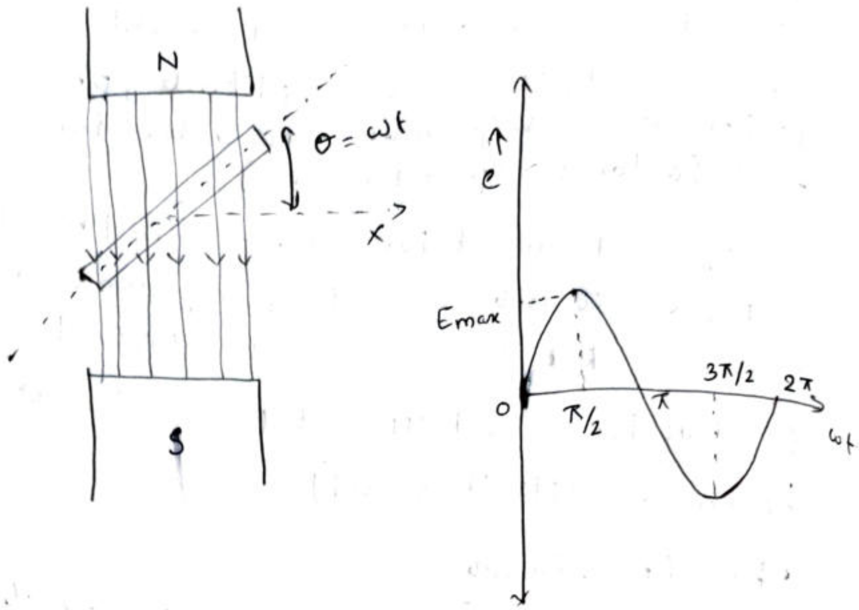
Generation of three phase AC quantity. Advantages and limitation, star and delta connection, relationship between line and phase quantities (excluding proof).

Equation of Alternating Voltage & Current (Generation of Alternating Voltage & Current)

An Alternating voltage may be generated by rotating a coil within a uniform magnetic field or by rotating a magnetic field within a stationary coil.

The magnitude of the emf generated depends on the number of turns in the ~~mag~~ coil, strength of the magnetic field & the speed at which the coil or magnetic field rotates.

Consider a coil of N turns rotating with an angular velocity of ω radians per second in a uniform magnetic field as shown in the fig. below.



When the plane of the coil coincides with the x axis, the flux ϕ linking with the coil has its maximum value ϕ_{max} & when it is perpendicular to the x axis the flux linkage is zero.

Hence, the flux linkage at an instant is $\phi = \phi_{max} \cos \omega t$

But according to Faraday's law of electro-magnetic induction

$$e = -N \frac{d\phi}{dt}$$

$$= -N \frac{d}{dt} (\phi_{max} \cos \omega t)$$

$$= N \omega \phi_{max} \sin \omega t$$

$$\boxed{e = E_{max} \sin \omega t} \quad \text{where} \quad \boxed{E_m = N \omega \phi_{max}}$$

The emf plotted against time will be a sinusoidal waveform as shown in fig.

Important definitions

Alternations & cycle

When an alternating quantity goes through one half cycle it completes an alternation.

When an alternating quantity goes through a complete set of +ve and -ve values, it is said to have completed one cycle.

Time Period (T)

The time taken in seconds to complete one cycle is called as Time period (T). Unit is seconds

frequency (f)

The number of cycles completed per seconds by an alternating quantity is known as frequency (f)
Unit of frequency is Hertz (Hz)

$$f = 1/T$$

In India, the standard frequency for power supply is 50Hz.

Amplitude

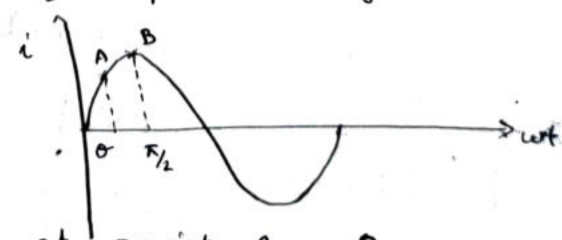
The maximum value attained by an alternating quantity is called as its amplitude or peak value.

Instantaneous Value

The value of an alternating quantity at any instant is called instantaneous value.

Phase

The phase of an alternating quantity at any instant is the fractional part of time period or cycle through which the quantity has advanced from the selected zero position of reference



Phase of current at point A = θ
Phase of current at point B = π/2

Average Value

The arithmetic average of all the values of an alternating quantity over one cycle is called as average value ie

$$\text{Average value} = \frac{\text{Average of 1 cycle}}{\text{base}}.$$

In the case of symmetrical wave, the +ve half is exactly equal to the -ve half cycle. So the average value over a complete cycle is equal to zero. Hence,

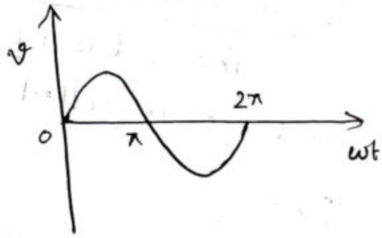
For a symmetrical wave, the average value ~~m~~ is the average of one half cycle or alternation.

ie
$$\boxed{\text{Average Value of a Symmetrical wave} = \frac{\text{Area of } \tfrac{1}{2} \text{ cycle}}{\text{base}}}$$

Average Value of an Alternating Quantity

The equation of a sinusoidally varying alternating quantity, voltage is given by

$$v = V_m \sin \omega t$$



$$V_{avg} = \frac{\text{Area of 1 alternation}}{\text{base}}$$

$$= \frac{1}{\pi} \int_0^{\pi} V_m \sin \omega t \, d(\omega t)$$

$$= \frac{V_m}{\pi} \left[-\cos \omega t \right]_0^{\pi} = \frac{V_m}{\pi} \times - \left[\cos \pi - \cos 0 \right]$$

$$= \frac{V_m}{\pi} \times 2 = 0.637 V_m$$

ie $\boxed{V_{avg} = 0.637 V_m}$

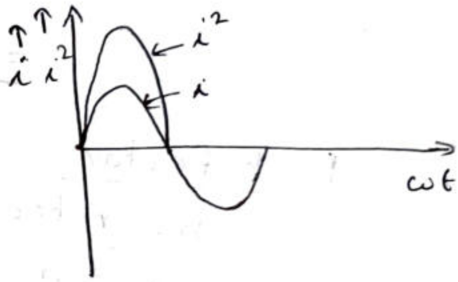
|||^{ly} $\boxed{I_{avg} = 0.637 I_m}$

RMS Value (Effective Value)

The rms value of an alternating ~~quantity~~ current is defined as that steady current which when flowing through a given resistance for a given time produces the same amount of heat as produced by the alternating current, when flowing through the same resistance for the same time.

