

MODULE - 4

Equilibrium and conditions of equilibrium

Equilibrium

A body is said to be in equilibrium when there is no translatory effect and rotational effect even after applying external forces.

$$\sum F_y = +5 - 5 = 0; \quad \sum F_x = 10 - 10 = 0;$$

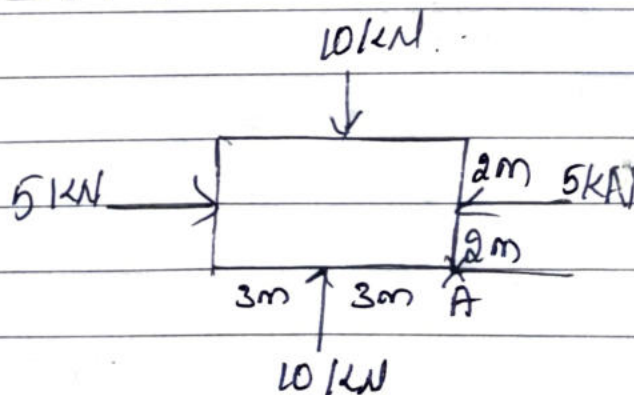
Conditions of Equilibrium

① Algebraic sum of forces in x-direction is zero
i.e., $\sum F_x = 0$.

② Algebraic sum of forces in y-direction is zero
i.e., $\sum F_y = 0$.

③ Algebraic sum of moments of system of forces is zero.
i.e., $\sum M = 0$.

Eg:-



$$\sum F_x = +10 - 10 = 0$$

$$\sum F_y = +5 - 5 = 0$$

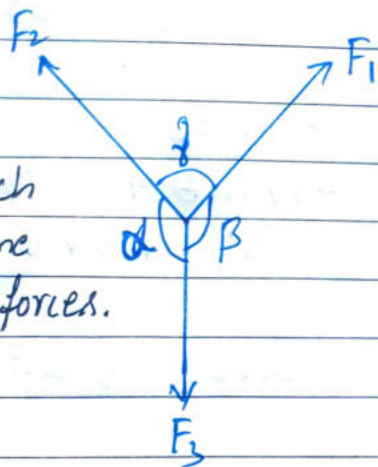
$$\sum M_A = (10 \times 2) - (5 \times 3) - (10 \times 2) + (5 \times 3) = 0$$

V. Imp

Lami's theoremStatement

If three forces acting at a point are in equilibrium then each force is directly proportional to sine of angle between the other two forces.

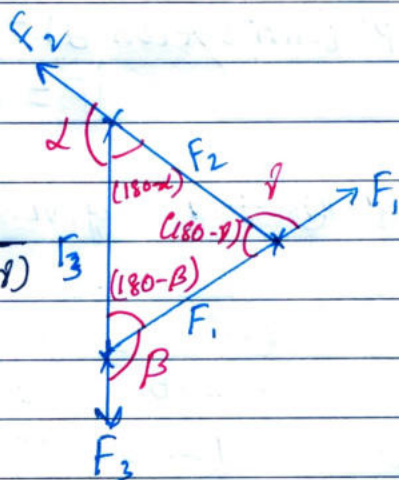
$$\frac{F_1}{\sin \alpha} = \frac{F_2}{\sin \beta} = \frac{F_3}{\sin \gamma}$$

Applying sine rule

$$\frac{F_1}{\sin(180-\alpha)} = \frac{F_2}{\sin(180-\beta)} = \frac{F_3}{\sin(180-\gamma)}$$

$$\therefore \sin(180-\theta) = \sin \theta$$

$$\boxed{\frac{F_1}{\sin \alpha} = \frac{F_2}{\sin \beta} = \frac{F_3}{\sin \gamma}}$$

Problems on equilibrium of forces

- ① Determine the magnitude and direction of force P , which keeps the concurrent system of forces in equilibrium

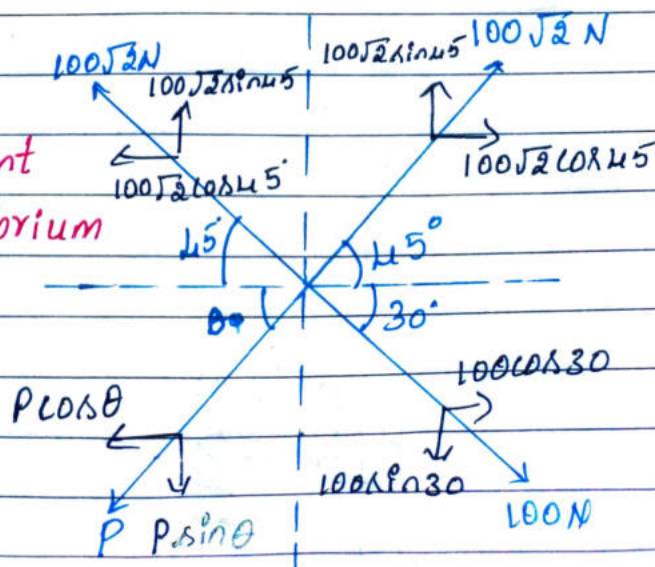
Sol:-

Applying eq^m condition

$$\sum F_{xc} = 0$$

$$-100\sqrt{2}\cos 45^\circ + 100\sqrt{2}\cos 45^\circ$$

$$+ 100\cos 30^\circ - P\cos \theta = 0$$



$$\therefore P \cos \theta = 86.6 \text{ N} \rightarrow \textcircled{1}$$

$$\Sigma F_y = 0$$

$$-P \sin \theta + 100\sqrt{2} \sin 45 + 100\sqrt{2} \sin 45 - 100 \sin 30 = 0$$

$$\therefore P \sin \theta = 150 \text{ N} \rightarrow \textcircled{2}$$

Solve Squaring and adding eqn ① & ②

$$P^2 \sin^2 \theta + P^2 \cos^2 \theta = 150^2 + 86.6^2$$

$$P^2 (\sin^2 \theta + \cos^2 \theta) = 29999.56$$

$$P = 173.2 \text{ N}$$

Eqn ② $\div$ ① gives

$$\frac{P \sin \theta}{P \cos \theta} = \frac{150}{86.6}$$

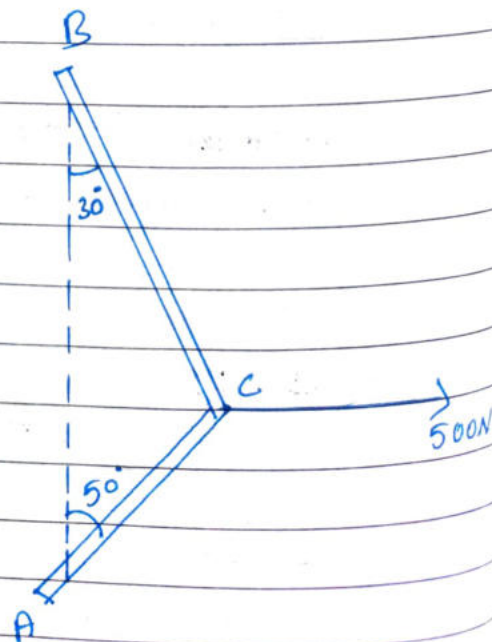
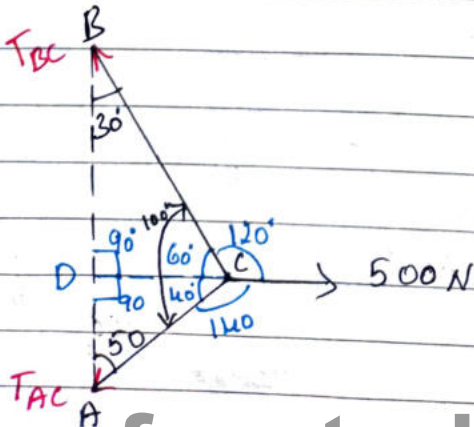
$$\tan \theta = \frac{150}{86.6}$$

$$\theta = \tan^{-1} \left(\frac{150}{86.6} \right)$$

$$\theta = 60^\circ$$

- ② Determine the forces induced in the members AC & BC of the structure shown in figure.

Soln:-



Applying Lami's Theorem

$$\frac{500}{\sin 100} = \frac{T_{BC}}{\sin 140} = \frac{T_{AC}}{\sin 120}$$

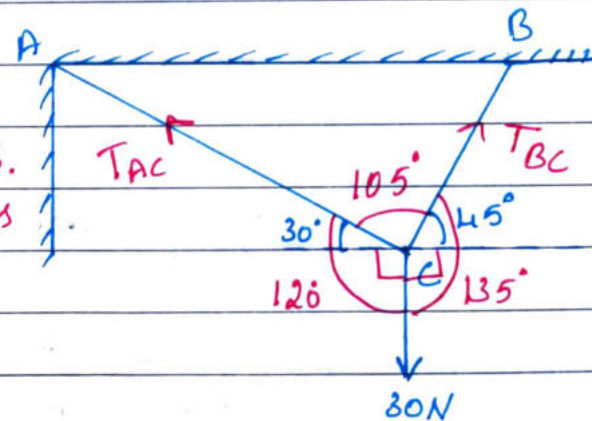
consider, $\frac{T_{BC}}{\sin 140} = \frac{500}{\sin 100}$

$$\therefore T_{BC} = 326.35 \text{ N}$$

consider, $\frac{T_{AC}}{\sin 120} = \frac{500}{\sin 100}$

$$\therefore T_{AC} = 439.69 \text{ N}$$

- ③ Two cables are connected at A & B as shown in figure. A force of 30 kN is applied at C. Determine the forces in the cables CA & CB.



Sol:- Applying Lami's theorem

$$\frac{T_{AC}}{\sin 135} = \frac{30}{\sin 105} = \frac{T_{BC}}{\sin 120}$$

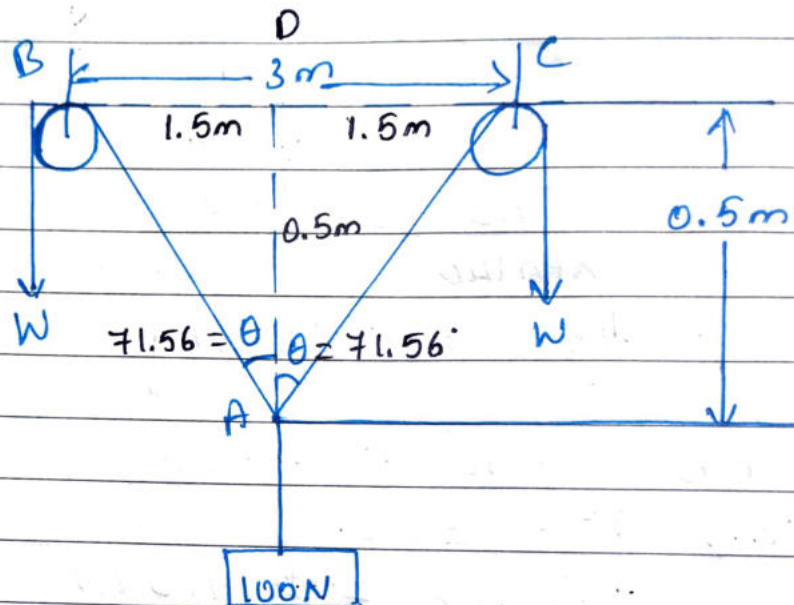
consider, $\frac{T_{AC}}{\sin 135} = \frac{30}{\sin 105}$

$$\therefore T_{AC} = 21.96 \text{ kN}$$

consider, $\frac{T_{BC}}{\sin 120} = \frac{30}{\sin 105}$

$$T_{BC} = 26.89 \text{ kN}$$

- ④ Find the value of w , which is required to maintain equilibrium configuration as shown in figure.



