APPLIED PHYSICS FOR CIVIL ENGINEERING (BPHYC102/202)
MODULE 1
OSCILLATIONS AND SHOCK WAVES

SYLLABUS

Module -I: Oscillations and Shock waves: Oscillations: Simple Harmonic motion (SHM), differential equation for SHM (No derivation), Sprigs: Stiffness Factor and its Physical Significance, series and parallel combination of springs(Derivation), Types of spring and their applications. Theory of damped oscillations (Qualitative), Types of damping (Graphical Approach). Engineering applications of damped oscillations, Theory of forced oscillations (Qualitative), resonance, and sharpness of resonance. Shock waves: Mach number and Mach Angle, Mach Regimes, definition and characteristics of Shock waves, Construction and working of Reddy shock tube, Applications of Shock Waves, Numerical problems.

Pre requisites: *Basics of Oscilations*

Self-learning*:Simple Harmonic motion, differential equation for SHM*

Oscillations and vibrations play a more significant role in our lives than we realize. When you strike a bell, the metal vibrates, creating a sound wave. All musical instruments are based on some method to force the air around the instrument to oscillate. Oscillations from the swing of a pendulum in a clock to the vibrations of a quartz crystal are used as timing devices. When you heat a substance, some of the energy you supply goes into oscillations of the atoms. Most forms of wave motion involve the oscillatory motion of the substance through which the wave is moving. Despite the enormous variety of systems that oscillate, they have many features in common with the simple system of a mass on a spring. The harmonic oscillators have close analogy in many other fields; mechanical example a weight on a spring, oscillations of charge flowing back and forth in an electrical circuit, vibrations of a tuning fork, vibrations of electrons in an atom generating light waves, oscillation of electrons in an antenna etc.,

Periodic Motion: Periodic motion is any motion that repeats itself in equal intervals of time.
Examples:

- a swing in motion
- a vibrating tuning fork
- the Earth in its orbit around the Sun
- simple pendulum

A special type of periodic motion is **simple harmonic motion**

Simple Harmonic Motion: SHM is the type of periodic motion in which the net restoring force F acting on the body is proportional to the displacement x from the equilibrium position and is directed opposite to the displacement, i.e., towards the equilibrium point. The body performing SHM is known as a simple harmonic oscillator (SHO).

There are two types of SHM

1. Linear SHM

Eg: 1. The vertical oscillations of a loaded spring suspended from a rigid support.

2. Motion of needle of sewing machine

2. Angular SHM

Eg : 1. Motion of pendulum

2. Vibration of tuning fork

Characteristics of SHM:

1. Motion must be periodic.
2. The acceleration developed in the motion due to the restoring force is directly proportional to its displacement from the equilibrium position.
3. The Force or acceleration is always directed opposite to the displacement i.e., towards the mean position (F α -x or F = – kx). The displacement can be represented by a sine or cosine function such as

   $x = a\ sin\omega t$, where, a is the amplitude and ω is the angular frequency.
4. The velocity of the body is maximum at the centre and minimum at extreme position.

Concepts of Simple Harmonic Motion (S.H.M):

Displacement (x): The distance covered by the body in SHM from its mean position.

Amplitude (a): The maximum displacement of the body from its equilibrium position or mean position is its amplitude.

Period (T): The time taken by the body to complete one oscillation is its period.

$$T = \frac{2\pi}{\omega}$$

Frequency (ν): Frequency of S.H.M. is the number of oscillations that a oscillating body performs per unit time.

$$\boldsymbol{\nu} = \frac{1}{T}$$

$$\boldsymbol{\nu} = \frac{\omega}{2\pi}$$

$$\omega = 2\pi\boldsymbol{\nu}$$

Angular frequency (ω): It is the angular displacement per unit time.

$$\omega = \frac{angle\ covered}{time\ taken} = \frac{2\pi}{T} = 2\pi\nu$$

Derivation of differential equation of SHM starting from Hooke's Law:

Consider a body of mass 'm' executing simple harmonic motion. Let the restoring force be F and the displacement of the body from its equilibrium position be *'x'*.

Then, for an oscillating body,the restoring force is directly proportional to the displacement from the mean position.

i.e., F = – kx --------(1)

which is commonly known as Hooke's Law where 'k' is the spring constant or force constant, [i.e. the restoring force/unit displacement]. The negative sign shows that restoring force acts in the direction opposite to the displacement 'x'.

The system must also obey Newton's second law of motion which states that the force is equal to mass 'm' times acceleration 'a', i.e. F = ma.

F = ma

$$F = m\frac{dv}{dt}$$

$F = m\frac{d^2x}{dt^2}$-----------(2)

Equating (1) and (2)

$$m\frac{d^2x}{dt^2} = -kx$$

$$\frac{d^2x}{dt^2} + \boldsymbol{\frac{k}{m}x = 0}$$

This equation is called as differential equation of SHM.

Solution of this equation is $x = a\sin wt$. where, 'a' is the amplitude and 'ω' is the angular frequency and 't' is the time elapsed. 'ω' is also called as natural frequency of vibration and is given as $\omega = \sqrt{\frac{k}{m}}$

The Time period & Frequency of oscillation: The time period of oscillation of a spring is dependent on the spring constant of the spring and the mass of the system. It is independent of the force of gravity. It is given by

$$T = \frac{2\pi}{\omega}$$

$$T = 2\pi\sqrt{\frac{m}{k}} \text{where} \omega^2 = \frac{k}{m}$$

And **frequency** is given by, $\nu = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

Sprigs

Stiffness Factor and its Physical Significance

Physically, force constant is a measure of stiffness. In the case of springs, it represents how much force it takes to stretch the spring over a unit length. If the spring is strong or stiff, spring constant **k** will be large, and **k** will be small for a weak spring.