RMS Value of an alternating quantity

A sinusoidal alternating current is represented by $i = I_m \sin \omega t$



$$I^2_{rms} = \frac{\text{Area of } \tfrac{1}{2} \text{ cycle}}{\pi}$$

$$= \frac{1}{\pi} \int_0^{\pi} i^2 \, d(\omega t)$$

$$= \frac{1}{\pi} \int_0^{2} (I_m \sin \omega t)^2 \, d(\omega t)$$

$$= \frac{I_m^2}{\pi} \int_0^{\pi} \sin^2 \omega t \, d(\omega t)$$

$$= \frac{I_m^2}{\pi} \int_0^{\pi} \left(\frac{1 - \cos 2\omega t}{2} \right) d\omega t$$

$$= \frac{I_m^2}{2\pi} \int_0^{\pi} (1 - \cos 2\omega t) \, d(\omega t)$$

$$= \frac{I_m^2}{2\pi} \left[(\omega t)_0^{\pi} - \left[\cancel{\frac{\sin 2\omega t}{2}} \right]_0^{\pi} \right]$$

$$= \frac{I_m^2}{2\pi} [\pi - 0] = \frac{I_m^2}{2}$$

ie $$I^2_{rms} = \frac{I_m^2}{2}$$

$$I_{rms} = \frac{I_m}{\sqrt{2}} = 0.707 \, I_m$$

$$\boxed{I_{rms} = 0.707 \, I_m}$$

|||ly $$\boxed{E_{rms} = 0.707 \, E_m}$$

Form factor

The ratio of rms value to the average value of an alternating quantity is known as form factor.

$$\text{Form factor} = \frac{\text{RMS Value}}{\text{Average value}}$$

For an alternating voltage or current

$$\text{Form factor} = \frac{0.707 \text{ max Value}}{0.637 \text{ max. value}}$$

$$= \underline{1.11}$$

Peak factor

The ratio of maximum value to the rms value of an alternating quantity is known as peak factor.

$$\text{Peak Value} = \frac{\text{max. value}}{\text{rms value}}$$

For an alternating quantity

$$\text{Peak factor} = \frac{\text{max. value}}{0.707 \times \text{max. value}}$$

$$= \underline{1.414}$$

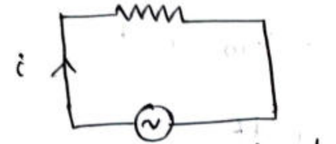
Single Phase AC Circuits

AC Circuits

The path for the flow of ~~alt~~ alternating current is known as ac circuits.

Pure Resistive (R) Circuit

Consider a circuit consisting of a pure resistance $R\ \Omega$ connected across an ac voltage source.



Here, $v = V_m \sin\omega t$

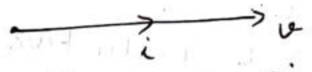
$i = \frac{v}{R} = \frac{V_m \sin\omega t}{R}$

$i = I_m \sin\omega t$

where $I_m = \frac{V_m}{R}$

i.e. $\boxed{\begin{array}{l} v = V_m \sin\omega t \\ i = I_m \sin\omega t \end{array}}$ —(1) —(2)

By observing equations (1) & (2), it can be concluded that the current is in phase with the voltage. It can be represented as shown below.

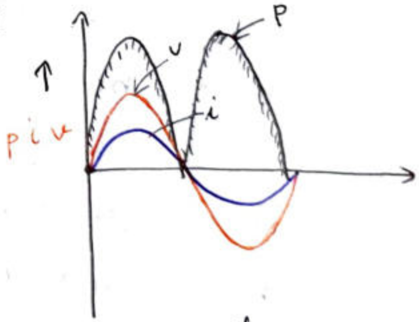


Instantaneous Power, $P = vi$

$= V_m \sin\omega t \cdot I_m \sin\omega t$

$= V_m I_m \sin^2\omega t$

$= V_m I_m \left(\frac{1-\cos 2\omega t}{2}\right)$



Average Power, $P = \frac{\text{Area of P over one cycle}}{\text{base}}$

$P_{avg} = \frac{1}{2\pi}\int_0^{2\pi} \frac{V_m I_m}{2} (1-\cos 2\omega t)\, d\omega t$

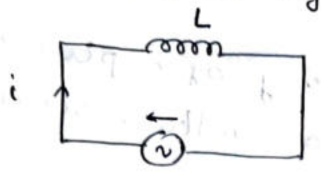
$= \frac{1}{2\pi}\frac{V_m I_m}{2}\int_0^{2\pi} (1-\cos 2\omega t)\, d\omega t$

$= \frac{1}{2\pi}\frac{V_m I_m}{2}\left[\omega t\right]_0^{2\pi} - \left[\frac{\sin 2\omega t}{2}\right]_0^{2\pi} = \frac{1}{2\pi}\frac{V_m I_m}{2}(2\pi)$

$\boxed{P_{avg} = \frac{V_m I_m}{2}}$ $\therefore P = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}}$ $\boxed{P = VI}$

Pure inductive circuit (L)

Consider an alternating voltage applied to a pure inductance of L Henri as shown in fig below.



$v = V_m \sin \omega t$

Here $v = V_m \sin \omega t$

But $v = L\frac{di}{dt}$

$\therefore L\frac{di}{dt} = V_m \sin \omega t$

$$di = \frac{1}{L} V_m \sin \omega t \, dt$$

Integrating both sides

$$i = \frac{V_m}{L}\int \sin \omega t \, dt$$

$$= \frac{V_m}{L}\left[\frac{-\cos \omega t}{\omega}\right]$$

$$i = \frac{V_m}{\omega L}\sin(\omega t - \pi/2)$$

$$i = I_m \sin(\omega t - \pi/2)$$

where ~~$X_L = V_m \omega t$~~

$X_L = \omega L$

$X_L = 2\pi f L$ Inductive reactance in Ohms.

So, we have $v = V_m \sin \omega t$ —— (1)

$i = I_m \sin(\omega t - \pi/2)$ —— (2)

From equations (1) & (2), it is clear that the current lags behind the applied voltage by 90°.

Instantaneous power, $p = vi$

$= V_m \sin \omega t \cdot I_m \sin(\omega t - \pi/2)$

$= V_m I_m \sin \omega t \sin(\omega t - \pi/2)$

$= -V_m I_m \sin \omega t \cos \omega t$

$= -\frac{V_m I_m}{2} \sin 2\omega t$

Average Power = Average of one cycle

$$= \frac{1}{2\pi}\int_0^{2\pi} -\frac{V_m I_m}{2} \sin 2\omega t \, d\omega t$$

$$= -\frac{1}{2\pi} \frac{V_m I_m}{2} \int_0^{2\pi} \sin 2\omega t \, d\omega t$$

$$= \underline{\underline{0}}$$

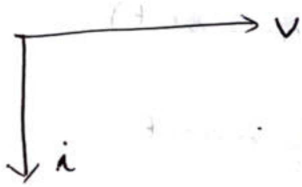
$\therefore$ Average Power over a complete cycle = 0

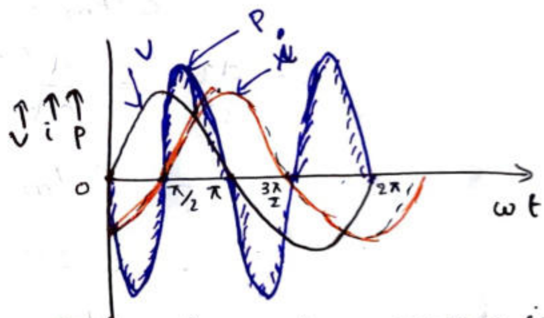
Hence the power absorbed by a pure inductance is zero.

In a pure inductive circuit

$$v = V_m \sin \omega t \quad \text{——— (1)}$$
$$i = I_m \sin(\omega t - \pi/2) \quad \text{——— (2)}$$

From these equations, current lags voltage by 90° ie $\pi/2$ in pure inductive circuit.

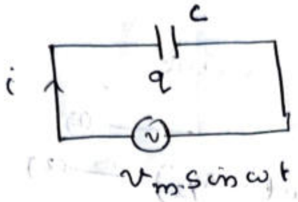




During the parts of the cycle, when power is +ve the energy is supplied from the source & is stored in the magnetic field of inductor. During the parts of the cycle when the power is negative, energy is being returned to the source. Since power supplied is equal to power returned over one cycle, net power absorbed by inductor is zero.