From Figure, $\tan \theta = \frac{1.5}{0.5}$

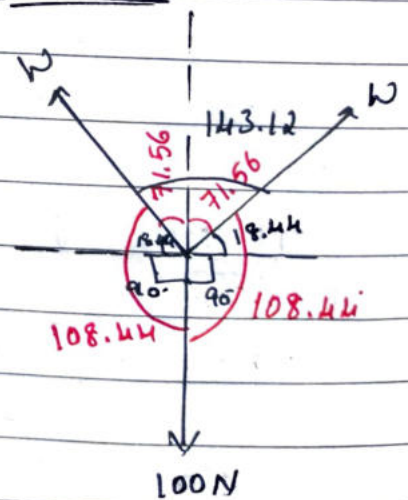
$$\therefore \theta = \tan^{-1} \left(\frac{1.5}{0.5} \right) = 71.56^\circ$$

Applying sine rule

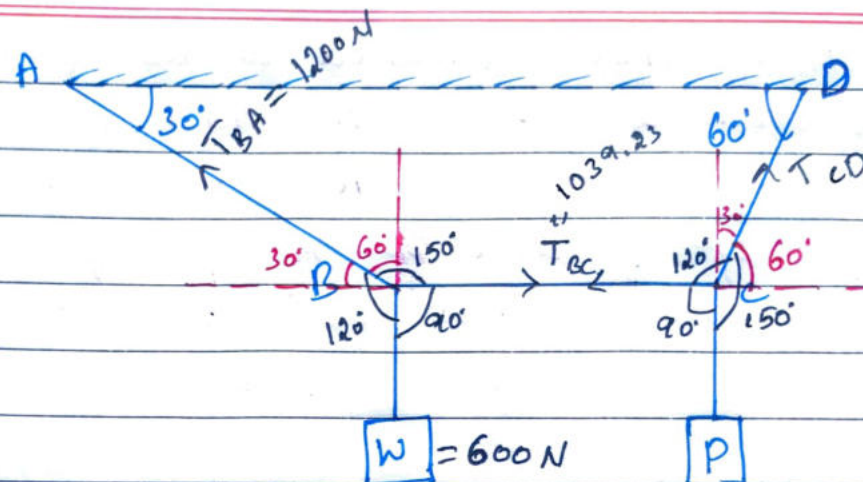
$$\frac{100}{\sin 143.12} = \frac{w}{\sin 108.44} = \frac{w}{\sin 108.44}$$

$$\therefore w = \frac{100 \sin 108.44}{\sin 143.12}$$

$$w = 158.10 \text{ N}$$



- ⑤ Figure below shows a rope supporting two loads w & P . If BC is horizontal and $w = 600 \text{ N}$, determine the load P . Also, find the tensile forces developed in the different segments of the rope.



Solⁿ:- consider the joint 'B' & apply Lami's theorem

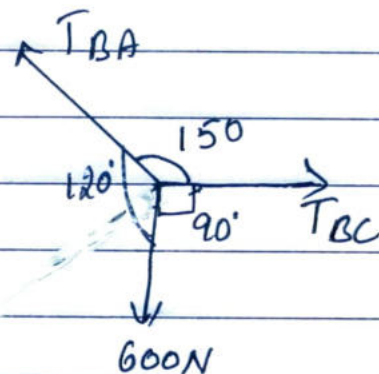
$$\frac{T_{BA}}{\sin 90^\circ} = \frac{600}{\sin 150^\circ} = \frac{T_{BC}}{\sin 120^\circ}$$

consider, $\frac{T_{BA}}{\sin 90^\circ} = \frac{600}{\sin 150^\circ}$

$$\therefore \boxed{T_{BA} = 1200 \text{ N}}$$

consider, $\frac{T_{BC}}{\sin 120^\circ} = \frac{600}{\sin 150^\circ}$

$$\therefore \boxed{T_{BC} = 1039.23 \text{ N}}$$



consider the joint 'C' & apply Lami's theorem

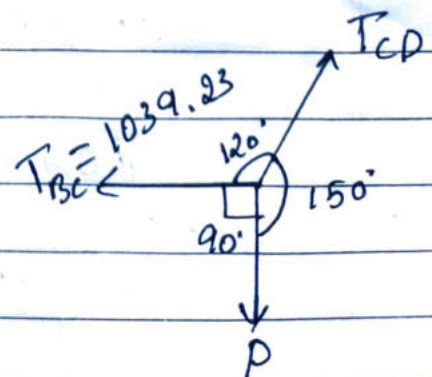
$$\frac{T_{CD}}{\sin 90^\circ} = \frac{1039.23}{\sin 150^\circ} = \frac{P}{\sin 120^\circ}$$

consider, $\frac{T_{CD}}{\sin 90^\circ} = \frac{1039.23}{\sin 150^\circ}$

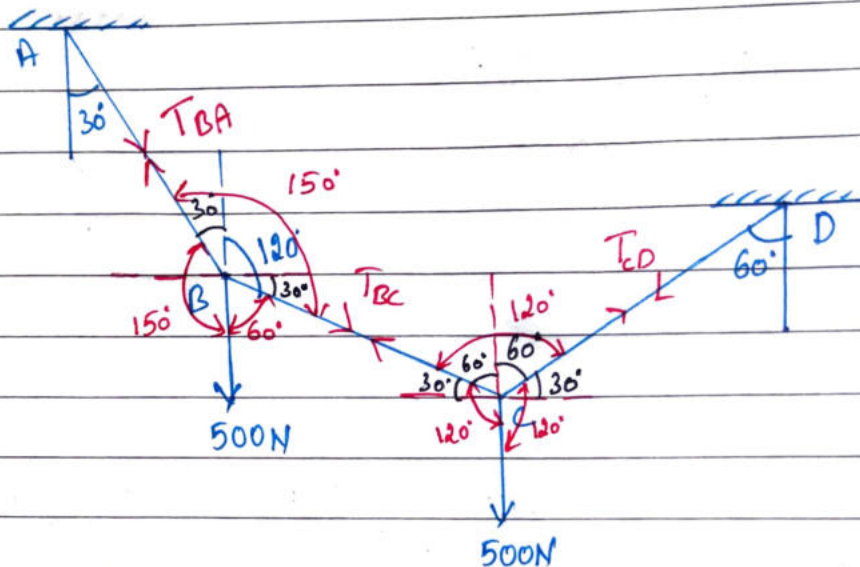
$$\therefore \boxed{T_{CD} = 2078.46 \text{ N}}$$

consider, $\frac{P}{\sin 120^\circ} = \frac{1039.23}{\sin 150^\circ}$

$$\therefore \boxed{P = 1800 \text{ N}}$$



A string ABCD attached to two fixed points A & D has two equal weights 500N attached to it at B and C. The weights rest with portions AB and CD inclined at angles of 30° and 60° respectively with the vertical. Find the tensions in the portions AB, BC and CD of the string. The inclination of BC with vertical is 120° .



consider joint 'B' & Apply Lami's theorem

$$\text{WKT, } \frac{T_{BA}}{\sin 60^\circ} = \frac{500}{\sin 150^\circ} = \frac{T_{BC}}{\sin 150^\circ}$$

$$\text{consider, } \frac{T_{BA}}{\sin 60^\circ} = \frac{500}{\sin 150^\circ}$$

$$\therefore T_{BA} = 866.02 \text{ N}$$

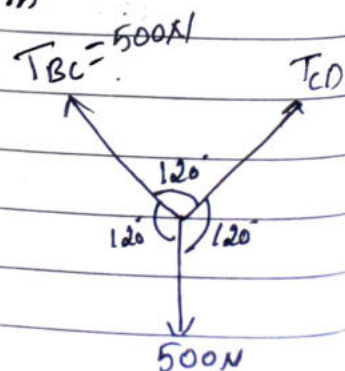
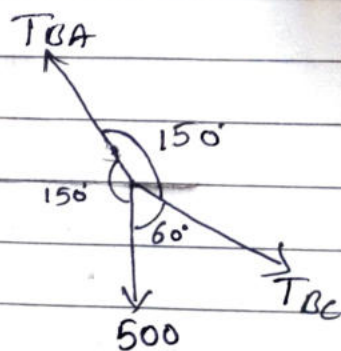
$$\text{consider, } \frac{T_{BC}}{\sin 150^\circ} = \frac{500}{\sin 150^\circ}$$

$$\therefore T_{BC} = 500 \text{ N}$$

consider joint 'C' & Apply Lami's theorem

$$\text{WKT, } \frac{500}{\sin 120^\circ} = \frac{500}{\sin 120^\circ} = \frac{T_{CD}}{\sin 120^\circ}$$

$$\therefore T_{CD} = 500 \text{ N}$$

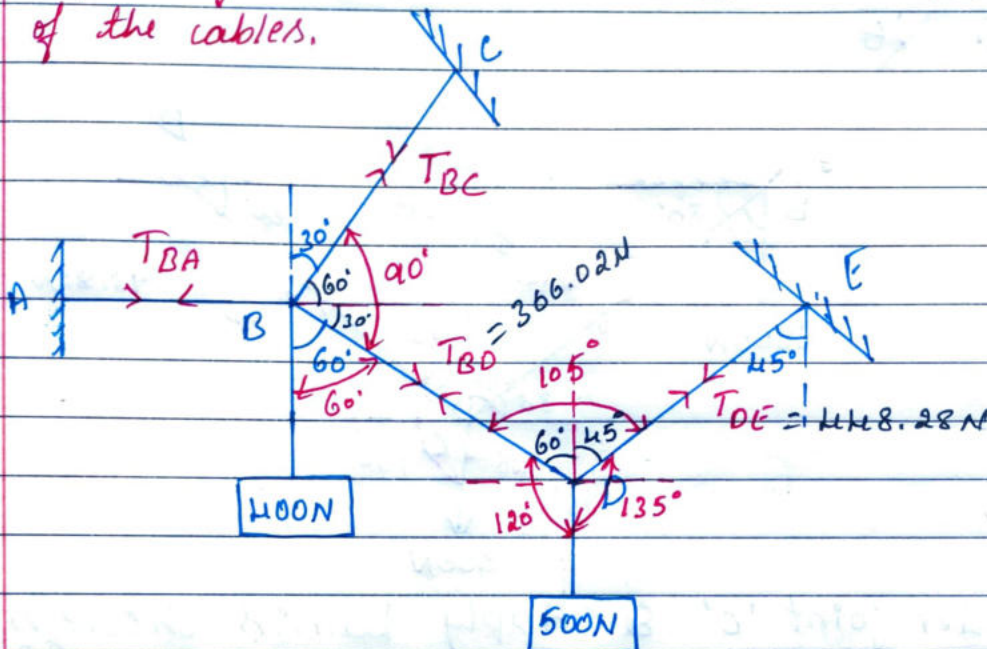


Lami's theorem is applicable only for
Three concurrent force system

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- ⑦ The system of connected flexible cables shown in figure is supporting 2 loads of 400N & 500N at points B & D, respectively. Determine the tensions in the various segments of the cables.