Expression for spring Constant for series combination of springs:

Consider two idealized springs with force constants k_1 and k_2 connected in series as shown in the figure. Let a body of load F = mg is suspended at the free end of these two springs in series combination. When the body is pulled downwards through a little distance 'x' , the two springs suffer different extensions say 'x_1' and 'x_2'. But the restoring force is same in each spring.

Following Hook's law, for the spring 1,

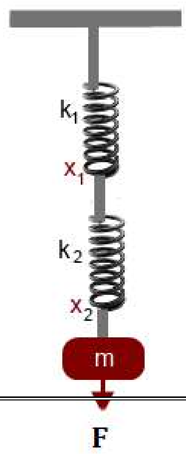
$F = -k_1x_1$

$mg = -k_1x_1$

or, $x_1 = -\frac{mg}{k_1}$ -----(1)

Similarly, for he spring 2,

$F = -k_2x_2$



$mg = -k_2x_2$

or, $x_2 = -\frac{mg}{k_2}$ -----(2)

For the combination,

$F = -k_s x$

$mg = -k_s x$

or, $x = -\frac{mg}{k_s}$-----(3)

But, the total extension, $x = (x_1 + x_2)$

Or, $-\frac{mg}{k_s} = -\frac{mg}{k_1} - \frac{mg}{k_2}$

$$\frac{1}{k_s} = \frac{1}{\mathrm{k}_1} + \frac{1}{\mathrm{k}_2}$$

Or, $\boldsymbol{k_s = \frac{k_1 k_2}{k_1 + k_2}}$

The time period is given as $\boldsymbol{T = 2\pi\sqrt{\frac{m}{k_s}}}$

Expression for spring Constant for parallel combination of springs:

Consider two idealized springs with force constants k_1 and k_2 connected in parallel as shown in figure. Let a body of load F = mg is suspended by these two springs in parallel combination. Let the body be pulled downwards through a small distance ‘x’. Each spring gets stretched by a length ‘x’. If F_1 and F_2 are the two restoring forces set up due to extension of springs, then

from Hook’s law, for the spring 1,

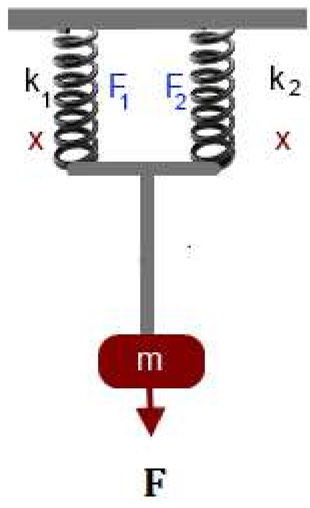
$F_1 = -k_1x$-----(1)

Similarly, for the spring 2,

$F_2 = -k_2x$-----(2)

∴For the combination,

$F_p = -k_px$-----(3)



The restoring force F_p is shared by the two springs.

Therefore,

$F_p = \mathrm{F}_1 + \mathrm{F}_2$

$-k_p x = -\mathrm{k}_1\mathrm{x} - \mathrm{k}_2\mathrm{x}$

Or $\boldsymbol{k_p} = \mathbf{k_1} + \mathbf{k_2}$

The time period is given as $\boldsymbol{T} = \boldsymbol{2\pi}\sqrt{\frac{\boldsymbol{m}}{\boldsymbol{k_p}}}$

Different types of springs

Helical Springs

Helical springs are the most common types of springs in product manufacturing. Wire coiled into a helix shape (hence the name) with different cross-sections can make helical springs. Below are the kinds of springs under category one.

1. Compression Springs

Compression springs are open-coil helical springs with a constant coiled diameter and variable shape that resists axial compression. The simplest example of its application is in the ballpoint pen, where it is responsible for the "popping" effect. It is also applicable in valves and suspension.

2. Extension Springs

Unlike compression springs, extension springs are closed coil helical springs. They are suitable for creating tension, storing the energy, and using the energy to return the spring to its original shape. A simple example of its applications is in garage doors. Others are in pull levers, jaw pliers, and weighing machines.

3. Torsion Springs

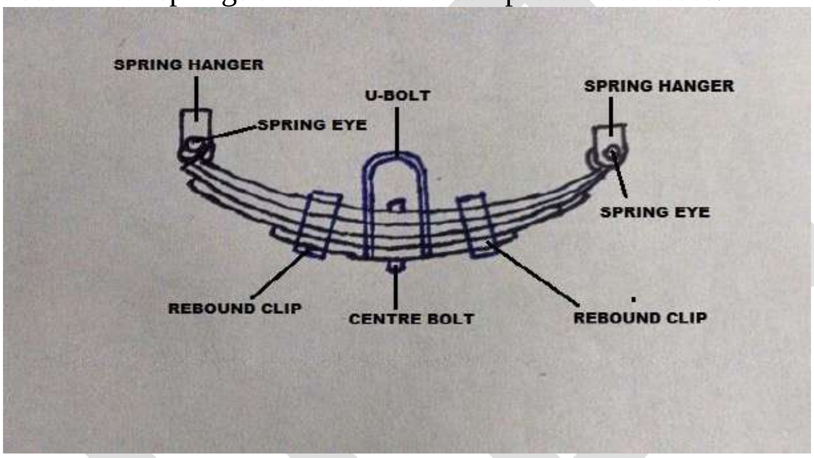
A torsion spring is attached to two different components using its two ends. This keeps the two components apart at a certain angle. These springs use radial direction when force is acting radially due to rotation.

