

Model Question Paper-I with effect from 2025

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1BMATS101

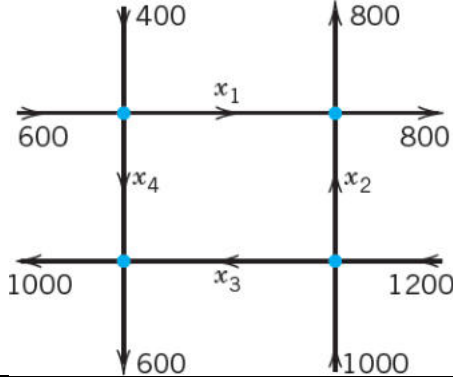
First Semester B.E./B.Tech. Degree Examination Calculus & Linear Algebra

TIME: 03Hours

Max.Marks:100

- Note: 1. Answer any **FIVE** full questions, choosing at least **ONE** question from each **MODULE**
 2. VTU Formula Hand Book is Permitted
 3. M: Marks, L: Bloom's level, C: Course outcomes

Module-1			M	L	C
Q 1.	a	Show that $u_x + u_y = u$, if $u = \frac{e^{x+y}}{e^x + e^y}$.	6	L2	1
	b	If $u = x + 3y^2 - z^3$, $v = 4x^2yz$, $w = 2z^2 - xy$, find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ at (1, -1, 0).	7	L2	1
	c	Find the extreme values of the function $f(x, y) = xy + \frac{a^3}{x} + \frac{a^3}{y}$.	7	L3	1
OR					
Q 2.	a	If $V = f(r, s, t)$ and $r = \frac{x}{y}$, $s = \frac{y}{z}$, $t = \frac{z}{x}$ show that $x \frac{\partial V}{\partial x} + y \frac{\partial V}{\partial y} + z \frac{\partial V}{\partial z} = 0$.	6	L2	1
	B	If $u = \frac{2yz}{x}$, $v = \frac{3zx}{y}$, $w = \frac{4xy}{z}$ then find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.	7	L2	1
	C	Apply Maclaurin's series, to expand $\cos x \cos y$ in powers of x and y up to second-degree terms.	7	L3	1
Module-2					
Q 3.	a	If $f = x^2yz$ and $g = xy - 3z^2$, calculate $\nabla(\nabla f \cdot \nabla g)$.	6	L2	1
	b	A vector field is given by $F = (6xy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (3xz^2 - y)\hat{k}$. Show that the field is irrotational and hence find its scalar potential.	7	L2	1
	c	Express the vector $\vec{F} = x\hat{i} + 2y\hat{j} + yz\hat{k}$ in spherical polar coordinates.	7	L3	1
OR					
Q 4.	a	Find the directional derivative of $f(x, y, z) = 4e^{2x-y+z}$ at the point (1, 1, -1) in the direction towards the point (-3, 5, 6).	6	L2	1
	b	Find $\text{div } \vec{F}$ and $\text{curl } \vec{F}$, where $F = \text{grad}(x^3 + y^3 + z^3 - 3xyz)$.	7	L2	1
	c	Express the vector $\vec{F} = 2x\hat{i} + 3y\hat{j} - z\hat{k}$ in cylindrical polar coordinates.	7	L3	1
Module-3					

Q 5.	A	Find the constant b if the rank of $\begin{bmatrix} 1 & 1 & -1 & 0 \\ 4 & 4 & -3 & 1 \\ b & 2 & 2 & 2 \\ 9 & 9 & b & 3 \end{bmatrix}$ is 3.	6	L2	2
	b	Find model matrix of $\begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$ and verify its diagonalization.	7	L2	2
	c	Write the system of linear equations of the traffic flow in the net of one-way street directions as shown in the figure and find its solution. 	7	L3	2
OR					
Q 6.	a	Investigate the values of λ and μ so that the equations $2x + 3y + 5z = 9$ $7x + 3y - 2z = 8$ $2x + 3y + \lambda z = \mu$ have (i) no solution (ii) unique solution (iii) infinite number of solutions.	6	L2	2
	b	Apply Gauss Jordan method to approximate the solutions of the system $83x + 11y - 4z = 95$ $7x + 52y + 13z = 104$ by choosing initial solution $(0,0,0)$. Perform four iterations.	7	L2	2
	C	Determine the eigenvalues and corresponding eigenvectors for the matrix $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$.	7	L3	2
Module-4					
Q 7.	a	Verify whether $v = (1, -2, 5)$ in $\mathbb{R}^3$ is a linear combination of the vectors $u_1 = (1, 1, 1)$, $u_2 = (1, 2, 3)$ and $u_3 = (2, -1, 1)$.	6	L2	3
	b	Determine whether $W = \{(a, b, c) / a + b + c = 0\}$ is a subspace of R^3 or not?	7	L2	3
	c	Find the basis and dimension of the row space, column space and null space of the matrix $\begin{bmatrix} 1 & -1 & 1 & 3 & 2 \\ 2 & -1 & 1 & 5 & 1 \\ 0 & 1 & -1 & -1 & -3 \end{bmatrix}$	7	L2	3

OR					
Q 8.	a	Find the basis and dimension of the subspace W spanned by $(1, 2, 3), (2, 4, 6), (0, 1, 1)$.	6	L2	3
	b	Find the inner products $\langle v_1, v_2 \rangle, \langle v_1, v_3 \rangle$ and $\langle v_2, v_3 \rangle$ where $v_1 = (1, 1, 1, 1)$, $v_2 = (1, 2, 4, 5)$ $v_3 = (1, -3, -4, -2)$.	7	L2	3
	c	Find the coordinates of the vector $v = (1, -3, 2)$ with respect to the basis $S = \{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}$.	7	L2	3
Module-5					
Q 9.	a	Verify whether the transformation $T: R^2 \rightarrow R^2$ which is defined by $T(x, y) = (3x + 4y, 10x - 4y + 3)$ is linear or not?	6	L2	3
	b	Prove that the transformation $F: R^2 \rightarrow R^2$ is singular and find its Kernal if the transformation $F(a, b) = (2a - 4b, 3a - 6b)$.	7	L2	3
	c	Find the rank and nullity of the transformation $T: R^3 \rightarrow R^3$ defined by $T(x, y, z) = (x + y, x - y, 2x + z)$.	7	L2	3
OR					
Q 10.	A	Check whether the transformation $T: V_1(R) \rightarrow V_3(R)$ defined by $T(x) = (x, x^2, x^3)$ is linear or not.	6	L2	3
	b	Consider the matrix $A = \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix}$ which defines a linear operator on $\mathbb{R}^2$. Find the matrix of the linear transformation relative to the basis $S = \{u_1, u_2\} = \left\{ \begin{bmatrix} 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 3 \\ -7 \end{bmatrix} \right\}$.	7	L2	3
		Let F be the linear transformation defined on a vector space R^2 through $F(x, y) = (2x + y, 3x + 2y)$, show that F is invertible and hence find F^{-1} .	7	L2	3