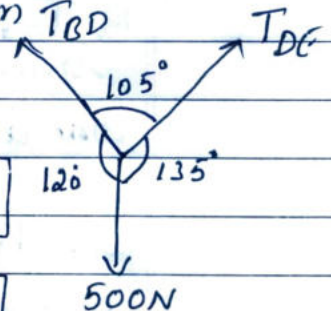


consider joint 'D' & apply Lami's theorem

WKT, $\frac{T_{BD}}{\sin 135} = \frac{500}{\sin 105} = \frac{T_{DE}}{\sin 120}$

consider, $\frac{T_{BD}}{\sin 135} = \frac{500}{\sin 105} \Rightarrow T_{BD} = 366.02N$

consider, $\frac{T_{DE}}{\sin 120} = \frac{500}{\sin 105} \Rightarrow T_{DE} = 448.28N$



consider joint 'B'

Applying eq^m condition

i.e, $\sum F_x = 0$

$-T_{BA} + T_{BC} \cos 60 + 366.02 \cos 30 = 0 \rightarrow \textcircled{1}$

Applying $\sum F_y = 0$

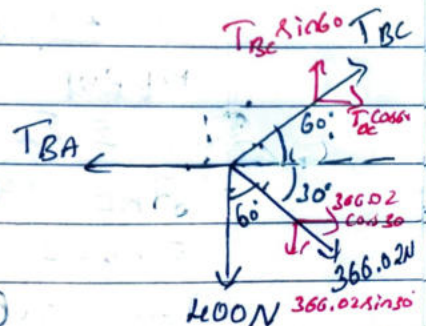
$T_{BC} \sin 60 - 366.02 \sin 30 - 400 = 0 \rightarrow \textcircled{2}$

$T_{BC} = 673.2N$

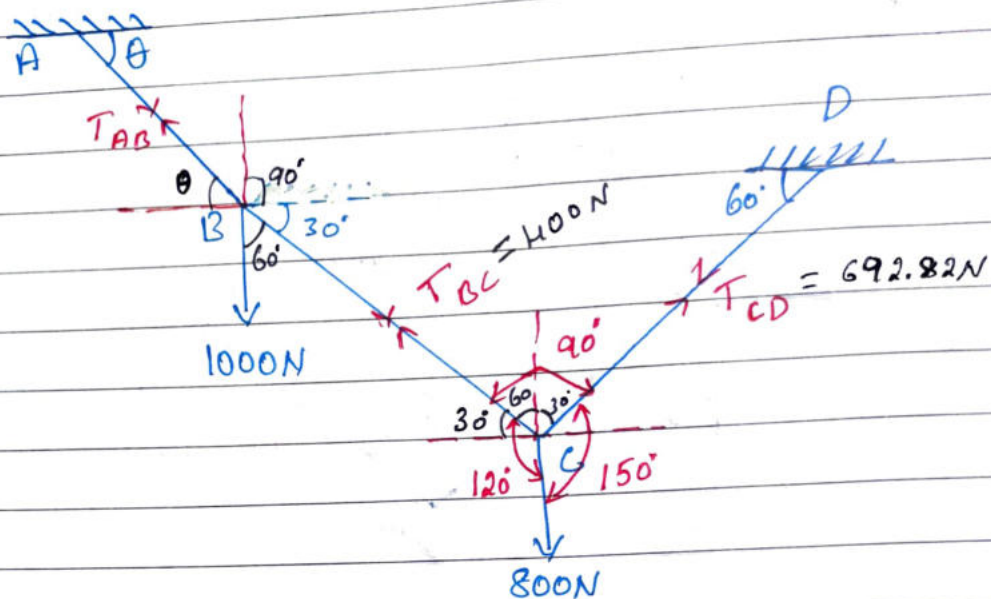
Substitute the value of T_{BC} in eqⁿ ①

$-T_{BA} + 673.2 \cos 60 + 366.02 \cos 30 = 0$

$T_{BA} = 653.58N$



⑧ Compute the tensions in the strings AB, BC and CD as shown in figure.

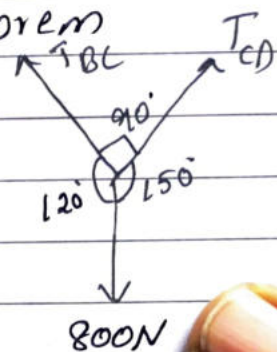


Solⁿ:- Consider joint 'C' & Apply Lami's theorem

$$\frac{T_{BC}}{\sin 150^\circ} = \frac{800}{\sin 90^\circ} = \frac{T_{CD}}{\sin 120^\circ}$$

consider, $\frac{T_{BC}}{\sin 150^\circ} = \frac{800}{\sin 90^\circ} \Rightarrow T_{BC} = 400 \text{ N}$

consider, $\frac{T_{CD}}{\sin 120^\circ} = \frac{800}{\sin 90^\circ} \Rightarrow T_{CD} = 692.82 \text{ N}$



consider joint 'B' & Apply Lami's theorem

Apply eq^m conditions

ie, $\Sigma F_x = 0$

$$-T_{AB} \cos \theta + 400 \cos 30^\circ = 0 \rightarrow \text{①}$$

$$\Sigma F_y = 0 \quad \therefore T_{AB} \sin \theta = 346.41 \text{ N} \rightarrow \text{②}$$

$$T_{AB} \sin \theta - 400 \sin 30^\circ - 1000 = 0$$

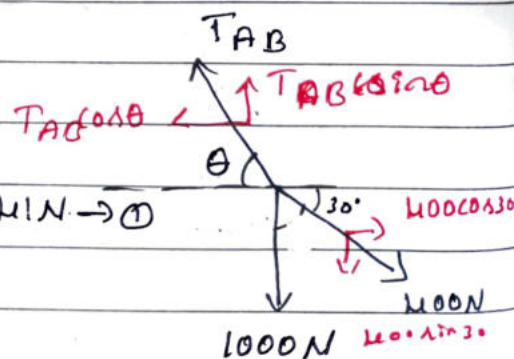
$$\therefore T_{AB} \sin \theta = 1200 \text{ N} \rightarrow \text{③}$$

Squaring & adding eqⁿ ① & ②

$$T_{AB}^2 \sin^2 \theta + T_{AB}^2 \cos^2 \theta = 1200^2 + 346.41^2$$

$$T_{AB}^2 (\sin^2 \theta + \cos^2 \theta) = 1200^2 + 346.41^2$$

$$\therefore T_{AB} = 1249 \text{ N}$$



eqn ② $\div$ eqn ① gives

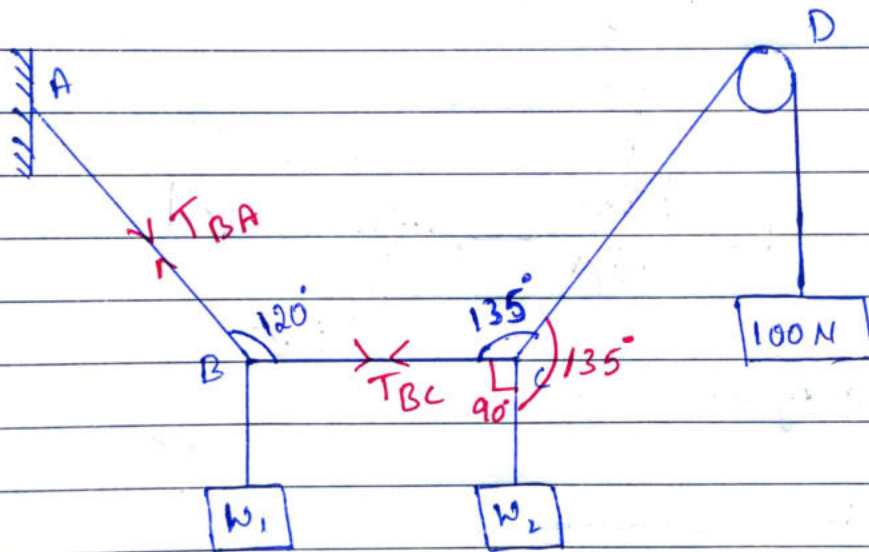
$$\frac{T_{AB} \sin \theta}{T_{AB} \cos \theta} = \frac{1200}{346.41}$$

$$\tan \theta = \frac{1200}{346.41}$$

$$\theta = \tan^{-1} \left(\frac{1200}{346.41} \right)$$

$$\therefore \theta = 73.9^\circ$$

- ⑨ Determine the tension in different parts of the string shown in figure. Also, find the value of w_1 & w_2 , if the portion BC is horizontal.

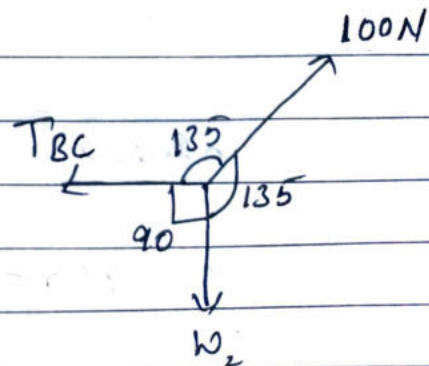


Sol:- Joint 'C' & Apply Lami's theorem

$$\frac{T_{BC}}{\sin 135} = \frac{w_2}{\sin 135} = \frac{100}{\sin 90}$$

$$T_{BC} = \frac{100 \sin 135}{\sin 90} = 70.71 \text{ N}$$

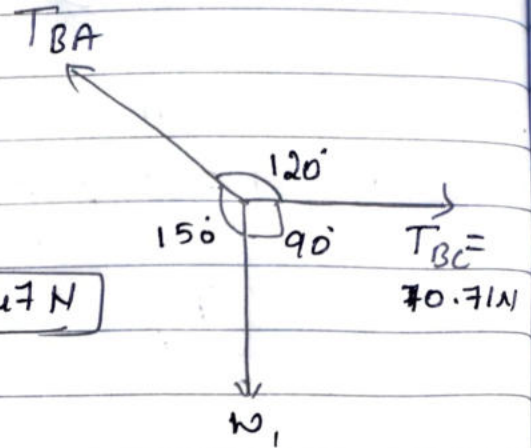
$$w_2 = \frac{100 \sin 135}{\sin 90} = 70.71 \text{ N}$$



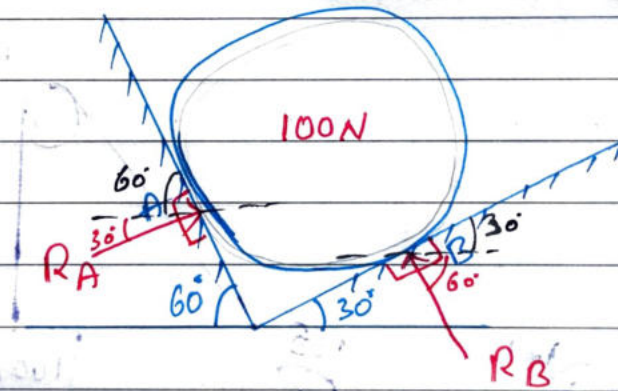
Joint 'B' & Apply Lami's theorem

$$\frac{T_{BA}}{\sin 90^\circ} = \frac{W_1}{\sin 120^\circ} = \frac{70.71}{\sin 150^\circ}$$

$$\therefore T_{BA} = 141.42 \text{ N} \quad \& \quad W_1 = 122.47 \text{ N}$$



- ⑩ A sphere weighing 100 N is fitted in a right-angled notch as shown in figure. If all contact surfaces are smooth, determine the reaction at contact surfaces.



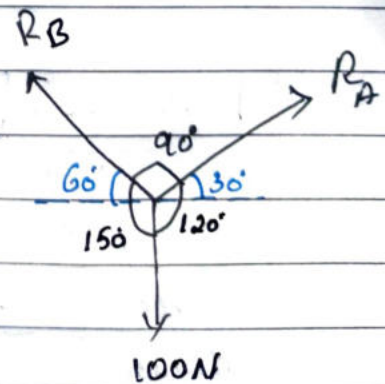
Solⁿ: Free body diagram

Apply Lami's theorem

$$\frac{R_B}{\sin 120^\circ} = \frac{R_A}{\sin 150^\circ} = \frac{100}{\sin 90^\circ}$$

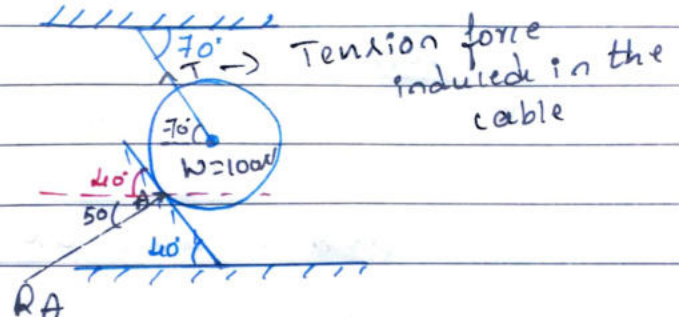
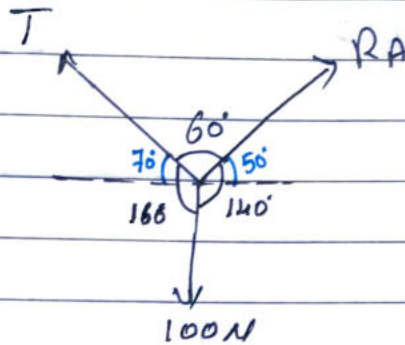
$$\therefore R_A = 50 \text{ N}$$

$$R_B = 86.6 \text{ N}$$



- (11) A sphere weighing 100 N is fitted in a right angled notch as shown in figure. If all contact surfaces are smooth, determine the reaction at contact surfaces.