Shock Absorber

A shock absorber is a mechanical or hydraulic device designed to absorb and damp shock impulses. This is achieved by converting the kinetic energy of the shock into another form of energy (typically heat) which is then dissipated. Shock absorber is to absorb or dampen the compression and rebound of the springs and suspension. Coil-over-shock absorber or spring coil shock absorber. Absorber with coil spring looks like a suspension strut.It helps to control unwanted and excess spring motion.

Leaf Springs (flat spring or carriage spring)

Leaf springs are types of springs made from rectangular metal plates, also known as leaves. The rectangular metal plates are normally bolted and clamped, and they have major use in heavy vehicles. Below are the different types of leaf springs and their applications. These are mostly used in automobiles. The major stresses produced in leaf springs are tensile and compressive stresses.



Application in railway/truck suspension

The main function of leaf springs in railways/truck suspension is to provide comfort to the passengers by minimizing the vertical vibration caused by the nonuniformity of road geometry. Leaf springs support the weight of the chassis, making them ideal for commercial vehicles. They also control axle damping. The chassis roll can be controlled more efficiently due to the high rear moment centre and wide spring base. If the springs are mounted wider apart, the roll tendencies will be less.

Different cases of SHM: Under SHM, we have the following 3 cases

1. Free oscillations
2. Damped oscillations
3. Forced oscillations

Free Oscillation: 'If an oscillating body oscillates with constant amplitude at its own natural frequency without the help of any external force is called free oscillation.

Natural Frequency: The natural frequency is the rate at which an object vibrates when it is not disturbed by an outside force.

Examples for free oscillations:

1. The vertical oscillations of a loaded spring suspended from a rigid support.
2. Motion of needle of sewing machine
3. Motion of pendulum
4. Vibration of tuning fork

Equation of motion for free oscillations:

The equation of motion of a free oscillation is given by

$$\frac{d^2x}{dt^2}+\frac{\mathrm{k}}{\mathrm{m}}\mathrm{x}=0$$

Where, *m* is the mass of the oscillating body, *k* is the force constant, *x* is the displacement at the instant *t* of an oscillating body.

Damped Oscillation: The oscillations of a body whose amplitude goes on decreases with time due to the presence of resistive forces is called damped oscillation.

Examples:

1. Mechanical oscillations of a simple pendulum
2. A swing left free to oscillate after being pushed once.

Theory of damped oscillations:

Consider a body of mass 'm' executing oscillations in a resistive medium. The oscillations are damped due to resistance offered by the medium.

The damping force, acts in a direction opposite to the movement of the body and velocity dependent

i.e.,

$F_{dam}\ \alpha\ -v$

$F_{dam} = -bv$, where 'b' is damping constant

$$F_{net} = F_{res} + F_{dam}$$

$$F_{net} = -kx - bv$$

$$F_{net} = -kx - b\frac{dx}{dt}$$

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

$$\frac{d^2x}{dt^2}+\left(\frac{b}{m}\right)\frac{dx}{dt}+\left(\frac{k}{m}\right)x=0 \text{-------(1)}$$

Where, $2\beta=\frac{b}{m}\ \beta=\frac{b}{2m}$, $\omega^2=\frac{k}{m}$

$$\frac{d^2x}{dt^2}+2\beta\frac{dx}{dt}+\omega^2x=0 \text{----------(2)}$$

The solution of the above equation is

$x=Ae^{\alpha t}$ --------(3) where, 'A' and 'α' are constants

$$\frac{dx}{dt}=A.\alpha.e^{\alpha t}=\alpha.x$$

$$\frac{d^2x}{dt^2}=A.\alpha.\alpha e^{\alpha t}=\alpha^2.x$$

$$\alpha^2x+2\beta.\alpha x+\omega^2x=0$$

$$x(\alpha^2+2\beta.\alpha+\omega^2)=0$$

$$\Rightarrow(\alpha^2+2\beta.\alpha+\omega^2)=0$$

The solution of this equation can be given using

$$\alpha=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$\alpha=\frac{-2\beta\pm\sqrt{4\beta^2-4\omega^2}}{2}$$

$$\alpha=\frac{-2\beta\pm2\sqrt{\beta^2-\omega^2}}{2}$$

$$\alpha=-\beta\pm\sqrt{\beta^2-\omega^2} \text{ -------(4)}$$

(4) in (3)

$$x=Ae^{\left(-\beta\pm\sqrt{\beta^2-\omega^2}\right)t}$$

Or, $x=A_1e^{\left(-\beta+\sqrt{\beta^2-\omega^2}\right)t}+A_2e^{\left(-\beta-\sqrt{\beta^2-\omega^2}\right)t}$ -------(5)

Where, A_1 and $A_2$are constants to be evaluated

Now, at t=0, $x = x_0$ (i.e., maximum displacement)

∴equation (5) becomes $x_0 = A_1 + A_2$ -------(6)

Also, at t=0, $x = x_0$, the velocity is zero. i.e.,$\frac{dx}{dt} = 0$

$$\therefore \frac{dx}{dt} = \left(-\beta + \sqrt{\beta^2 - \omega^2}\right)A_1 e^{\left(-\beta + \sqrt{\beta^2 - \omega^2}\right)t} + \left(-\beta - \sqrt{\beta^2 - \omega^2}\right)A_2 e^{\left(-\beta - \sqrt{\beta^2 - \omega^2}\right)t} = 0$$