Pure Capacitive Circuit

Consider a capacitor of pure capacitance C across which alternating voltage $v = V_m \sin\omega t$ is applied as shown below, due to which an alternating current i flows charging the plates of the capacitor with a charge of q Coulombs



Here $v = V_m \sin\omega t$ ——— (1)

$$q = CV$$
$$= C V_m \sin\omega t$$

$$i = \frac{dq}{dt} = \frac{d}{dt}(C V_m \sin\omega t)$$

$$= C V_m \frac{d}{dt} \sin\omega t$$

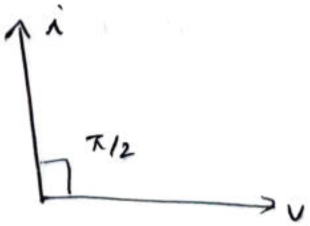
$$= \omega C V_m \cos\omega t$$

$$i = \frac{V_m}{(1/\omega C)} \sin(\omega t + \pi/2)$$

$$i = I_m \sin(\omega t + \pi/2) \quad \text{——— (2)}$$

where $X_C = \frac{1}{2\pi f C} = \frac{1}{\omega C}$ Capacitive reactance in Ohms

From eqn. (1) & (2), it is clear that the current leads the voltage by an angle $\pi/2$



Instantaneous Power,

$$P = vi$$
$$= V_m \sin\omega t \, I_m \sin(\omega t + \pi/2)$$

$P = V_m I_m \sin\omega t \cos\omega t$

$= \frac{V_m I_m}{2} \sin 2\omega t$

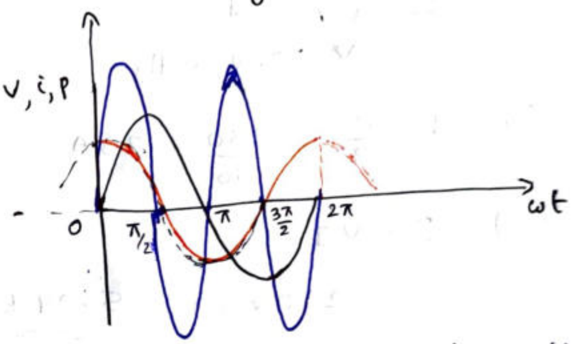
Average power, P = Average power over a complete cycle

$$= \frac{1}{2\pi} \int_0^{2\pi} \frac{V_m I_m}{2} \sin 2\omega t \, d\omega t$$

$$= \frac{1}{2\pi} \frac{V_m I_m}{2} \int_0^{2\pi} \sin 2\omega t \, d\omega t$$

$$= \underline{\underline{0}}$$

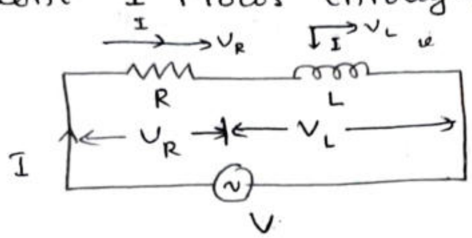
Hence the Power absorbed by a pure capacitor is zero



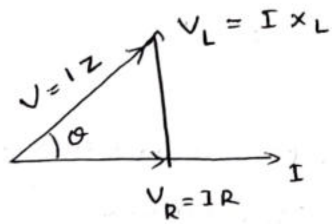
During the parts of the cycle when the power is +ve the capacitor will be charged to supply voltage. During the parts of the cycle, when the power is negative, energy being returned to the source. Since the power supplied is equal to the power returned over one cycle the net power absorbed by a capacitor is zero.

R-L Series Circuit

Consider an RL series circuit as shown in fig. An alternating voltage of rms Value V is applied due to which an rms value of current I flows through the circuit.



Taking current as the reference vector, the phasor diagram of the circuit can be drawn as shown in fig.



The voltage drop, V_R is in phase with the current I & the voltage drop V_L leads the current by 90°. But

$$V^2 = (V_R)^2 + (V_L)^2$$

$$V^2 = (IR)^2 + (IX_L)^2$$

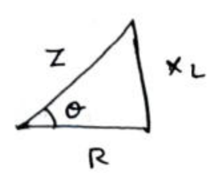
$$\theta = \tan^{-1}\left(\frac{X_L}{R}\right)$$

$$V = I\sqrt{R^2 + X_L^2}$$

$$= IZ$$

where $Z = \sqrt{R^2 + X_L^2} = R + jX_L$

$Z \rightarrow$ Impedence



Impedence triangle

The impedence (Z) of an ac circuit may be defined as the opposition offered for the flow of alternating current in the circuit. Impedence is measured in Ohms.

From the vector diagram, we can observe that current lags voltage by an angle ~~90~~ θ

$v = V_m \sin \omega t$
$i = I_m \sin(\omega t - \theta)$

Instaneous Power, $P = vi$

$= V_m \sin\omega t \cdot I_m \sin(\omega t - \theta)$

$= V_m I_m \cdot \sin\omega t \sin(\omega t - \theta)$

$\cancel{= V_m I_m \cdot \frac{1}{2}\left[\cos\theta - \cos 2\omega t\right]}$

$\left[\because \sin A \sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]\right]$

$= V_m I_m \cdot \frac{1}{2}\left[\cos\theta - \cos(2\omega t - \theta)\right]$

$= \frac{1}{2} V_m I_m \cos\theta - \frac{1}{2} V_m I_m \cos(2\omega t - \theta)$

The second term is cosine terms & its average power over a complete cycle is zero.

$\therefore$ Average Power, $P = \frac{1}{2} V_m I_m \cos\theta$

$= \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos\theta$

$$\underline{\underline{P = VI\cos\theta}}$$

where $\cos\theta$ is known as the Power factor of the circuit.

The Power factor of a circuit can be defined in the following ways.

1) Power factor $= \cos\phi$

The power factor of a circuit is defined as the cosine of the angle between the voltage & current.

2) $\cos\phi = \frac{R}{Z}$

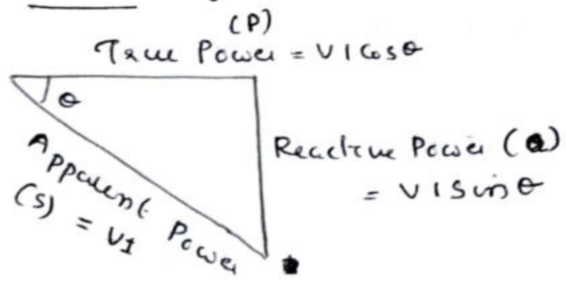
The Power factor of a circuit is defined as the ratio of Resistance to the impedence of the circuit.

3) $\cos\phi = \cancel{VI}\ \frac{P}{VI}$

The power factor $\cos\theta$ is defined as the ratio of true power to the apparent power.

i.e. $\cos\theta = \frac{P}{VI}$

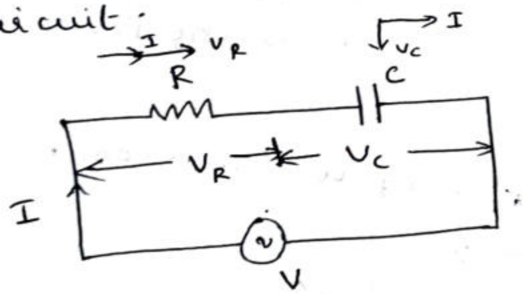
Power triangle



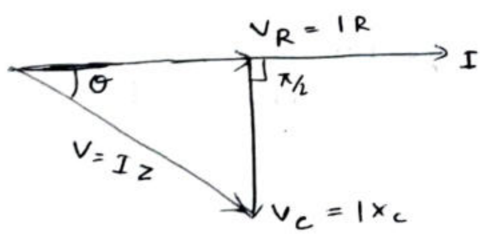
The maximum value of power factor is unity. If current lags voltage, power factor is said to be lagging. If the current leads voltage, power factor is said to be leading.