Solⁿ:- Free body diagram



Apply Lami's theorem

$$\frac{T}{\sin 140} = \frac{R_A}{\sin 160} = \frac{100}{\sin 60}$$

$$\therefore R_A = 177.77 \text{ N} \quad \& \quad T = 334 \text{ N}$$

$$39.49$$

- (12) A horizontal shaft with inner clearance of 1000 mm carries two spheres of radius 350 mm & 250 mm as shown in figure. The weights are 600 N & 500 N respectively. Find the reactions at all the points of contact.

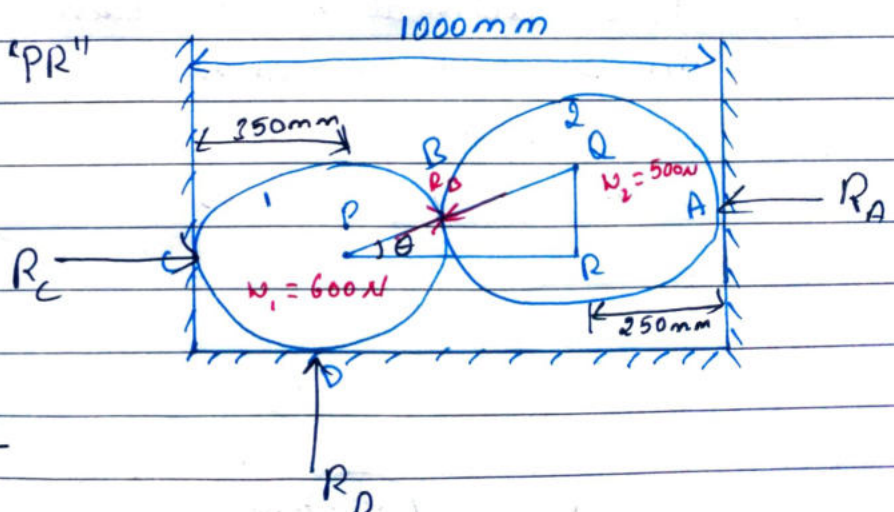
Solⁿ:- From figure length "PR"
 $= 1000 - 350 - 250$
 $= 400\text{ mm}$

$$PQ = 350 + 250$$

$$PQ = 600\text{ mm}$$

$$\cos \theta = \frac{PR}{PQ} = \frac{400}{600}$$

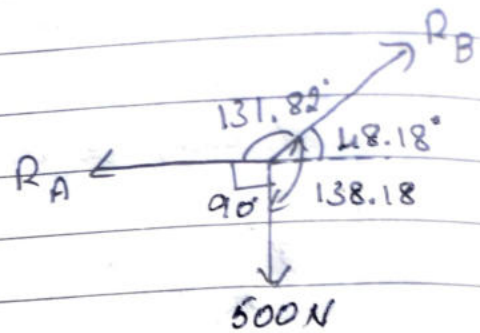
$$\theta = 48.18^\circ$$



consider sphere - 2

Apply Lami's theorem

$$\frac{R_A}{\sin 138.18} = \frac{R_B}{\sin 90} = \frac{500}{\sin 131.82}$$



$$\therefore \begin{cases} R_A = 447.36 \text{ N} \\ R_B = 670.92 \text{ N} \end{cases}$$

consider sphere - 1

Apply $\Sigma F_x = 0$

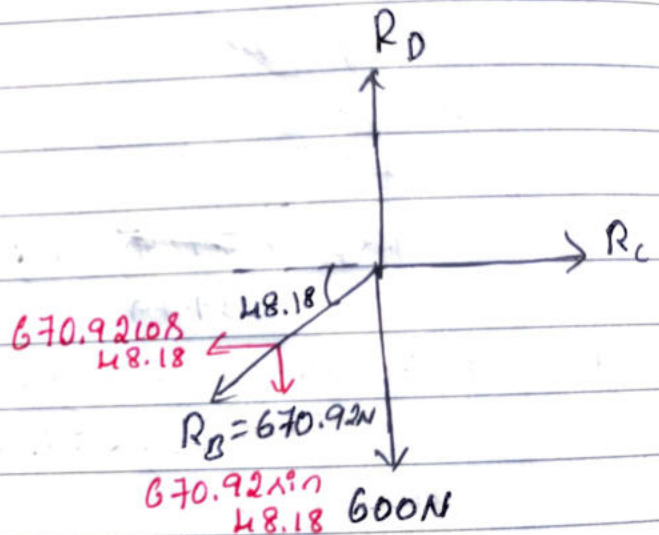
$$-670.92 \cos 48.18 + R_C = 0$$

$$R_C = 447.36 \text{ N}$$

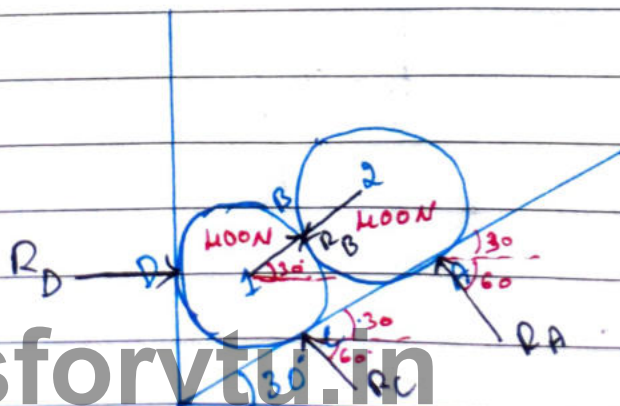
$$\Sigma F_y = 0$$

$$R_D - 600 - 670.92 \sin 48.18 = 0$$

$$R_D = 1100 \text{ N}$$



- (13) Two identical rollers, each weighing 400 N, are placed in a trench as shown in figure. Assuming that all contact surfaces are smooth, determine the reactions at contact points A, B, C and D.



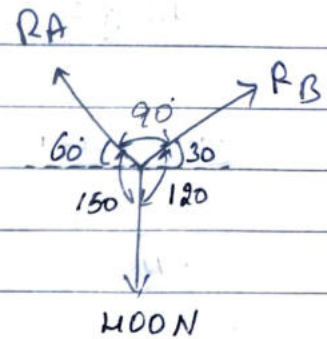
Solⁿ:- consider Sphere-2

Apply Lami's theorem

$$\frac{R_A}{\sin 120} = \frac{R_B}{\sin 150} = \frac{400}{\sin 90}$$

$$\therefore R_A = 346.41 \text{ N}$$

$$R_B = 200 \text{ N}$$



consider Sphere-1

$$\sum F_x = 0$$

$$-R_C \cos 60 - 200 \cos 30 + R_D = 0 \rightarrow (1)$$

$$\sum F_y = 0$$

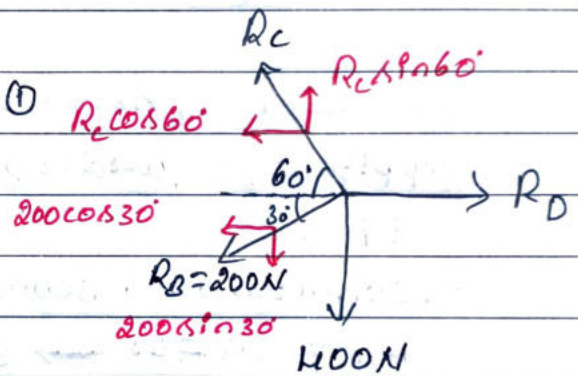
$$R_C \sin 60 - 200 \sin 30 - 400 = 0$$

$$\therefore R_C = 577.35 \text{ N}$$

Substitute the value of R_C in eqⁿ (1)

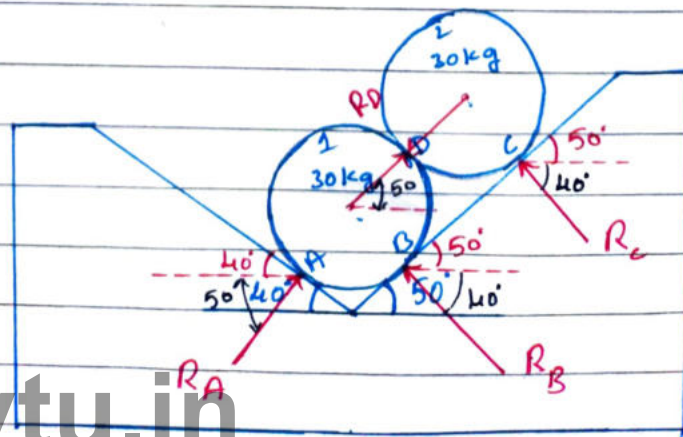
$$-200 \cos 30 - 577.35 \cos 60 + R_D = 0$$

$$R_D = 461.88 \text{ N}$$



- (14) Calculate the reaction exerted at the contact points A, B, C and D as shown in figure.

Solⁿ:-

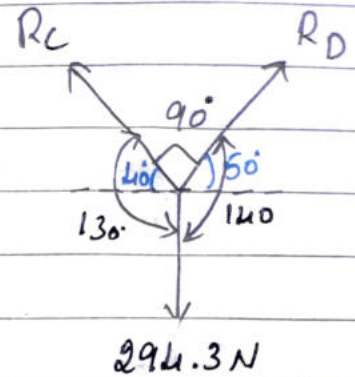


$$1 \text{ kg} = 9.81 \text{ N} \Rightarrow 30 \text{ kg} = 30 \times 9.81 = \underline{294.3 \text{ N}}$$

consider Sphere-2

$$\frac{R_C}{\sin 140} = \frac{294.3}{\sin 90} = \frac{R_D}{\sin 130}$$

$$\therefore \begin{cases} R_C = 189.17 \text{ N} \\ R_D = 225.44 \text{ N} \end{cases}$$



consider Sphere-1

Apply Eqm condition

$$\sum F_x = 0$$

$$-225.44 \cos 50 - R_B \cos 40$$

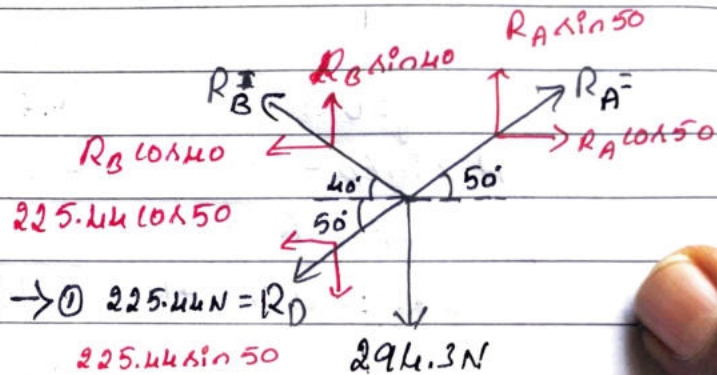
$$+ R_A \cos 50 = 0 \Rightarrow \textcircled{1}$$

$$0.642 R_A - 0.766 R_B = 144.91 \rightarrow \textcircled{1} \quad 225.44 \text{ N} = R_D$$

$$\sum F_y = 0$$

$$-225.44 \sin 50 + R_B \sin 40 + R_A \sin 50 - 294.3 = 0$$

$$0.766 R_A + 0.642 R_B = 467 \rightarrow \textcircled{2}$$



$$R_A = 451.3 \text{ N}$$

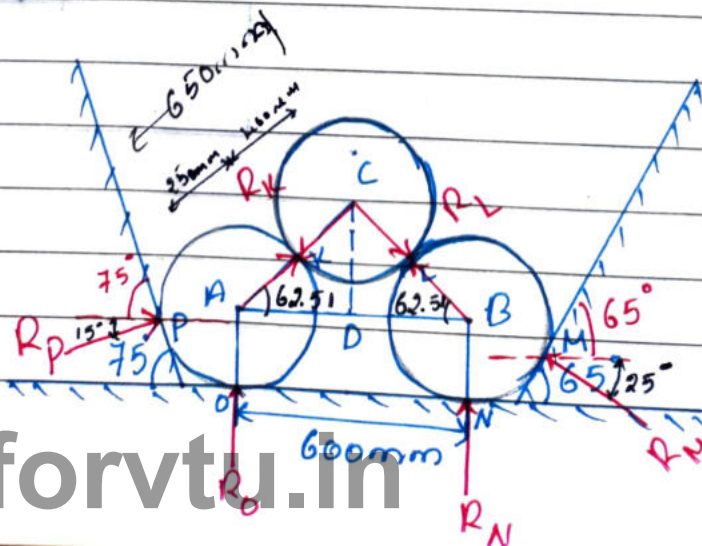
$$R_B = 189 \text{ N}$$

- (15) Determine the reactions at contact points for spheres A, B and C as shown in figure. It is given that: $W_A = W_B = W_C = 6 \text{ kN}$, $d_A = d_B = 500 \text{ mm}$, $d_C = 800 \text{ mm}$.

$$AC = 650 \text{ mm}$$

$$AB = 600 \text{ mm}$$

$$CB = 650 \text{ mm}$$



UNIT-6

Equilibrium of Non-concurrent System of Forces and Support Reactions.