Since t=0,

$$\left(-\beta + \sqrt{\beta^2 - \omega^2}\right)A_1 + \left(-\beta - \sqrt{\beta^2 - \omega^2}\right)A_2 = 0$$

$$-\beta(A_1 + A_2) + \sqrt{\beta^2 - \omega^2}(A_1 - A_2) = 0$$

$$-\beta x_0 + \sqrt{\beta^2 - \omega^2}(A_1 - A_2) = 0$$

$$\frac{\beta x_0}{\sqrt{\beta^2 - \omega^2}} = A_1 - A_2 \text{-------(7)}$$

Adding (6) and (7)

$$2A_1 = x_0 + \frac{\beta x_0}{\sqrt{\beta^2 - \omega^2}}$$

$$2A_1 = x_0 \left(\frac{1 + \beta}{\sqrt{\beta^2 - \omega^2}} \right)$$

$$\boxed{A_1 = \frac{x_0}{2}\left(1 + \frac{\beta}{\sqrt{\beta^2 - \omega^2}}\right)} \quad \text{----------(8)}$$

Subtracting, (6) - (7)

$$2A_2 = x_0 - \frac{\beta x_0}{\sqrt{\beta^2 - \omega^2}}$$

$$2A_2 = x_0 \left(1 - \frac{\beta}{\sqrt{\beta^2 - \omega^2}}\right)$$

$$A_2 = \frac{x_0}{2}\left(1 - \frac{\beta}{\sqrt{\beta^2 - \omega^2}}\right) \text{-------(9)}$$

$\therefore$ Equation (5) is given as

$$\boxed{x = \frac{x_0}{2}\left(1 + \frac{\beta}{\sqrt{\beta^2 - \omega^2}}\right).e^{\left(-\beta + \sqrt{\beta^2 - \omega^2}\right)t} + \frac{x_0}{2}\left(1 - \frac{\beta}{\sqrt{\beta^2 - \omega^2}}\right).e^{\left(-\beta - \sqrt{\beta^2 - \omega^2}\right)t}} \text{--------(10)}$$

This is the expression for decay amplitude.

Depending upon the strength of damping force the quantity $(\beta^2 - \omega^2)$ can be positive /negative /zero giving rise to three different cases.

We have the general equation of damped oscillation as (eqn (5))

$$x = A_1 e^{\left(-\beta + \sqrt{\beta^2 - \omega^2}\right)t} + A_2 e^{\left(-\beta - \sqrt{\beta^2 -}\omega^2\right)t}$$

Or, $x = e^{-\beta t}(A_1 e^{\left(\sqrt{\beta^2 - \omega^2}\right)t} + A_2 e^{\left(-\sqrt{\beta^2 -}\omega^2\right)t}$ ---------**

Case 1: If $\beta^2 > \omega^2$, over damping

When $\beta^2 > \omega^2$, $\sqrt{\beta^2 - \omega^2}$ is positive.

Let $\alpha = \sqrt{\beta^2 - \omega^2}$

Then the eqn ** becomes

$$x = e^{-\beta t}(A_1 e^{\alpha t} + A_2 e^{-\alpha t})$$

That means, there is an exponential decay of the displacement w.r.t time.

i.e., the body, without oscillating, returns from its maximum displacement to the equilibrium position very slowly and rests there. This is referred as over damped motion.

Eg: The motion of simple pendulum in a highly viscous medium.

Case 2 : If $\beta^2 = \omega^2$, critical damping

When $\beta^2 = \omega^2$,

Then the eqn ** will be

$$x = e^{-\beta t}(A_1 + A_2)$$

i.e., the body doesn't oscillate and returns to its equilibrium position very rapidly. This is referred as critical damping.

Critical damping provides the quickest approach to zero amplitude compared to other two cases.

Eg: The spring of the automobiles, the speedometers of the vehicles.

Case 3: If $\beta^2 < \omega^2$, under damping

When $\beta^2 < \omega^2$, $\sqrt{\beta^2 - \omega^2}$ is negative.

If $\sqrt{\beta^2 - \omega^2} = \omega_1$

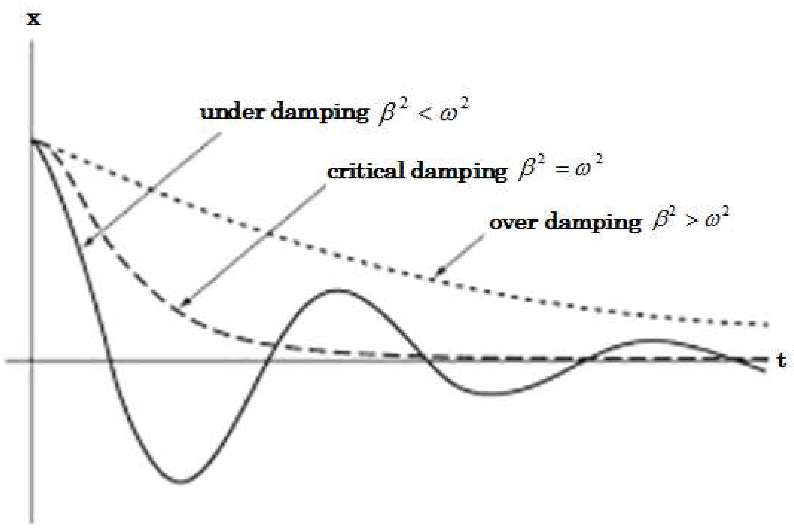
Then, $\sqrt{\omega^2 - \beta^2} = i\omega_1$

$\therefore$the eqn ** will be

$$x = e^{-\beta t}(A_1 e^{i\omega_1 t} + A_2 e^{-i\omega_1 t})$$

i.e., the amplitude of the oscillating system will not be constant and decreases exponentially with time, till the oscillation dies out. This is referred as under damping

Eg: The motion of simple pendulum in air.