RC series Circuit

Consider an RC series circuit as shown in fig. to which an alternating voltage of rms value V is applied, due to which an rms value of current flows through the circuit:



Taking current as the reference vector, the phasor diagram of the circuit can be drawn as given below.



$$V^2 = V_R^2 + V_C^2$$

$$V^2 = (IR)^2 + (IX_c)^2$$

$$V = I\sqrt{R^2 + X_c^2} = IZ$$

where $Z \rightarrow$ impedance of the circuit

$$Z = \sqrt{R^2 + X_c^2} = R - jX_c$$

Phase angle, $\theta = \tan^{-1}(-X_c/R)$

From the vector diagram, we observe that, the current leads the voltage by an angle θ.

$\therefore$ $v = V_m \sin \omega t$

$i = I_m \sin(\omega t + \theta)$

Instantaneous power, $p = vi$

$$= V_m \sin \omega t \cdot I_m \sin(\omega t + \theta)$$

$$= V_m I_m \sin \omega t \sin(\omega t + \theta)$$

$$= V_m I_m \frac{1}{2}\left[\cos(\theta) - \cos(2\omega t + \theta)\right]$$

$$= \frac{V_m I_m}{2} \cos\theta - \underbrace{\frac{V_m I_m}{2} \cos(2\omega t + \theta)}_{=0}$$

The average value for the second term over a complete cycle is 0

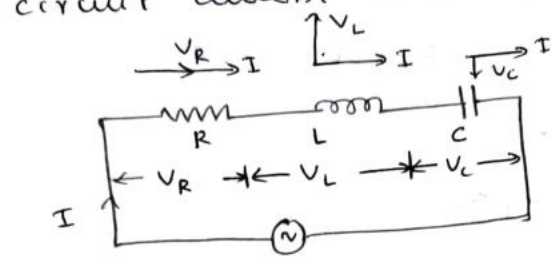
$\therefore$ Average Power, $P = \frac{1}{2} V_m I_m \cos\theta$

$$= \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\theta$$

$$\boxed{P = VI\cos\theta}$$

RLC series Circuit

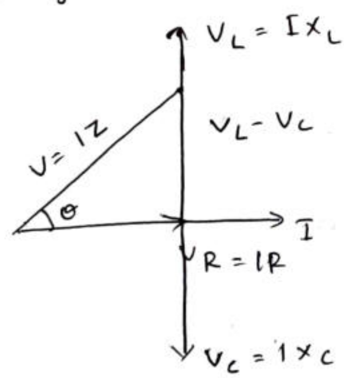
Consider a RLC circuit as shown below, to which an alternating voltage of rms value V is applied. The resulting circuit current is I.



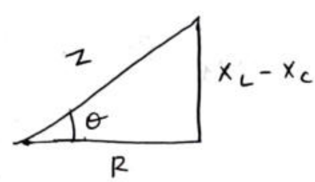
For this circuit 3 cases can be studied.

Case I : When $X_L > X_C$

When the inductive reactance is more than the capacitive reactance, the vector diagram of the circuit is as shown below.



Impedence triangle



Current $I = \dfrac{V}{Z}$

$$= \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}}$$

ie $Z = R + j(X_L - X_C)$

The circuit is similar to an RL circuit.

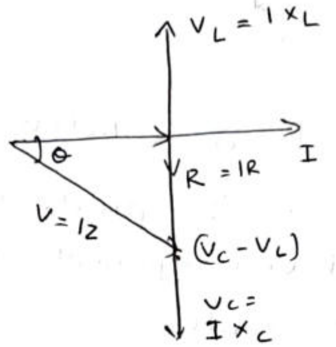
Here $v = V_m \sin \omega t$

$i = I_m \sin(\omega t - \theta)$

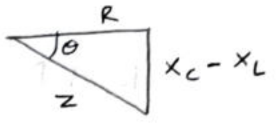
Hence $P = VI\cos\theta$

case (ii) When $X_C > X_L$

When the capacitive Reactance is greater than the inductive reactance, the vector diagram is as shown in the fig.



Impedence triangle



From the vector diagram, it is observed that the current leads the voltage by angle θ.

It is similar to RC circuit.

$$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + (X_C - X_L)^2}}$$

$$Z = R - j(X_C - X_L)$$

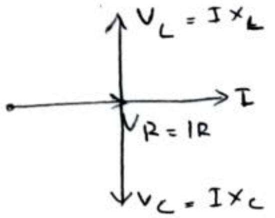
Here $v = V_m \sin \omega t$

$i = I_m \sin(\omega t + \theta)$

$\therefore$ Power Consumed, $P = VI \cos \phi$

case 3 When $X_C = X_L$

When inductive reactance is equal to the capacitive reactance, the vector diagram is as shown below. V_L & V_C get canal with each other. So the current is in phase with voltage. & the circuit behaves as a pure resistive circuit



Hence $Z = R$

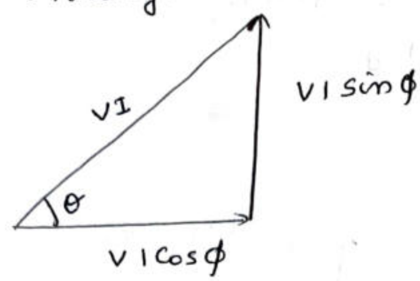
Here $v = V_m \sin \omega t$

$i = I_m \sin \omega t$.

Power consumed, $P = VI$

Power and Power triangle

Power triangle



Real Power (True Power or Active Power) (P)

It is defined as the product of applied voltage & the active component of power

$P = VI \cos\phi$ watts.

It is measured in watts. or in kW

Apparent Power (S)

It is defined as the product of rms value of voltage & current.

$S = VI$

It is measured in Volt Ampere or kVA

Reactive Power (Q)

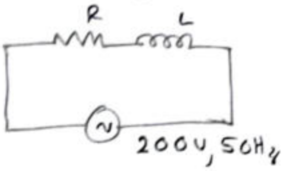
It is defined as the product of applied voltage and reactive component of current.

$Q = VI \sin\phi$ VAR

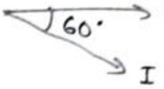
It is measured in VAR or kVAR. (volt-ampere Reactive)

Problems

1) A choke coil takes a current of 2A lagging 60° behind the applied voltage of 200V at 50Hz. Calculate the inductance, resistance and impedence of the coil. Also determine the power consumed when it is connected across 100V, 25Hz supply.



Solution



Impedence

(i) $Z = \frac{V}{I} = \frac{200}{2} = 100\,\Omega$

$R = Z\cos\theta$

$= 100\cos 60 = 50\,\Omega$

Inductive Reactance $X_L = Z\sin\theta$

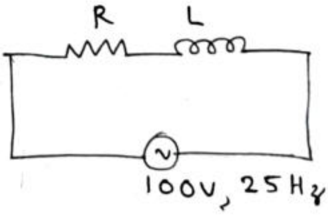
$= 100\sin 60 = 86.6\,\Omega$

$X_L = 2\pi f L$

$\therefore$ Inductance, $L = \frac{X_L}{2\pi f} = \frac{86.6}{2\pi \times 50}$

$= \frac{86.6}{2 \times 3.14 \times 50} = 0.276\,H$

Case (ii) When connected across 100V, 25Hz



$X_L = 2\pi f L = 2 \times 3.14 \times 25 \times 0.276$

$= 43.33\,\Omega$

$R = 50\,\Omega$

$Z = R + jX_L$

$= 50 + j43.3$

$Z = \sqrt{R^2 + X_L^2} = 66.16$

$\cos\theta = \frac{R}{Z} = \frac{50}{66.16} = 0.7557$

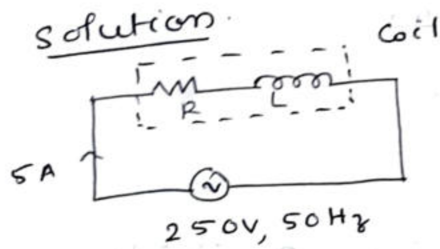
$\theta = \cos^{-1}(R/Z) = 40.9°$

$$I = \frac{V}{Z} = \frac{100}{66.16} = \underline{1.51\,A}$$

$$P = VI\cos\theta$$
$$= 100 \times 1.51 \times 0.7557$$
$$= \underline{114.2\,W}$$

2 An inductor coil is connected to the supply of 250V, 50Hz and takes a current of 5A. The coil dissipates 750W. Calculate (i) Power factor (ii) Resistance of the coil (iii) Inductance of the coil.