Equilibrium Equations:-

A body is said to be in equilibrium when it does not have any translatory or rotational moment. This means [when a body is in equilibrium under the action of coplanar non-concurrent forces the following equations are satisfied:-

- (i) The algebraic sum of the component of forces along any two mutually perpendicular direction is zero.

$$R_x = \sum F_x = 0$$

$$R_y = \sum F_y = 0$$

- (ii) The algebraic sum of the moments of the forces about any point in the plane is zero.

$$\sum M = 0$$

Types of supports:-

- 1.) Simple supports:-



Simple supports are those which exert reactions perpendicular to the plane of support. They restrict translation of the body in one direction only, but do not restrict rotation.

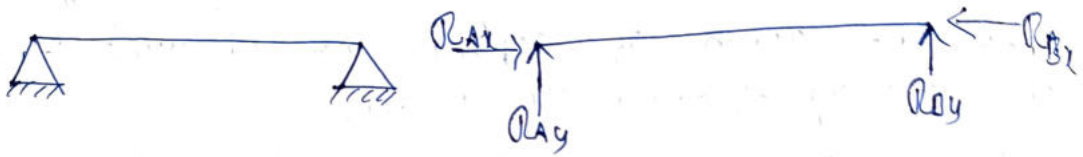
- 2.) Roller supports:-

Roller supports are those which exert reactions in any direction, but free ~~are~~ perpendicular to the plane of the support. They restrict translation of the body along one direction only, and, rotation is allowed.



3. > Hinged or pinned supports:

Hinged supports are those which exert reactions in any direction, but from one ~~case~~ both H/E and V/E direction. Therefore hinged supports resist translation in both directions. But rotation is possible.



4. > Fixed supports:

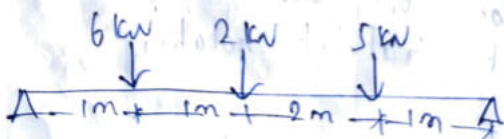
Fixed supports are those which exert reactions in H/E & V/E direction and also restrict the rotation.



Types of loads (Types of forces)

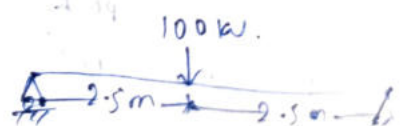
1. > Concentrated load

A load which is concentrated at a point in a beam is known as concentrated load.



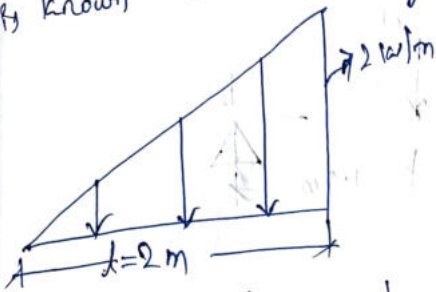
2. > Uniformly distributed load :-

A load which is distributed uniformly throughout the length is called (UDL)

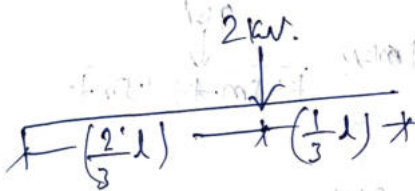


3. Uniformly varying load:-

A load which varies with the length of the beam is known as uniformly varying load.

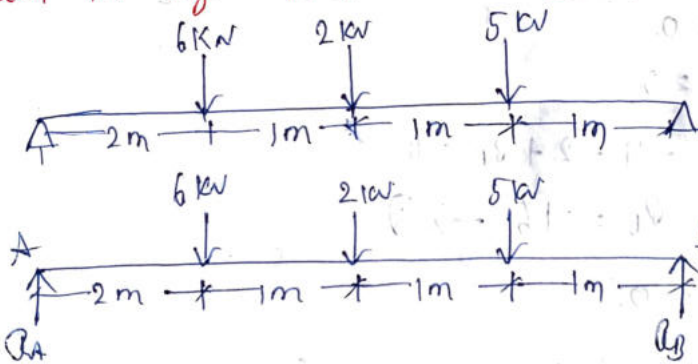


Area of $\Delta l = \frac{1}{2} \times 2 \times 2 = 2 \text{ kN}$, ~~$\frac{2}{3} \times 2 =$~~



Problems:-

1. A simply supported beam of length 5m is loaded as shown in fig. Find the reactions at A & B.



from Eqm condition

$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$R_A - 6 - 2 - 5 + R_B = 0$$

$$R_A + R_B = 13 \rightarrow (1)$$

$$\sum M_A = 0$$

$$6 \times 2 + 2 \times (2+1) + 5(2+1+1) - R_B(2+1+1+1) = 0$$

$$12 + 6 + 20 - 5R_B = 0$$

$$38 = 5R_B$$

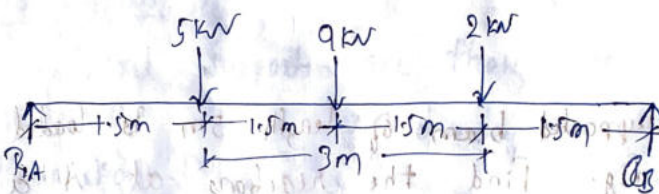
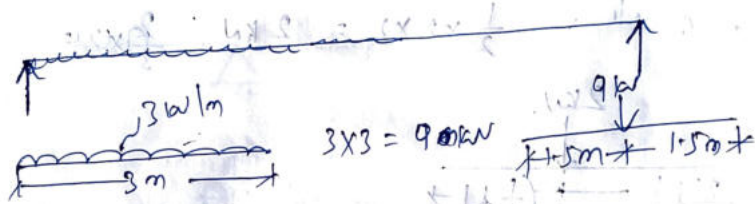
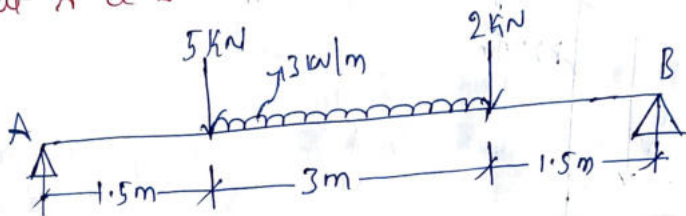
$$R_B = 7.6 \text{ kN}$$

Sub. R_B in (1)

$$R_A + 7.6 = 13$$

$$R_A = 5.4 \text{ kN}$$

2.8 A simply supported beam of span 6m is subjected to loading as shown in fig. Determine the reactions at A & B.



$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$R_A - 5 - 9 - 2 + R_B = 0$$

$$R_A + R_B = 16 \rightarrow (1)$$

$$\sum M_A = 0$$

$$5 \times 1.5 + 9(1.5 + 1.5) + 2(1.5 + 1.5 + 1.5) - R_B \times 6 = 0$$

$$7.5 + 27 + 9 - 6R_B = 0$$

$$43.5 = 6R_B$$

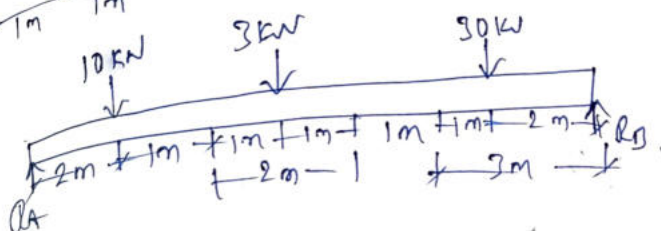
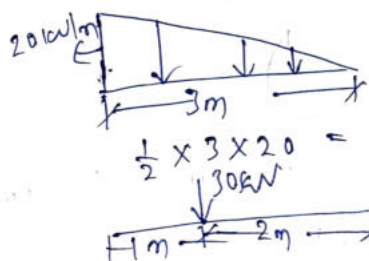
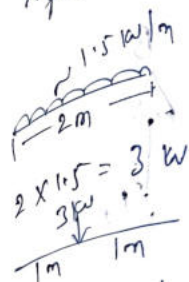
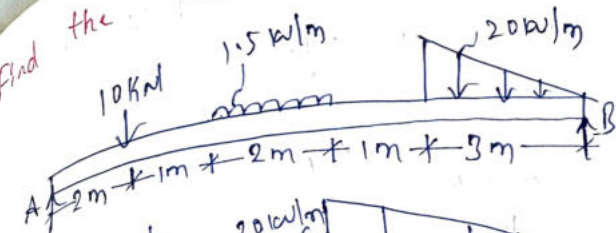
$$R_B = 7.25 \text{ kN}$$

Sub. R_B in (1)

$$R_A + 7.25 = 16$$

$$R_A = 8.75 \text{ kN}$$

Find the reactions for the beam shown in fig.



$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$R_A - 10 - 3 - 30 + R_B = 0$$

$$R_A + R_B = 43 \rightarrow (1)$$

$$\sum M_A = 0$$

$$10 \times 2 + 3 \times 4 + 30 \times 7 - R_B \times 9 = 0$$

$$20 + 12 + 210 = 9R_B$$

$$242 = 9R_B$$

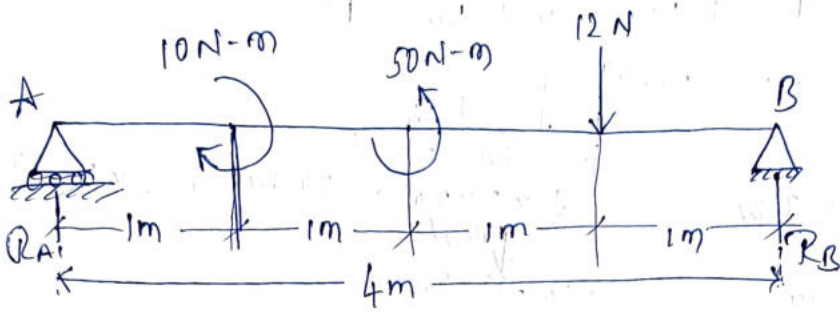
$$R_B = 26.89 \text{ kN}$$

Sub. R_B in (1)