Quality Factor& its significance: The rate of energy loss of a weakly damped harmonic oscillator is convenientlycharacterized bya single parameter Q, called the **quality factor of the oscillator.**

Quality factor is defined to be 2π timesthe energy stored in the oscillator divided by the energy lost in a single oscillation period.

$$Q = 2\pi \frac{Energy\ stored\ in\ the\ oscillator}{Energy\ Loss\ per\ period}$$

$Q = \frac{\omega}{2\beta}$, where, $= \frac{2\pi}{T}$, $2\beta = \frac{b}{m}$

Forced Oscillation: The oscillation in which a body oscillates under the influence of an external periodic force (also called as driving force) is known as forced oscillation.Here, the amplitude of oscillation, experiences damping but remains constant due to the external energy supplied to the system.

Eg: Oscillations of a swing which is pushed periodically by a person.

Theory of Forced Oscillations: Consider a body of mass 'm' executing oscillations in a damping medium, acted upon by an external periodic sinusoidal force '$F sin \omega_o t$'. Then its equation of motion is

$$F_{net} = F_{res} + F_{dam} + F_{forced}$$

$$F_{net} = -kx - bv + F \sin \omega_o t$$

$$F_{net} = -kx - b\frac{dx}{dt} + F \sin \omega_o t$$

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F \sin \omega_o t$$

$$\frac{d^2x}{dt^2} + \left(\frac{b}{m}\right)\frac{dx}{dt} + \left(\frac{k}{m}\right)x = \frac{F}{m}\sin \omega_o t$$

$$\frac{d^2x}{dt^2} + 2\beta\frac{dx}{dt} + \omega^2 x = \frac{F}{m}\sin \omega_o t \text{ ------(1)}$$

The solution of this equation is

$$x = a\sin(\omega_o t - \alpha) \text{ --------(2)}$$

Where, 'a' is the amplitude and $'\alpha'$ is the phase of the oscillating body to be determined.

$$\frac{dx}{dt} = a\omega_o \cos(\omega_o t - \alpha)$$

$$\frac{d^2x}{dt^2} = -a\omega_o\omega_o \sin(\omega_o t - \alpha)$$

$$\frac{d^2x}{dt^2} = -a\omega_o{}^2 \sin(\omega_o t - \alpha)$$

$$\therefore (1) \Rightarrow -a\omega_o^{\ 2}\sin(\omega_o t-\alpha)+2\beta a\omega_o\cos(\omega_o t-\alpha)+\omega^2 a\sin(\omega_o t-\alpha)=\frac{F}{m}\sin\omega_o t$$

$\frac{F}{m}\sin(\omega_o t)$ can be written as $\frac{F}{m}\sin[(\omega_o t-\alpha)+\alpha]=\frac{F}{m}\left(\sin(\omega_o t-\alpha).\cos\alpha+\cos(\omega_o t-\alpha).\sin\alpha\right)$

$\therefore (1) \Rightarrow$

$$-a\omega_o^{\ 2}\sin(\omega_o t-\alpha)+2\beta a\omega_o\cos(\omega_o t-\alpha)+a\omega^2\sin(\omega_o t-\alpha)=\frac{F}{m}\sin(\omega_o t-\alpha).\cos\alpha+\frac{F}{m}\cos(\omega_o t-\alpha).\sin\alpha$$

By equating the coefficients of $\sin(\omega_o t-\alpha)$ from both the sides, we get

$$-a\omega_o^{\ 2}+a\omega^2=\frac{F}{m}.\cos\alpha$$

$$a(\omega^2-\omega_o^2)=\frac{F}{m}.\cos\alpha \text{ --------(2)}$$

Similarly, by equating the coefficients of $cos(\omega_o t-\alpha)$ from both the sides, we get

$$2\beta a\omega_o=\frac{F}{m}.\sin\alpha \text{ --------(3)}$$

Squaring and adding eqn (2) and (3) we get,

$$\left(a\left(\omega^2-\omega_0^2\right)\right)^2+\left(2\beta a\omega_0\right)^2=\left(\frac{F}{m}\right)^2(\cos^2\alpha+\sin^2\alpha)$$

$$a^2\left[\left(\omega^2-\omega_0^2\right)^2+4\beta^2\omega_0^2\right]=\left(\frac{F}{m}\right)^2$$

$$a^2=\frac{\left(\frac{F}{m}\right)^2}{\left(\omega^2-\omega_0^2\right)^2+4\beta^2\omega_0^2}$$

$$\boxed{a=\frac{\left(\frac{F}{m}\right)}{\sqrt{\left(\omega^2-\omega_0^2\right)^2+4\beta^2\omega_0^2}}}$$

This equation represents the amplitude of the forced vibrations.

Phase of the forced vibrations:

Dividing eqn (3) by eqn (2)

$$\tan\alpha=\frac{2\beta a\omega_0}{a(\omega^2-\omega_0^2)}=\frac{2\beta\omega_0}{(\omega^2-\omega_0^2)}$$

$\therefore$the Phase of the forced vibration is given by

$$\boxed{\alpha = \tan^{-1}\left(\frac{2\beta\omega_0}{(\omega^2-\omega_0^2)}\right)}$$

Dependence of amplitude and phase on the frequency of the applied force:

Case 1: $\omega_o \ll \omega$, when the frequency of the force is low.

For $\omega_o \ll \omega$, ω_o^2 will be very small and damping is small ($\beta \to 0$)

i.e.,$\omega^2 - \omega_o^2 \approx \omega^2 \; and \; 2\beta\omega_o \approx 0$

∴The amplitude,

$$a = \frac{\left(\frac{F}{m}\right)}{\omega^2} = \frac{F}{m\omega^2}$$

The Phase α is given as

$$\alpha = \tan^{-1}\left(\frac{2\beta\omega_0}{\omega^2-\omega_0^2}\right)$$

since$\omega_o$is very small and damping $(\beta)$is also small

$$\omega^2 - \omega_o^2 \approx \omega^2 \; and \; \frac{2\beta\omega_o}{\omega^2} \approx 0$$

$$\therefore \alpha = \tan^{-1}(0)$$

$$\alpha = 0$$

This shows that the amplitude of vibration is independent of the frequency of force. The amplitude depends on the magnitude of the applied force. The displacement and force are always in phase.