Solution.



$I = 5A$

$V = 250V$

$$Z = \frac{V}{I} = \frac{250}{5} = \underline{50\Omega}$$

$$P = I^2R = 750\,W$$
$$R = \frac{750}{I^2} = \frac{750}{5^2} = \underline{30\Omega}$$

(i) $\text{Power factor} = \cos\theta = \frac{R}{Z}$

$$= \frac{30}{50} = \underline{0.6}\ (\text{lag})$$

(ii) $R = \underline{30\Omega}$

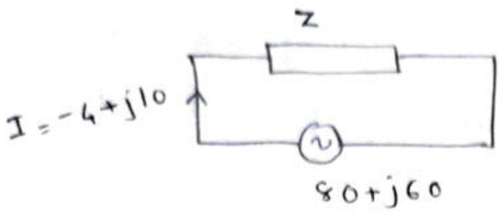
(iii) $Z = \sqrt{R^2 + X_L^2}$

$$X_L = \sqrt{Z^2 - R^2} = \sqrt{50^2 - 30^2} = \underline{40\Omega}$$

$$X_L = 2\pi f L$$
$$L = \frac{X_L}{2\pi f}$$
$$= \frac{40}{2 \times 3.14 \times 50} = \underline{0.127}$$

3. An alternating voltage $(80+j60)$ is applied to a circuit and the current flowing is $(-4+j10)$ A. Find the (i) impedance of the circuit (ii) Phase angle (iii) Power consumed.

Soln



$$Z = \frac{80+j60}{-4+j10} = \frac{100\angle 36.86}{10.77\angle 111.8}$$

$$= 9.29\angle -74.94 \ \Omega$$

(i) $Z = 9.29$

(ii) Phase angle $(\phi) = 74.94$ (~~lead~~)

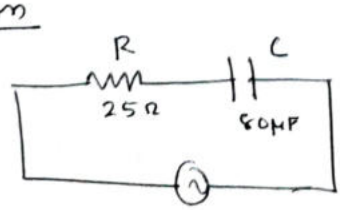
(iii) $P = VI\cos\theta$

$$= 100 \times 10.77 \cos(74.94)$$

$$P = 279.84W$$

4. A voltage $v = 100\sin 314t$ is applied to a circuit consisting of a 25Ω resistor & an $80\mu F$ capacitor in series. Determine (i) peak value of current. (ii) Power factor (iii) Total Power consumed in the circuit.

solution



$v = V_m \sin \omega t = 100 \sin 314t$

$V = \frac{V_m}{\sqrt{2}} = \frac{100}{\sqrt{2}} = 70.71V$

$\omega = 314$

$2\pi f = 314$

$f = \frac{314}{2 \times 3.14} = 50\,Hz$

$\therefore X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 80 \times 10^{-6}}$

$= \underline{\underline{39.78\,\Omega}}$

$R = 25\,\Omega$

$Z = \sqrt{R^2 + X_C^2}$

$= \underline{\underline{47\,\Omega}}$

$Z = R - jX_C$

$= 25 - j\,39.78$

$= \underline{\underline{47 \angle -57.85}}$

(i) Peak value of current

$I = \frac{V}{Z} = \frac{70.71}{47} = \underline{\underline{1.5}}\,A$

$I_m = \sqrt{2}\,I = \sqrt{2} \times 1.5 = \underline{\underline{2.12}}\,A$

or, $Z = \frac{V}{I} = \frac{V_m}{I_m}$

$I_m = \frac{V_m}{Z}$

$= \underline{\underline{2.12\,A}}$

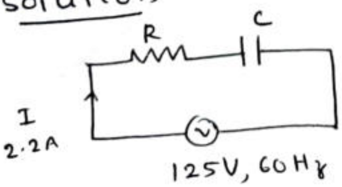
(iii) Power $= VI\cos\theta$

$= 70.71 \times 1.5 \times \cos(-57.85)$

$= \underline{\underline{56.44\,W}}$

5. ~~The~~ 125 V at 60 Hz is applied across a capacitance connected in series with a non-inductive resistor. The combination carries a current of 2.2 A & causes a power loss of 96.8 W in the resistor. Power loss in the capacitor is negligible. Calculate the resistance & capacitance.

solution



$P = I^2 R$

(i) $R = \frac{P}{I^2} = \frac{96.8}{(2.2)^2} = \underline{\underline{20\,\Omega}}$

(ii) $Z = \frac{V}{I} = \frac{125}{2.2} = 56.82\,\Omega$

$X_C = \sqrt{Z^2 - R^2} = \sqrt{(56.82)^2 - (20)^2}$

$= 53.18\,\Omega$

$X_C = \frac{1}{2\pi f C}$ $\quad C = \frac{1}{2\pi f X_C} = \underline{\underline{4.99 \times 10^{-5}\,F}}$

6. The instantaneous values of current and voltage in ac circuit are given by

$i = 28.28 \sin(314t - \pi/3)$ amp,

$v = 282.8 \sin 314t$ volts. Find (i) rms value of current and voltage (ii) Average value of current and voltage (iii) frequency of supply voltage (iv) Power in the circuit

—— × ——

Solution

Given $i = 28.28 \sin(314t - \pi/3)$

$v = 282.8 \sin 314t$

(i) RMS value of voltage and current

$V_m = 282.8$

$$V = \frac{V_m}{\sqrt{2}} = \frac{282.8}{\sqrt{2}} = 200V$$

$$I = \frac{I_m}{\sqrt{2}} = \frac{28.28}{\sqrt{2}} = \underline{\underline{20A}}$$

(ii) Average value of voltage and current

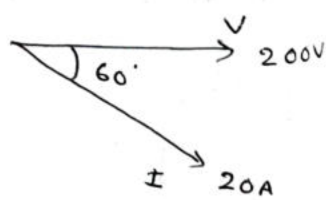
$V_{av} = 0.637 V_m$

$= 0.637 \times 282.8 = \underline{\underline{180V}}$

$I_{av} = 0.637 I_m = 0.637 \times 28.28 = \underline{\underline{18A}}$

(iii) $\omega = 2\pi f = 314$

$$f = \frac{314}{2\pi} = 50Hz$$



(iv) Power $= VI \cos\theta$

$= 200 \times 20 \cos 60°$

$= \underline{\underline{2kW}}$

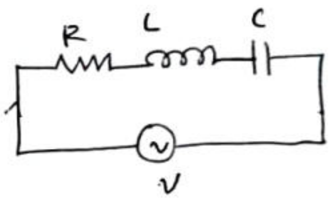
7. A circuit consists of a resistance of 10 Ω, an inductance of 16 mH & capacitance of 150 μF connected in series. A supply voltage of 100 V, 50 Hz is given to the circuit. Find the current, pf and power consumed by the circuit. Draw the vector diagram.