$$R_A + 26.89 = 43$$

$$R_A = 16.11 \text{ kN}$$

4. Determine the reactions at the supports A and B for the beam shown in fig.



$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$R_A + R_B - 12 = 0$$

$$R_A + R_B = 12 \text{ kN}$$

$$\sum M_A = 0$$

$$10 - 50 + 12 \times 3 - R_B \times 4 = 0$$

$$-4 = 4 R_B$$

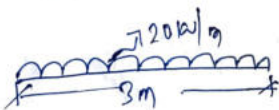
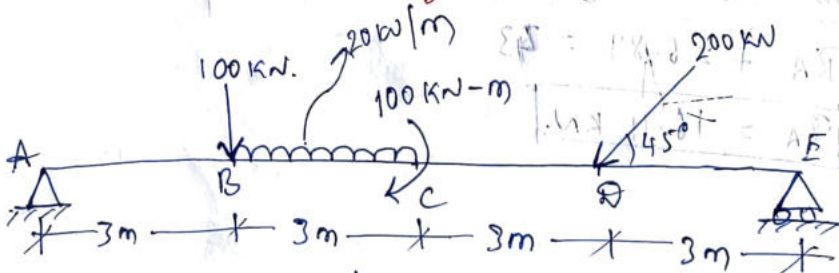
$$R_B = -1 \text{ kN}$$

$$R_A + R_B = 12$$

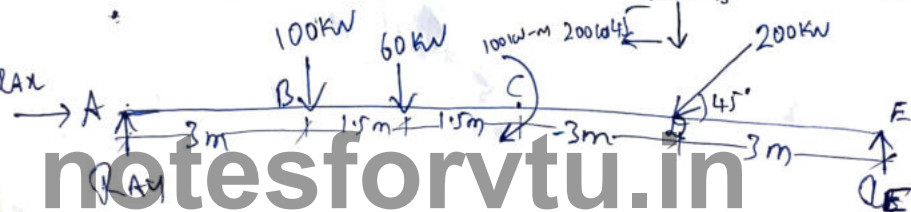
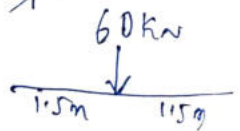
$$R_A - 1 = 12$$

$$R_A = 13 \text{ kN}$$

5. Determine the reactions at A & E for the beam shown in fig.



$$20 \times 3 = 60 \text{ kN}$$



$$\sum F_x = 0.$$

$$R_{Ax} - 200 \cos 45 = 0.$$

$$R_{Ax} = 141.421 \text{ kN}$$

$$\sum F_y = 0.$$

$$R_{Ay} - 100 - 60 - 200 \sin 45 + R_E = 0.$$

$$R_{Ay} + R_E = 301.42 \text{ kN} \rightarrow \textcircled{1}$$

$$\sum M_A = 0.$$

$$100 \times 3 + 60 \times 4.5 + 100 + 200 \sin 45 \times 9 - R_E \times 12 = 0.$$

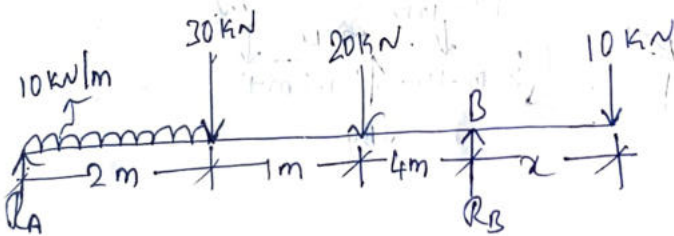
$$R_E = 161.89 \text{ kN}$$

Sub. R_E in $\textcircled{1}$

$$R_{Ay} + 161.89 = 301.42$$

$$R_{Ay} = 139.53 \text{ kN}$$

Ex. Determine the distance x such that R_A & R_B (reactions) are equal, for the beam shown in fig.

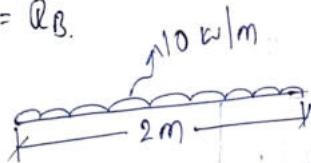


Given data $R_A = R_B$

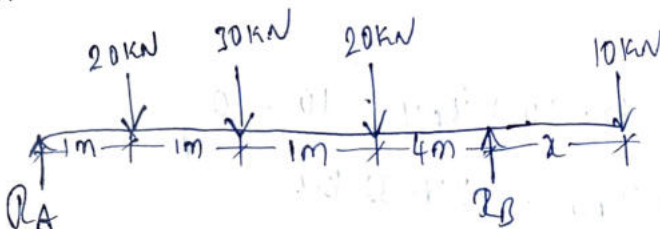
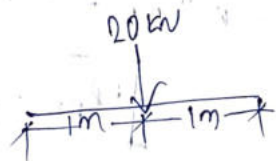
$$\sum F_x = 0$$

$$\sum F_y = 0.$$

①



$$10 \times 2 = 20 \text{ kN}$$



$$\sum F_x = 0$$

$$\sum F_y = 0.$$

$$R_A - 20 - 30 - 20 + R_B - 10 = 0$$

$$R_A + R_B = 80 \text{ kN}$$

$$R_A + R_A = 80$$

$$2R_A = 80 \Rightarrow$$

$$R_A = 40 \text{ kN} \Rightarrow R_A = R_B = 40 \text{ kN}$$

$$\sum M_A = 0$$

$$20 \times 1 + 30 \times 2 + 20 \times 3 - R_B \times 7 + 10(7+x) = 0$$

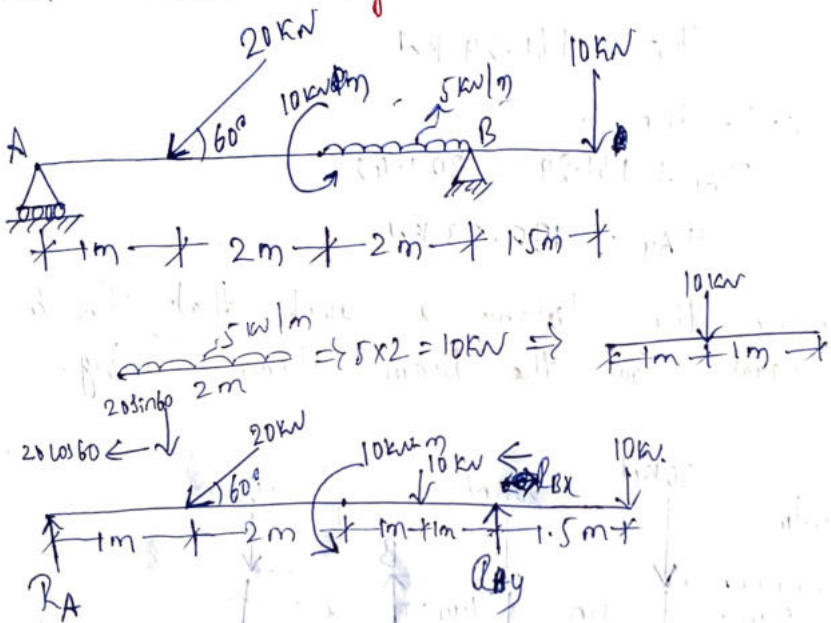
$$20 + 60 + 60 - 40 \times 7 + 10(7+x) = 0$$

$$10(7+x) = 140$$

$$7+x = 14$$

$$\boxed{x = 7 \text{ m}}$$

7. Determine the reactions at the support for the beam loaded shown in fig.



$$\sum F_x = 0$$

$$-R_{Bx} - 20 = 0$$

$$\boxed{R_{Bx} = -20 \text{ kN}}$$

$$\sum F_y = 0$$

$$R_A - 20 \sin 60 - 10 + R_{By} - 10 = 0$$

$$R_A + R_{By} = 37.32 \text{ kN}$$

$$\sum M_A = 0$$

$$20 \sin 60 \times 1 - 10 + 10 \times 4 - R_{By} \times 5 + 10 \times 6.5 = 0$$

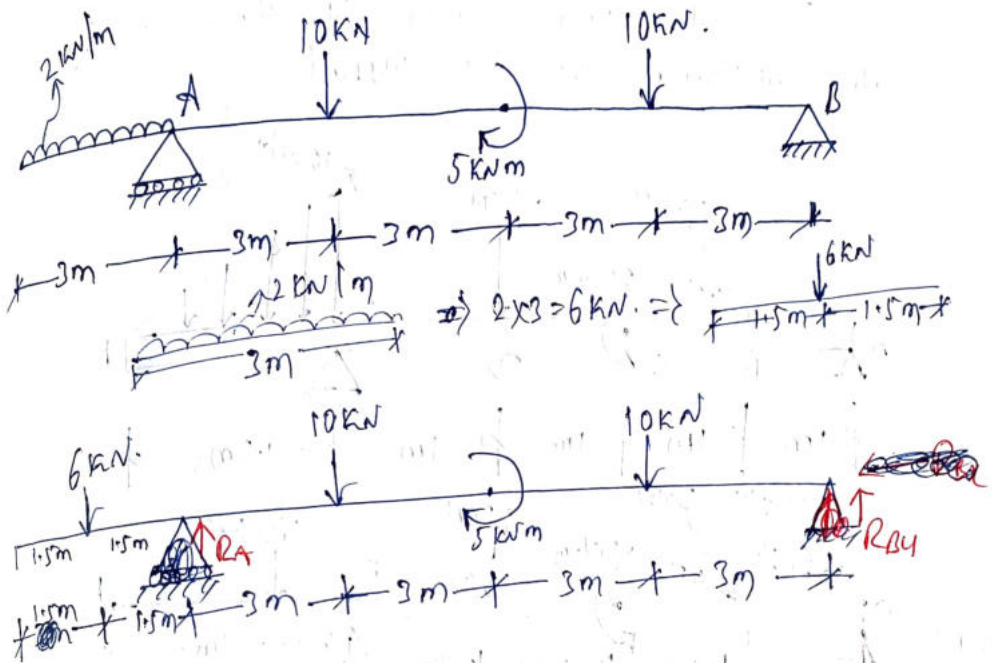
$$112.32 = 5 R_{By}$$

$$\boxed{R_{By} = 22.46 \text{ kN}}$$

$$R_A + 22.46 = 37.32$$

$$R_A = 14.86 \text{ kN}$$

8.8 Determine the support reactions for the beam shown in fig.



$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$-6 + R_A - 10 - 10 + R_B = 0$$

$$R_A + R_B = 26 \text{ kN}$$

$$\sum M_A = 0$$

$$-6 \times 1.5 + 10 \times 3 + 5 + 10 \times 9 - R_B \times 12 = 0$$

$$186 = 12 R_B$$

$$R_B = 9.67 \text{ kN}$$

$$R_A + R_B = 26 \text{ kN}$$

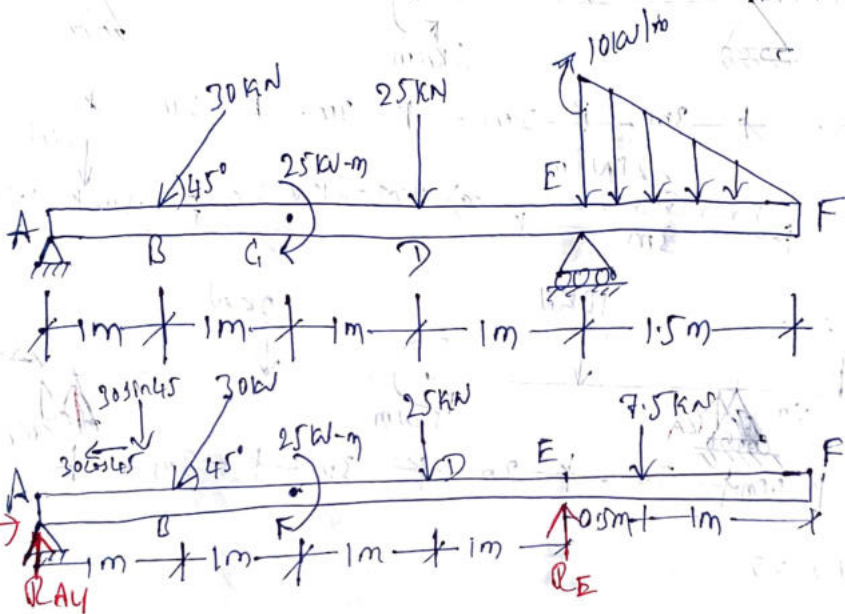
$$R_A = 26 - 9.67$$

$$R_A = 16.33 \text{ kN}$$

Q.8 Find the support reactions and member forces for the pin-jointed plane truss shown in fig.