Case 2: $\omega_o = \omega$, the frequency of force is equal to frequency of the body.

For $\omega_0 = \omega, (\omega^2 - \omega_0^2) = 0$

∴amplitude $a = \frac{F/m}{2\beta\omega_0} = \frac{F/m}{2\left(\frac{b}{2m}\right)\omega_0} = \frac{F}{b\omega_0}$

$$a = \frac{F}{b\omega_0}$$

The phase, $\alpha = \tan^{-1}\left(\frac{2\beta\omega_0}{0}\right)$

$$\alpha = \tan^{-1}(\infty) = \frac{\pi}{2}$$

This shows that the amplitude of vibration is governed by the damping and for small damping forces the amplitude of vibration is quite large. The displacement lags behind the force by $(\frac{\pi}{2})$.

Case 3: $\omega_o \gg \omega$, the frequency of force is greater than frequency of the body.

For $\omega_o \gg \omega$, and damping is small ($\beta \to 0$)

$$(\omega^2 - \omega_0^2)^2 \approx (\omega_o^2)^2$$

$$\therefore a = \frac{F/m}{\sqrt{4\beta^2\omega_0^2 + \left(\omega_0^4\right)}}$$

Since β is very small, $4\beta^2\omega_o^2 \langle\langle \omega_o^4$

$$a = \frac{F/m}{\sqrt{\omega_0^4}} = \frac{F/m}{\omega_0^2} = \frac{F}{m\omega_0^2}$$

The phase, $\alpha = \tan^{-1}\left[\frac{2\beta\omega_0}{\omega^2 - \omega_0^2}\right] = \tan^{-1}\left[\frac{-2\beta}{\omega_0}\right]$

Since β is very small $\frac{2\beta}{\omega_o} \approx 0$

$$\alpha = \tan^{-1}(-0) = \pi$$

In this case, the amplitude goes on decreasing and phase difference tends towards π.

Resonance

The amplitude of the forced vibrations is given as

$$a = \frac{\left(\frac{F}{m}\right)}{\sqrt{\left(\omega^2 - \omega_0^2\right)^2 + 4\beta^2\omega_0^2}} \text{ ---------(1)}$$

Conditions for resonance: For ‘a’ to be maximum, the denominator in the above equation must be minimum. It is possible when,

i) $\beta = \frac{b}{2m}$ is minimum, i.e., when the damping caused by the medium is made minimum.

ii) $\omega_o = \omega$, i.e., when frequency of the applied force (ω_o) becomes equal to the natural frequency of vibration of the body (ω).

Therefore eqn (1) reduces to

$$a = \frac{\left(\frac{F}{m}\right)}{\sqrt{4\beta^2\omega^2}} = \frac{\left(\frac{F}{m}\right)}{2\beta\omega} \quad \text{--- (2)}$$

This is the expression for maximum amplitude (resonance).

"When the frequency of periodic force acting on a vibrating body is equal to the natural frequency of vibrations of the body, the body vibrates with maximum amplitude. This phenomenon is called resonance".

At resonance, the energy transfer from the periodic force to the vibrating body is maximum.

Eg: 1. Helmholtz resonator,

2. the vibrations caused by an excited tuning fork in another nearby identical tuning fork.

Sharpness of resonance: -

The amplitude is maximum at resonance frequency which decreases rapidly as the frequency increases or decreases from the resonant frequency.

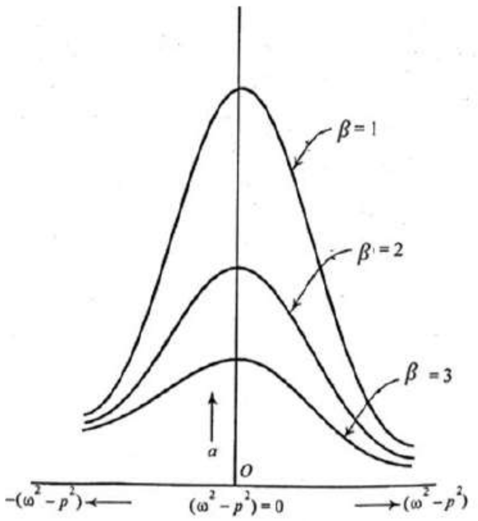
The rate at which the amplitude decreases with the frequency of the applied external force on either side of resonant frequency is termed as "sharpness of resonance".

$$Sharpness\ of\ resonance = \frac{change\ in\ amplitude}{change\ in\ frequency}$$

Note: The sharpness of a resonance is measured by its Q-factor.

Significance of sharpness of resonance: The rate at which the change in amplitude occurs near resonance depends on damping. For small damping, the rate is high and the resonance is said to be sharp. For heavy damping it will be low, and resonance is said to be flat.

Effect of damping: The response of amplitude to various degrees of damping is as shown in the graph. From the graph, it can be observed that, for larger values of damping coefficient 'β', the curve is flat and hence the resonance is flat. On the other hand, for smaller values of damping coefficient β the curve is sharp and it refers to sharp resonance.



SHARPNESS OF RESONANCFE

The sharpness of resonant peak depends on the damping. If the damping is small, the resonant peak is sharp, if the damping is large, it is less sharp.