Solution

$R = 10\,\Omega$ $\quad L = 16\,mH = 16 \times 10^{-3}\,H$ $\quad C = 150\,\mu F = 150 \times 10^{-6}\,F$

$X_L = 2\pi f L = 2 \times 3.14 \times 50 \times 16 \times 10^{-3} = \underline{\underline{5.024\,\Omega}}$

$X_C = \dfrac{1}{2\pi f C} = \dfrac{1}{2 \times 3.14 \times 50 \times 150 \times 10^{-6}} = 21.23\,\Omega$



$Z = R + jX_L - jX_C$

$Z = 10 + j5.024 - 21.23j$

(8) $Z = 10 - j16.82 = 19.57 \angle{-58.23}$

(i) $I = V/Z = 100/19 = 5.26\,A$

(ii) $\cos\theta = \cos(58.23) = \underline{\underline{0.526}}$

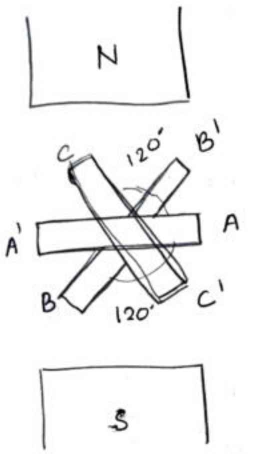
(iv) $P = VI\cos\theta = 100 \times 5.26 \times 0.526$

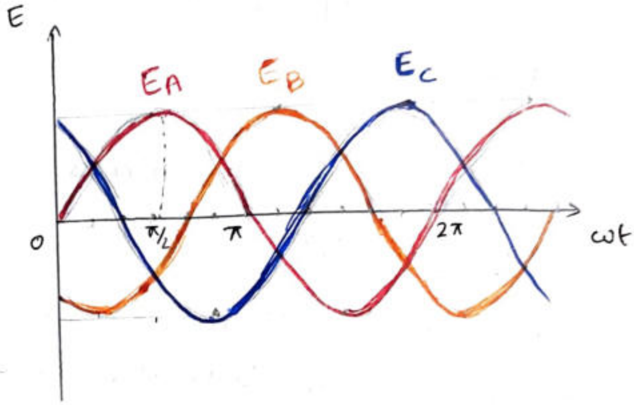
$= \underline{\underline{277\,W}}.$

Three Phase Circuits

In a 3ϕ system, there are 3 separate but identical windings displaced from each other by an angle of 120° acted upon by the common magnetic field.

Each winding or phase produces an alternating voltage of same magnitude and frequency, but they are displaced from each other by an angle of 120° electrical.





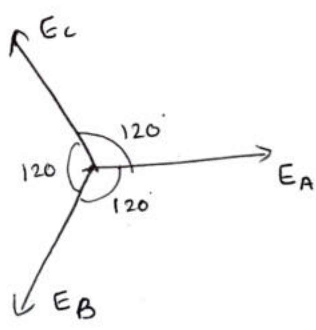
The equation for the induced emf are

$$E_A = E_m \sin \omega t$$

$$E_B = E_m \sin(\omega t - 120)$$

$$E_C = E_m \sin(\omega t - 240)$$

$$= E_m \sin(\omega t + 120)$$



The sum of the emf at every instant is zero

$$\vec{E_A} + \vec{E_B} + \vec{E_C} = 0$$

$$= E_m \sin \omega t + E_m \sin(\omega t - 120) + E_m \sin(\omega t + 120)$$

$$= E_m [\sin \omega t + \sin(\omega t - 120) + \sin(\omega t + 120)]$$

$$= E_m [\sin \omega t + \sin \omega t \cos 120 - \cancel{\cos \omega t \sin 120} + \sin \omega t \cos 120 + \cancel{\cos \omega t \sin 120}]$$

$$= E_m [\sin \omega t - \tfrac{1}{2} \sin \omega t - \tfrac{1}{2} \sin \omega t]$$

$$= 0$$

Phase Sequence

The order in which the voltages ~~attain~~ in the three phases or coil reaches their maximum positive values is called phase sequence.

The emf waveform shows that E_A reaches its maximum value first, then E_B and then E_C. Here the order in which the emfs reach their maximum values is A, B, C (considering rotation in anticlockwise direction)

Advantages of 3 Phase Systems over single Phase system

(1) To transmit a given amount of power over a given distance, less copper and other material is required for 3 ϕ systems.

(2) Rotating magnetic field can be produced by using 3 ϕ system. Hence 3 ϕ motors are self starting while 1 ϕ motors are not.

(3) For a given rating & voltage a 3 ϕ system occupies less space & less cost compared to single phase system.

(4) In case of 3 ϕ star system, it is possible to obtain lower voltage between phases & neutral for domestic use and higher voltage for industrial use without extra cost whereas only one voltage can be obtained in single phase systems.

(5) Three phase motors produce uniform torque whereas the torque produced by the 1 ϕ motors are pulsating.

(6) 3ϕ systems give steady output.

3ϕ connections

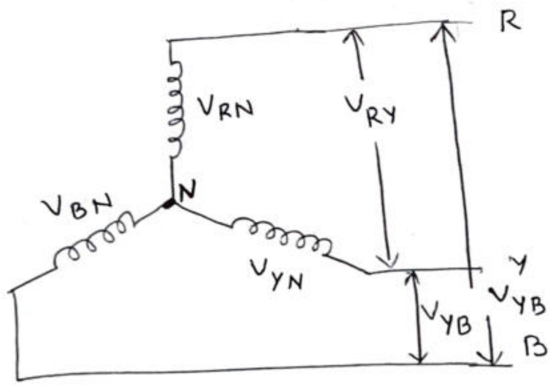
In a 3ϕ system, to make the system less expensive, the three phases are interconnected by using

(i) Star Connection

(2) Delta Connection.

(i) Star Connection

In this system, similar ends of 3 coils are joined together to form a common junction or neutral point N as shown below.



The voltage between any line and neutral point is called the phase voltage.

The voltage between any two lines is ~~ta~~ called Line voltage.

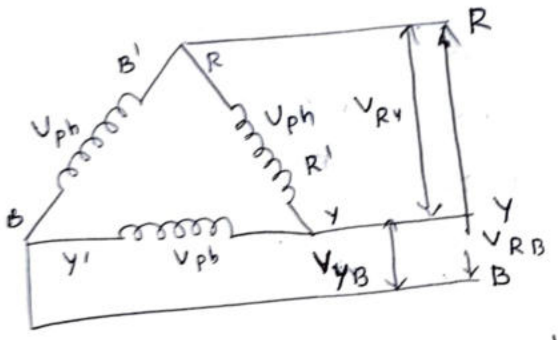
The current flowing in the phase is called phase current.

The current flowing in the lines are called line currents.

In star connection, if the neutral point is connected, then it is called as 3ϕ, 4 wire system. Otherwise it is called as 3ϕ 3 wire system.

(2) <u>Delta or Mesh Connection</u>

In this system, the dissimilar ends of the phase windings are joined together. The starting end of one coil is connected to the finishing end of the other & so on. Thus the coils are connected in series to form a closed mesh. This system is called as 3ϕ 3 wire system.



<u>Relation between Line ~~voltage~~ and Line values of voltage & currents in balanced</u>

<u>Star Connection</u>

In Star Connection,

Line Voltage = $\sqrt{3}$ Phase Voltage

$$\boxed{V_L = \sqrt{3}\, V_{ph}}$$

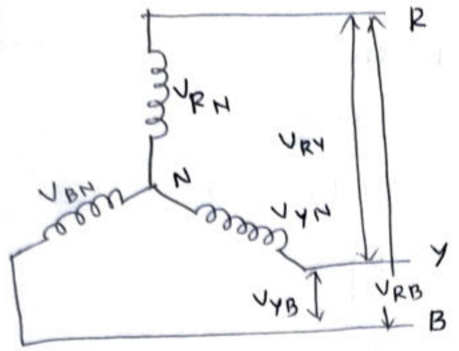
Line Current = Phase Current

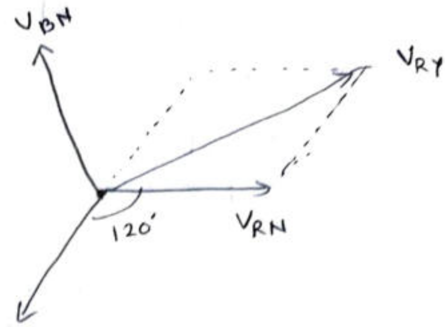
$$\boxed{I_L = I_{ph}}$$

<u>Proof</u>

Consider the balanced 3 phase star connection as shown

Note: As per syllabus proof is excluded.