Q.9 Determine the reactions at the supports A & B for

Q.10 A beam ABCDEF is hinged at A, supported on rollers at E and carries loads as shown in fig. Determine the reactions at the support.



$$\sum F_x = 0$$

$$R_{Ax} - 30 \cos 45^\circ = 0$$

$$R_{Ax} = 21.21 \text{ kN}$$

$$\sum F_y = 0$$

$$R_{Ay} - 30 \sin 45^\circ - 25 - 7.5 = 0$$

$$R_{Ay} + R_E = 53.71 \text{ kN}$$

$$\sum M_A = 0$$

$$30 \sin 45^\circ \times 1 + 25 + 25 \times 3 - R_E \times 4 + 7.5 \times 4.5 = 0$$

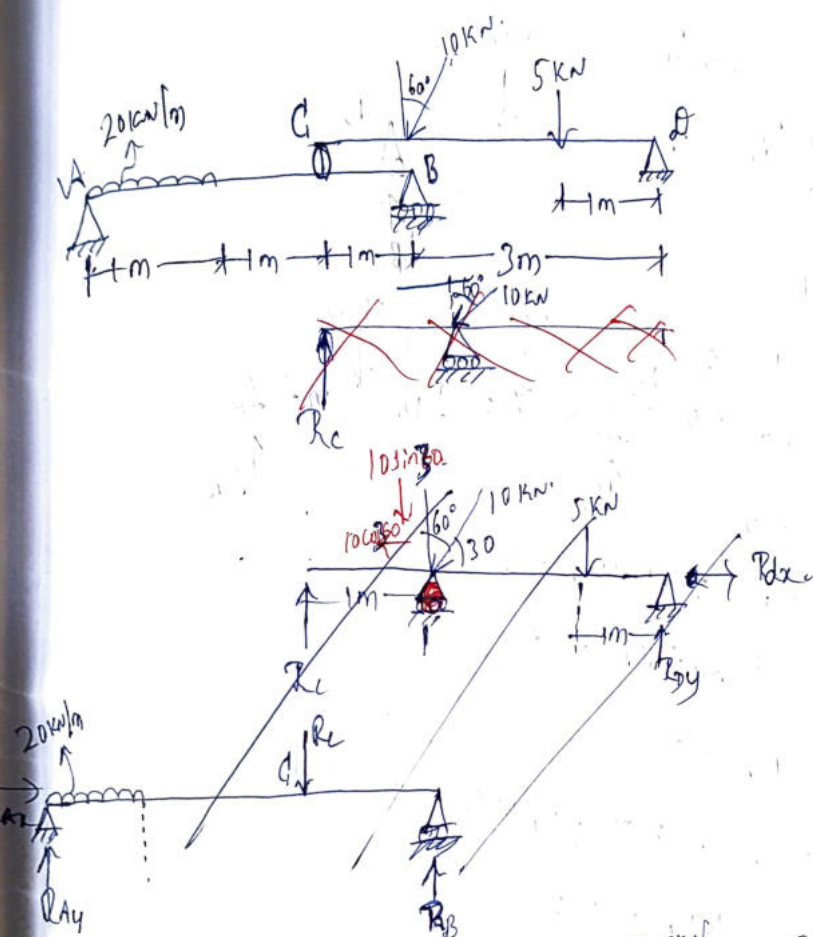
$$4 R_E = 154.90$$

$$R_E = 38.74 \text{ kN}$$

$$R_{Ay} + 38.74 = 53.71$$

$$R_{Ay} = 14.97 \text{ kN}$$

10. Determine the reactions at the supports for the system shown in fig.



Consider 1st beam

$$\sum F_x = 0$$

$$-10 \cos 30 + R_{Dx} = 0$$

$$R_{Dx} = 8.66 \text{ kN}$$

$$\sum F_y = 0$$

$$R_C - 10 \sin 30 - 5 + R_{Dy} = 0$$

$$R_C + R_{Dy} = 10$$

$$\sum M_C = 0$$

$$10 \sin 30 \times 1 + 5 \times 3 - R_{Dy} \times 4 = 0$$

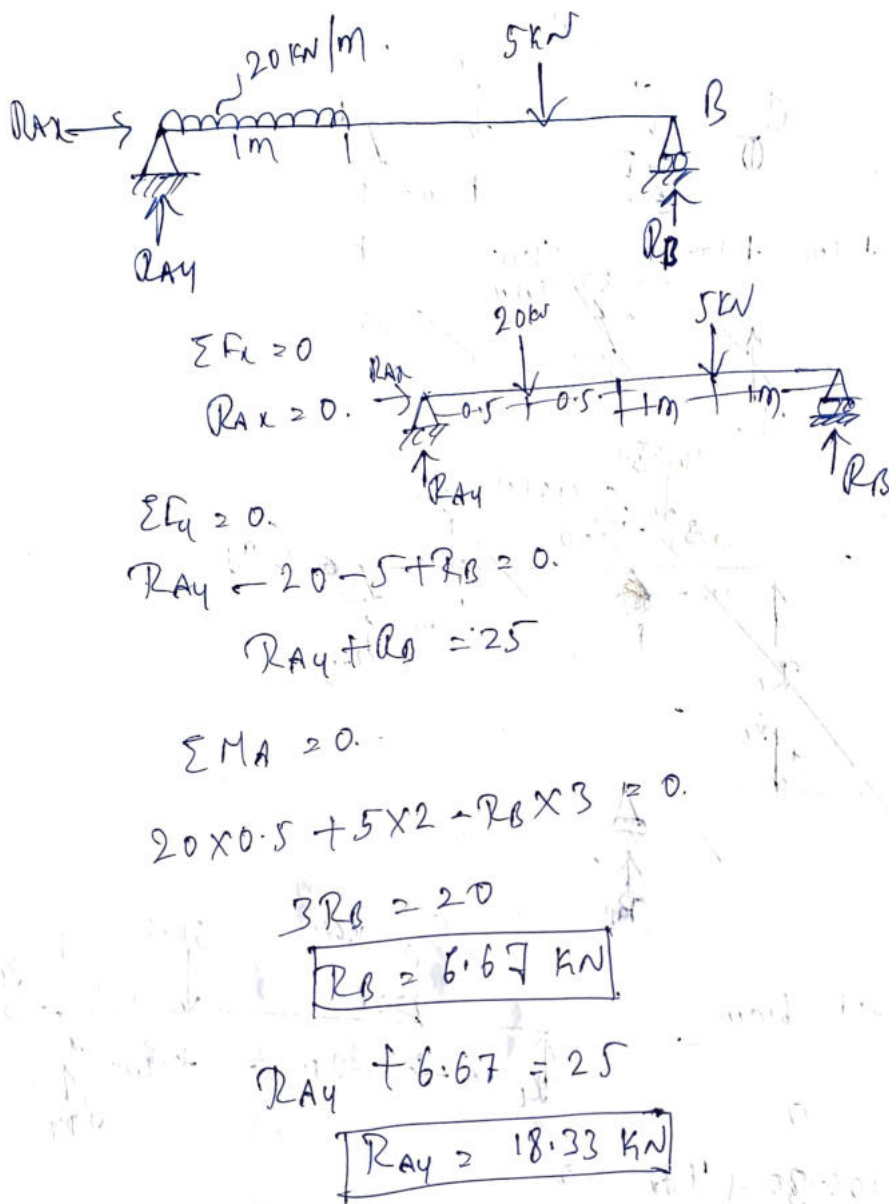
$$R_{Dy} = 5 \text{ kN}$$

notesforvtu.in

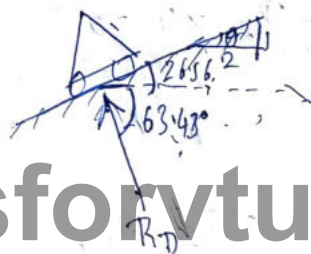
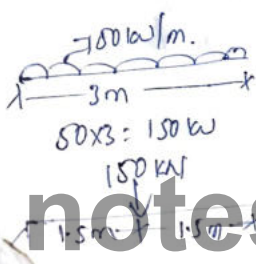
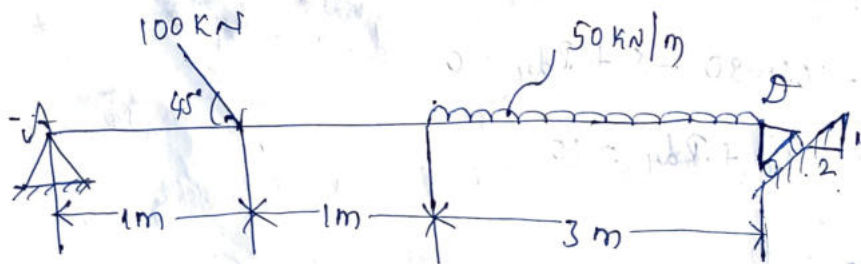
$$R_C + R_{Dy} = 10$$

$$R_C + 5 = 10 \Rightarrow R_C = 5 \text{ kN}$$

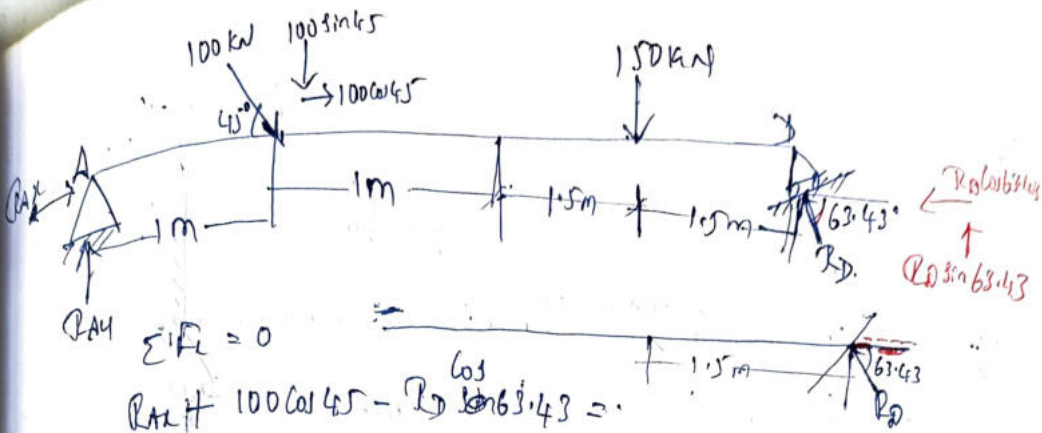
Apply the value of R_c at point 'c' of the second beam.



11. > For the beam with loading shown in fig. determine the reactions at the supports.