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Shock waves

In Aerodynamics, the speed of the bodies moving in a fluid medium can be classified into different categories on the basis of following terms

Mach number: it is defined as ratio of the speed of the object to the speed of the sound in a given medium. i.e.,

$$\textit{Mach number} = \frac{\textit{object speed}}{\textit{speed of the sound in a given medium}}$$

It is denoted as M. If v is the speed of object and ‘a’ is the speed of sound in the medium ,

then $\mathbf{M = \frac{v}{a}}$ and speed of the sound is given by $\mathbf{a=\sqrt{\gamma RT}}$

γ – ratio of specific heats

R – specific gas constant

T – local temperature in Kelvin

Mach number gives a measure of how fast a body is moving with respect to the speed of sound. It is a very important quantity in compressible flow theory (where density of the fluid changes).

Acoustic, ultrasonic (based on frequency), Subsonic, Supersonic waves

1) **Acoustic wave**

Acoustic wave is simply a sound wave which moves with the speed of 330m/s, in air at STP. they have frequencies between 20 -20,000 Hz. Amplitude of acoustic wave is very small.

2) **Ultrasonic waves**

These are pressure waves having frequencies beyond 20,000 Hz. but they travel with the same speed as that of sound. Amplitude of ultrasonic wave is very small.

Mach number regimes

1) **Subsonic waves**

If the speed of mechanical wave or body moving in a fluid is lesser than that of sound. Then, such a speed is referred to as subsonic and the wave is subsonic wave. All the subsonic waves have Mach number less than 1. For a body moving with subsonic speed sound emitted by it manages move ahead and away from the body since it is faster than the body.

Eg : speed of car and train

2) Supersonic waves

Supersonic waves are mechanical waves which travel with speeds greater than that of sound i.e., with speeds for which Mach no. >1. A body with supersonic speed moves ahead leaving behind series of expanding sound waves. Amplitude of supersonic wave will be very high and it effects medium in which it is travelling.

Eg: fighter planes.

3) **Transonic:** As the speed of the object approaches the speed of sound, the flight Mach number is nearly equal to one, **M = 1**, and the flow is said to be transonic. At some places on the object, the local speed exceeds the speed of sound.

4) **Hypersonic:** For speeds greater than five times the speed of sound, **M > 5**, the flow is said to be hypersonic. At these speeds, some of the energy of the object now goes into exciting the chemical bonds which hold together the nitrogen and oxygen molecules of the air.

Mach cone

A number of common tangents drawn to the expanding sound waves emitted from a body at supersonic speed constitute a cone called Mach cone.

Mach angle

The angle made by the tangent with the axis of the Mach cone is called the Mach angle (μ)

$\mu = \sin^{-1}(\frac{1}{M})$ where M is the Mach number.

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Shock waves
Any fluid that propagates at supersonic speeds gives rise to shock wave.

- Shock waves are produced by a sudden dissipation of mechanical energy in a medium enclosed in a small space. The Shock waves depends on pressure, temperature and density of the medium through which it propagates.
- Ex: Shock waves are produced in nature during earth quakes called Seismic waves (2km/s – 8km/s)

There are two kinds of shock waves

1) Strong Shock waves: It is a compressed region possessing very high pressure and temperature having Mach number greater than 1

Ex: Lighting thunder or bombing, etc.,

2) Weak Shock waves: It is a compressed region possess Low pressure and temperature having Mach number less than or closer to 1

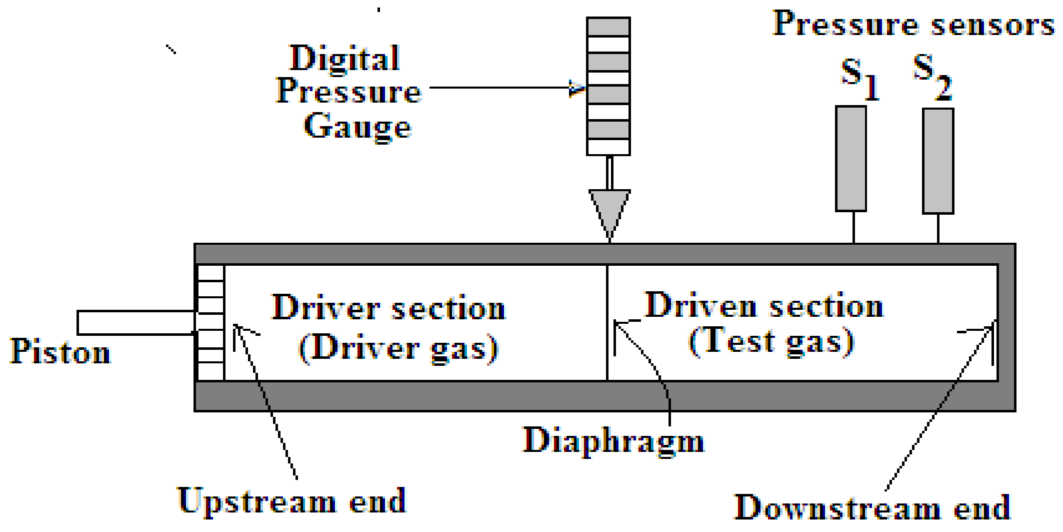
Ex: Explosion of a Cracker, Bursting of aTyre, etc.,

Properties of shock waves:

1) They travel in the medium with Mach number exceeding 1.
2) Shock waves obey the laws of fluid mechanics.
3) The effect caused by the shock waves result in increase of entropy.
4) Across the shock wave supersonic flow is decelerated into subsonic flow.
5) Shock wave exists in a very thin space of thickness not exceeding one micro meter.