Considering the lines R & Y, the line voltage V_{RY} is equal to the phasor difference of V_{RN} & V_{YN}

ie $\overrightarrow{V_{RY}} = \overrightarrow{V_{RN}} + \overrightarrow{V_{NY}}$

$\overrightarrow{V_{RY}} = \overrightarrow{V_{RN}} - \overrightarrow{V_{YN}}$

$$(V_{RY})^2 = (V_{RN})^2 + (V_{YN})^2 + 2V_{RN}V_{YN}\cos 60$$

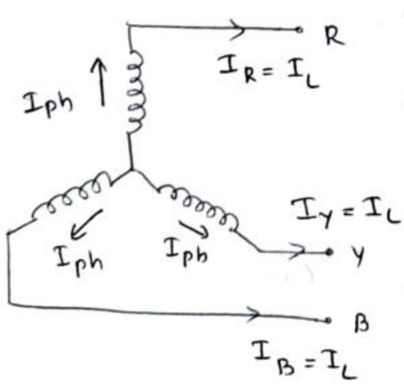
$$(V_L)^2 = (V_{ph})^2 + (V_{ph})^2 + 2V_{ph}V_{ph} \times 0.5$$

$$(V_L)^2 = 3(V_{ph})^2$$

$\boxed{V_L = \sqrt{3}\,V_{ph}}$ in star connected.

Line and Phase Current

From the fig, it is clear that the current flowing through the phase winding will be the same as that of line Current.



∴ Line Current = Phase Current

$$\boxed{I_L = I_{ph}}$$

Power

For a balanced load, the power consumed in each load phase is same

$\therefore$ Total Power, $P = 3 \times (1\phi \text{ Power})$

$$\boxed{P = 3 V_{ph} I_{ph} \cos\phi}$$

$$= 3 \frac{V_L}{\sqrt{3}} I_L \cos\phi$$

$$\boxed{P = \sqrt{3} V_L I_L \cos\phi}$$

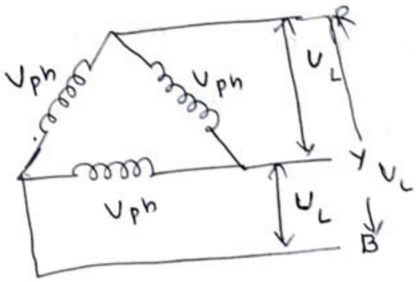
Relationship between Line and Phase values of Voltages & Currents in Balanced Delta Connection

Line and Phase Voltages

From the fig, we observe that, the voltage between the phases and lines are same

ie Phase Voltage = Line Voltage

$$\boxed{V_{ph} = V_L}$$



Line and Phase Currents

Line Current = $\sqrt{3}$ Phase Current

$$\boxed{I_L = \sqrt{3} I_{ph}}$$

as per syllabus proof is excluded.

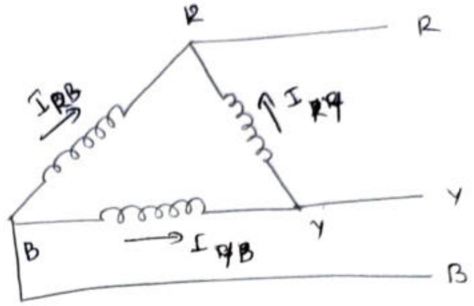
Proof

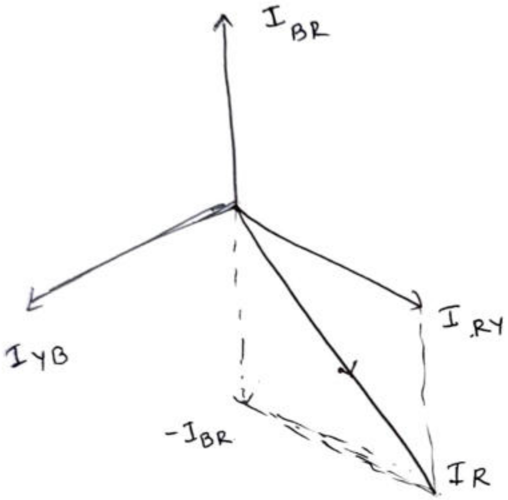
The current in line is equal to the phasor difference of the currents in the two phases attached to the line

$$\vec{I_R} = \vec{I_{RY}} - \vec{I_{BR}}$$

$$\vec{I_Y} = \vec{I_{RY}} - \vec{I_{YB}}$$

$$I_B = \vec{I_{YB}} - \vec{I_{BR}}$$





$$(I_R)^2 = (I_{(RY)})^2 + (I_{BR})^2 + 2 I_{RY} I_{BR} \cos 60$$

$$(I_L)^2 = (I_{ph})^2 + (I_{ph})^2 + 2 I_{ph} I_p \times \frac{1}{2}$$

$$(I_L)^2 = 3 (I_{ph})^2$$

$$\boxed{I_L = \sqrt{3} I_{ph}}$$

Power $P = 3 \times (1\phi \text{ Power})$

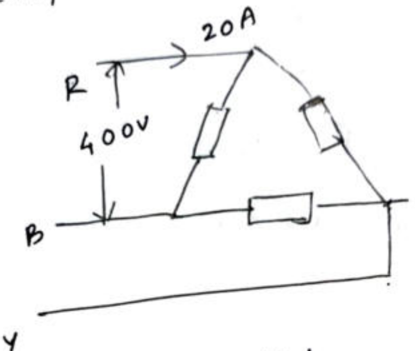
$$\boxed{P = 3 V_{ph} I_{ph} \cos \phi}$$

$$P = 3 V_L \frac{I_L}{\sqrt{3}} \cos \phi$$

$$\boxed{P = \sqrt{3} V_L I_L \cos \theta}$$

3 Phase Circuit Problems

1. A balanced delta connected 3ϕ load is fed from 3ϕ, 400V, supply. The line current is 20A & the total power absorbed by the load is 10kW. Calculate
   (i) The impedence in each branch
   (2) The Power factor
   (3) The total power consumed if the same impedence are star connected.



For delta Connected

$V_{ph} = V_L = 400V$

$I_{ph} = I_L/\sqrt{3} = \frac{20}{\sqrt{3}}$

$= 11.55A$

(i) $Z_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{400}{11.55} = 34.63\Omega$

(ii) $\cos\phi = \frac{P}{\sqrt{3} V_L I_L} = \frac{10 \times 10^3}{\sqrt{3} \times 400 \times 20} = 0.7216$

For Star Connection

$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 231 V$

$I_{ph} = I_L = \frac{V_{ph}}{Z_{ph}} = \frac{231}{34.63} = 6.67A$

$P = \sqrt{3} V_L I_L \cos\theta$

$= \sqrt{3} \times 400 \times 6.67 \times 0.7216$

$=$

2. A balanced star connected load is supplied from symmetrical 3ϕ, 400V (L-L) supply. The current in each phase is 50A & lags 30° behind the phase voltage. Find (i) Phase

voltage. (ii) Phase impedance (iii) Active & reactive power drawn.

Soln

Given $V_L = 400V$, $I_{ph} = 50A$ $\theta = 30°$

For star connection

(i) $V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94V$

(ii) $Z_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{230.94}{50} = 4.62\Omega$

(iii) Active Power, $P = \sqrt{3} V_L I_L \cos\theta$

$= \sqrt{3} \times 400 \times 50 \times \cos 30$

$P = 30kW$

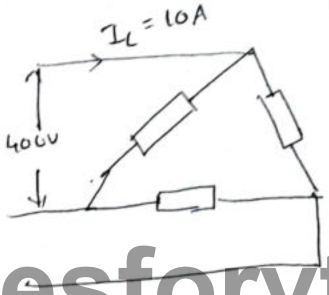
(iv) Reactive Power, $Q = \sqrt{3} V_L I_L \sin\theta$

$= \sqrt{3} \times 400 \times 50 \sin 30$

$= 11.32 kVAR.$

3. A 3ϕ load of 3 equal impedences connected in delta across a balanced 400V supply takes a line current of 10A at a power factor of 0.7 lagging. Calculate

(i) the phase current
(ii) The total Power
(iii) the total Reactive volt ampere



(i) $I_{ph} = \frac{I_L}{\sqrt{3}} = 5.77A$

(ii) $P = \sqrt{3} V_L I_L \cos\theta$

$= \sqrt{3} \times 400 \times 10 \times 0.7$

$= 4.85kW$

$\theta = \cos^{-1}(0.7)$

(iii) $Q = \sqrt{3} V_L I_L \sin\theta$

$= \sqrt{3} \times 400 \times 10 \times \sin(45.57)$

$Q = 4.95\ kVAR$

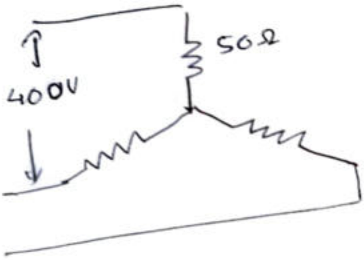
(4) Three 50Ω resistors are connected in star across 400V, 3 phase supply

(i) Find phase current, line current & power drawn from supply.

(ii) What would be the above values if one of the resistors were disconnected.

Solution

For star connection



(i) $V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94V$

(ii) $I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{230.94}{50}$

$= 4.62A$

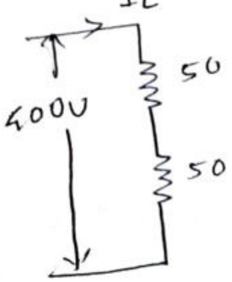
(ii) $I_L = I_{ph} = 4.62A$

(iii) $P = \sqrt{3} V_L I_L \cos\theta$

$= \sqrt{3} \times 400 \times 4.62 \times 1$

$= 3.2\ kW$ ($\cos\theta = 1$ resistive load)

(ii) If one resistor is disconnected



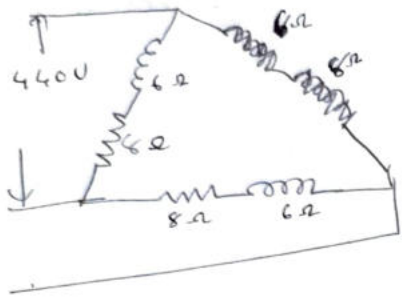
$I_L = \frac{V}{50+50} = \frac{400}{100} = 4A$

$P = VI$

$= 400 \times 4$

$= 1600W$

5. For the circuit shown below find (i) Vph (ii) Iph (iii) I_L (iv) cos θ (v) Power/Phase (vi) Total Power



(i) $V_L = V_{ph} = 440V$

(ii) $I_{ph} = \frac{V_{ph}}{Z_{ph}}$

$= \frac{440}{\sqrt{8^2+6^2}} = \frac{440}{10}$

$= 44A$

$I_L = \sqrt{3} I_{ph} = 44 \times \sqrt{3} = 76.21A$

(iii) $\cos\theta = \frac{R}{Z} = \frac{8}{10} = 0.8$

(iv) Power/Phase $= V_{ph} I_{ph} \cos\theta$

$= 440 \times 44 \times 0.8 = 15.49kW$

(v) Total Power $= \sqrt{3} V_L I_L \cos\theta$

$= \sqrt{3} \times 440 \times 76.21 \times 0.8$

$= 46.46 kW$

6. A balanced star connected load of (8+6j) Ω per phase is connected to the 3ϕ, 230V supply. Find the line current, power factor, power, reactive volt ampere & total volt ampere

soln For star connection

(i) $V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{230}{\sqrt{3}} = 132.79V$

(ii) $I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{132.79}{\sqrt{8^2+6^2}} = \frac{132.79}{10} = 13.28A$

Now $I_L = I_{ph} = 13.28A$

(ii) $\cos\theta = \frac{R}{Z} = \frac{8}{10} = 0.8$

(iii) Power, $P = \sqrt{3} V_L I_L \cos\theta$

$= \sqrt{3} \times 230 \times 13.28 \times 0.8 = 4.2kW$

(iv) Reactive Power $Q = \sqrt{3} V_L I_L \sin\theta$

$= \sqrt{3} \times 230 \times 13.28 \times 0.6$

$= 3.17$ kVAR

v) Total Volt amper

$S = \sqrt{3} V_L I_L$

$= \sqrt{3} \times 230 \times 13.28$

$= 5.29kW$

7) A 3ϕ delta connected balanced load consumed a power of 60kW taking a lagging current of 200 A at a line voltage of 400V, 50Hz. Find parameters of each Phase.

Soln

$P = 60kW \quad I_L = 200A \quad V_L = 400V$

$Z_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{400}{115.47} = 3.464\Omega$

$I_{ph} = \frac{I_L}{\sqrt{3}} = \frac{200}{\sqrt{3}} = 115.47$

$P = \sqrt{3} V_L I_L \cos\theta$

$\cos\theta = \frac{P}{\sqrt{3} V_L I_L} = \frac{60 \times 10^3}{\sqrt{3} \times 400 \times 200} = 0.433$

Since current is lagging, it is RL circuit

$R = Z_{ph} \cos\theta = 3.464 \times 0.433$

$= 1.5 \Omega$

$X_L = Z_{ph} \sin\theta = 3.464 \times 0.901 = 3.12\Omega$

But $X_L = 2\pi f L \qquad L = \frac{X_L}{2\pi f} = \frac{3.12}{2\pi \times 50}$

$= 9.94 mH$

8. A balanced 3 Phase star connected load of 100 kW takes a leading current of 80A when connected to a 3 phase 1.1 kW, 50 Hz. Find R, Z, C per phase & $\cos\phi$

Soln

Given λ connected, $P = 100\,kW$ $I_L = 80A$

$V_L = 1.1\,kV$ $f = 50\,Hz$

$I_L = I_{ph} = 80A$

$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{1.1 \times 10^3}{\sqrt{3}} = 635.1\,V$

(i) $Z_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{635.1}{80} = 7.94\,\Omega$

$P = \sqrt{3} V_L I_L \cos\theta$

$\cos\theta = \frac{P}{\sqrt{3} V_L I_L} = \frac{100 \times 10^3}{\sqrt{3} \times 1.1 \times 10^3 \times 80}$

$= 0.656$

(iii) $R = Z \cos\theta$

$= 7.94 \times 0.656$

$= 5.2\,\Omega$

(iv) $X_C = Z \sin\theta$

$= 7.94 \times 0.754 = 5.98\,\Omega$

$X_C = \frac{1}{2\pi f C}$

$C = \frac{1}{2\pi f X_C}$

$= \frac{1}{2 \times 3.14 \times 50 \times 5.98}$

$= 532.56\,\mu F$