Reddy tube or Reddy shock tube

Reddy tube is hand operated shock tube capable of producing shock waves by using human energy. It is a long cylindrical tube with two sections separated by a diaphragm. Its one end is fitted with the piston and the other end is closed or open to the surroundings.



REDDY TUBE

Description

- Reddy tube consists of a cylindrical stainless steel tube of about 30mm diameter. And of length nearly 1 m.
- It is divided into 2 sections each of length about 50 cm. one is the **driver tube** other is **driven tube** separated by a 0.1mm thick aluminum or paper diaphragm.
- It has a piston fitted at the far end of the driver section. Whereas the end of the driven section is closed.
- A digital pressure gauge is mounted in the driver section next to the diaphragm.
- Two piezoelectric sensors S_1 and S_2 are mounted on the driven section.
- The driver section is filled with gas termed as driver gas (high pressure) and the gas in the driven section is termed **driven gas.**

Working:

- The driver gas is compressed by pushing the piston hard into the driver tube until the diaphragm ruptures.
- Following the rupture, the driver gas rushes into the driven section and pushes the driven gas towards the downstream end which generates a moving shock wave that traverses the length of the driven section.
- The shock wave instantaneously raises the temperature and the pressure of the driven gas.
- If 'x' and 't' is the distance travelled and time taken by the shock waves between the two sensors, then velocity of shock waves traveling between the two sensors are given by $v = \frac{x}{t}$ and Mach number can be calculated as $M = \frac{v}{a}$

Characteristics of a Reddy's tube.

- The Reddy's tube operates on the principle of free piston driven shock tube(FPST)
- It is a hand operated shock producing device
- It is capable of producing Mach no exceeding 1.5
- The rupture pressure is a function of the thickness of the diaphragm
- Temperatures exceeding 900 K can ben be easily obtained by the reddytube. By using He as the driver gas and Ar as the driven gas.

Applications of shock waves

1. Cell information

By passing shock wave of suitable strength, DNA can be pushed inside a cell without affecting the functionality of DNA. This has wide biological applications.

2. Wood preservation

By using shock waves, chemical preservatives in the form of solutions could be pushed into the interior of wood samples which helps in withstanding the microbial attacks. By this method the life of ordinary wood can be increased.

3. Use in Pencil Industry:

In the manufacture of pencils, in the industry, the wood needs to be softened by soaking it in a polymer at 700C for about 3 hours and then dried. It takes days for the wood to dry.
In the modern process, the liquid is passed into the wood almost instantaneously by placing it in a liquid and sending a shock wave. The wood is then taken out and it will not take longer time to dry. The treated wood is ready for the next process without any delay.

4. Kidney stone treatment:

Shock wave is used in a therapy to crush the kidney stones into smaller pieces after which, they are passed out of the body smoothly through the urinary tracts.

5. Gas dynamics studies:

The extreme conditions of pressure and temperature that can be produced in the shock tube, helps us to the study of high temperature gas dynamics. This knowledge is useful in the study of supersonic motion of bodies & hypersonic re-entry of space vehicles into the atmosphere.

6. Shock wave assisted needleless drug delivery:

By using shock waves, drugs can be injected into the body without using needles. The drug is filled into a cartridge which is kept pressed on the skin & the shock wave is sent into the body using high pressure. The drug enters the body directly through pores of the skin. In this process, the patients do not experience any pain.

7. Treatment of dry bore wells:

Water will be available in the bore wells when water accumulates in the bore well through a number of seepage points which are porous. Sometimes, such seepage points are blocked by sand particles. A shockwave sent through such a dry bore well, clears the blockages and makes the bore well into a water source.

Numerical problem

1) The distance between the two pressure sensors in a shock tube is 100mm. The time taken by a shock wave to travel this distance is 200 microseconds. If the velocity of sound under the same conditions is $340ms^{-1}$, find the Mach number of the shock wave. (4 marks)

QUESTIONS:

1. Define simple harmonic motion, Mention the characteristics of SHM, Derive the equation for simple harmonic motion using Hooke's law.
2. Obtain the expression for period of oscillations of two springs in series and parallel combination.
3. What are damped oscillations? Give the theory of damped oscillations and hence discuss the case of under damping.
4. Give the theory of forced oscillations.
5. Define shock waves. Explain the experimental method of producing shock waves and measuring its Mach number using Reddy Shock tube.
6. Define Mach Number. Distinguish between subsonic, supersonic and hypersonic waves.

NUMERICALS

1) A mass of $25x10^{-2}$ kg is suspended from the lower end of vertical spring having a force constant 25N/m. What should be the damping constant of the system so that motion is critically damped?
2) A spring undergoes an extension of 5 cm for a load of 50 g. Find its force constant, angular frequency and frequency of oscillation, if it is set for vertical oscillations with a load of 200 g attached to its bottom. Ignore the mass of the spring.
3) A free particle is executing SHM in a straight line with a period of 25 seconds, after 5 seconds it has crossed the equilibrium point, the velocity is found to be 0.7m/s. find the displacement at the end of 10 seconds, and also the amplitude of oscillations.
4) The distance between the two pressure sensors in a shock tube is 100mm. The time taken by a shock wave to travel this distance is 200 microsecond. If the velocity of sound under the same conditions is $340ms^{-1}$, find the Mach number of the shock wave.

5) A mass of 0.5kg causes an extension of 0.003m in a spring and the system is set for oscillations. Find i) The force constant for the spring ii) Angular frequency and iii) Time period of the resulting oscillation.
6) Calculate the peak amplitude of vibration of system whose natural frequency is 1000Hz when it oscillates in a resistive medium for which the value of damping/unit mass is 0.008rad/s under the action of an external periodic force /unit mass of amplitude 5N/kg, with tunable frequency.

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