$\tan(\theta) = \frac{1}{2}$
 $\theta = 26.56^\circ$



$$\sum F_x = 0$$

$$R_{Ax} + 100 \cos 45 - R_D \sin 63.43 = 0$$

$$R_{Ax} - 0.447 R_D = -70.71 \rightarrow (1)$$

$$\sum F_y = 0$$

$$R_{Ay} - 100 \sin 45 - 150 + R_D \sin 63.43 = 0$$

$$R_{Ay} + 0.894 R_D = 220.71 \rightarrow (2)$$

$$\sum M_A = 0$$

$$100 \sin 45 \times 1 + 150 \times 3.5 - R_D \sin 63.43 \times 5 = 0$$

$$5 R_D \sin 63.43 = 595.7$$

$$\boxed{R_D = 133.21 \text{ kN}}$$

Sub. R_D in (1) & (2)

$$R_{Ax} - 0.447 \times 133.21 = -70.71$$

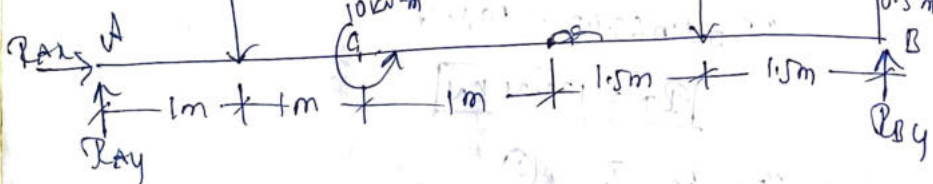
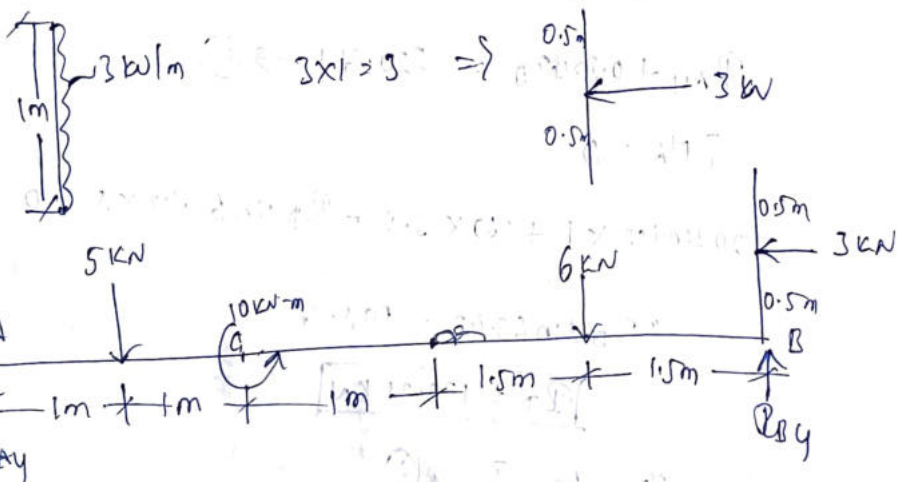
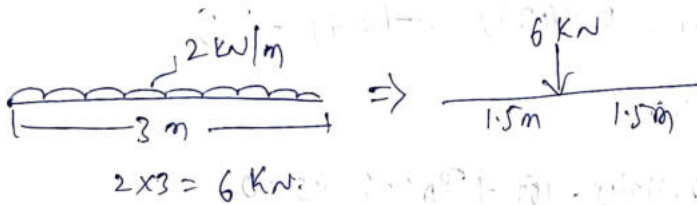
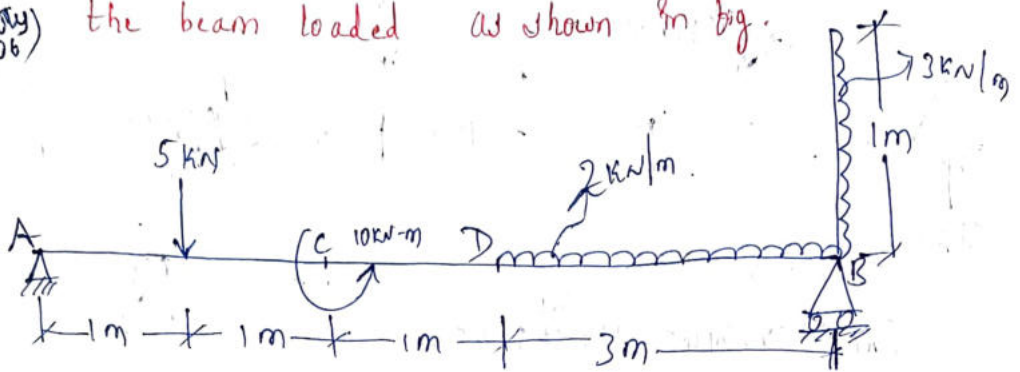
$$R_{Ax} = 130.25 \text{ kN}$$

$$R_{Ay} + 0.894 \times 133.21 = 220.71$$

$$R_{Ay} = 101.63 \text{ kN}$$

12.} Find the support reactions at A & B for the beam loaded as shown in fig.

(July 06)



Soln:

$$\sum F_x = 0$$

$$R_{Ax} - 3 = 0$$

$$R_{Ax} = 3 \text{ kN}$$

$$\sum F_y = 0$$

$$R_{Ay} - 5 - 6 + R_{By} = 0$$

$$R_{Ay} + R_{By} = 11 \rightarrow (1)$$

$$\sum M_A = 0$$

$$5 \times 1 - 10 + 6 \times 4.5 + R_{By} \times 6 - 3 \times 0.5 = 0$$

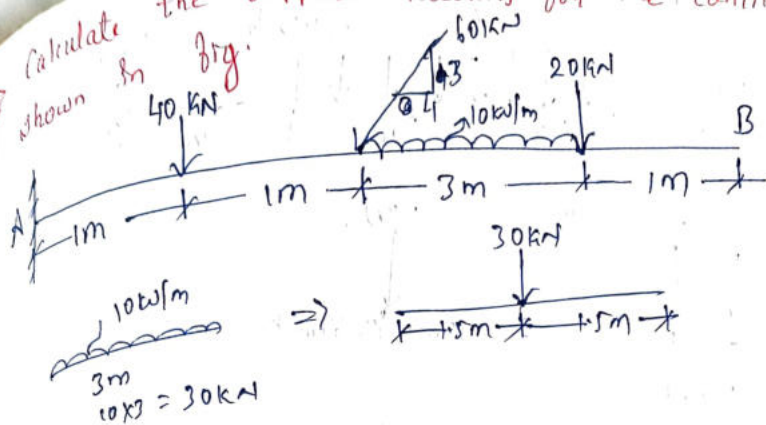
$$6 R_{By} = 20.5$$

$$R_{By} = 3.417 \text{ kN}$$

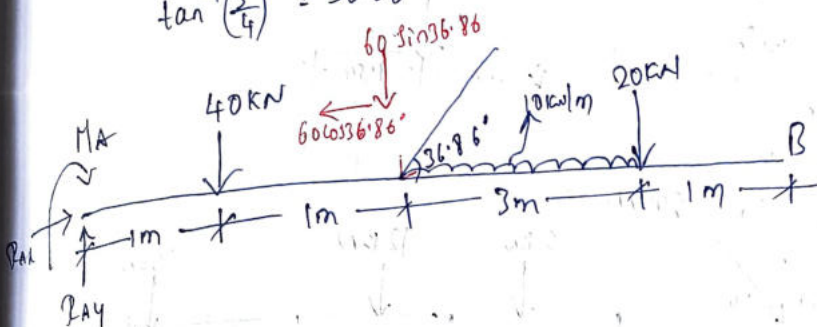
Sub. R_{By} in (1)

$$R_{Ay} = 7.583 \text{ kN}$$

13.8 Calculate the support reactions for the cantilever beam shown in fig.



$$\tan^{-1}\left(\frac{3}{4}\right) = 36.86^\circ$$



$$\sum F_x = 0$$

$$R_{Ax} - 60 \cos 36.86 = 0$$

$$\boxed{R_{Ax} = 48 \text{ kN}}$$

$$\sum F_y = 0$$

$$R_{Ay} - 40 - 60 \sin 36.86 - 30 - 20 = 0$$

$$R_{Ay} = 126 \text{ kN}$$

$$\sum M_A = 0$$

$$M_A + 40 \times 1 + 60 \sin 36.86 \times 2 + 30 \times 3.5 + 20 \times 5 = 0$$

$$\boxed{M_A = -316.98 \text{ kN-m}}$$

for moment indication assumption

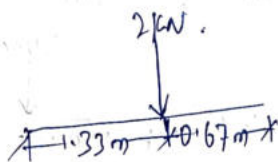
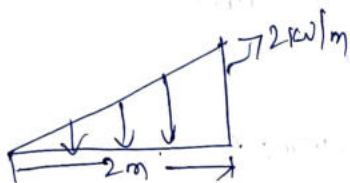
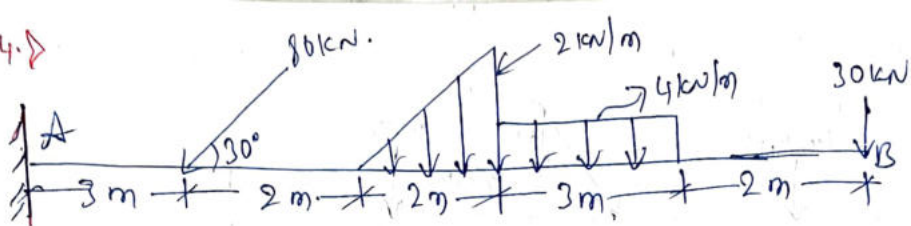
By using

sign

$$M_A = 316.98 \text{ kN-m (Anticlockwise)}$$

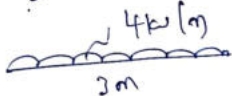
wrong.

14.8

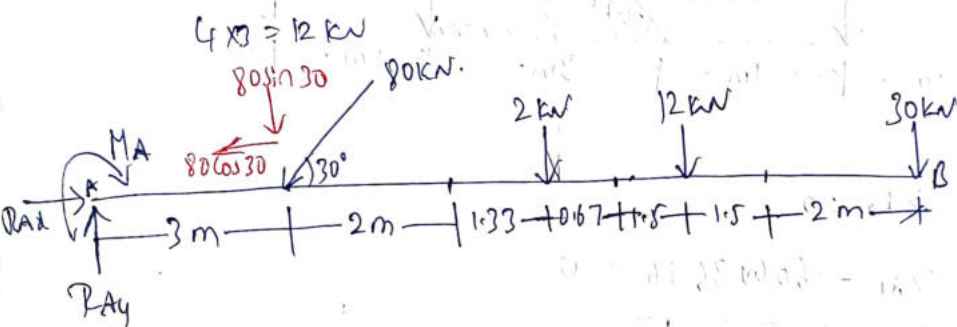
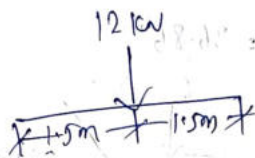


$$\frac{1}{2} \times 2 \times 2 = 2 \text{ kN}$$

$$\frac{2}{3} \times 2 = 1.33$$



$$4 \times 3 = 12 \text{ kN}$$



$$\sum F_x = 0$$

$$R_{Ax} - 80 \cos 30 = 0$$

$$\boxed{R_{Ax} = 69.28 \text{ kN}}$$

$$\sum F_y = 0$$

$$R_{Ay} - 80 \sin 30 - 2 - 12 - 30 = 0$$

$$\boxed{R_{Ay} = 84 \text{ kN}}$$

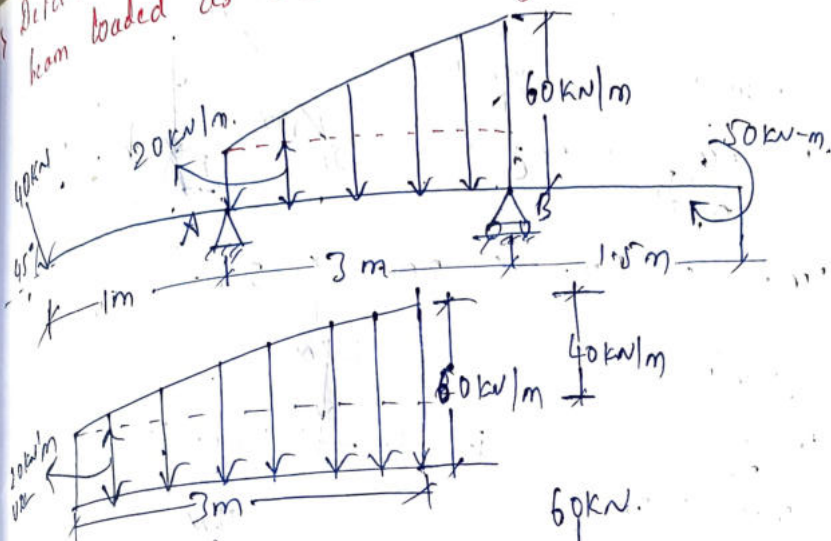
$$\sum M_A = 0$$

$$M_A + 80 \sin 30 \times 3 + 2 \times (5 + 1.33) + 12 \times (5 + 1.33 + 0.67 + 1.5) + 30 \times 10 = 0$$

$$M_A = -594.67 \text{ kN-m}$$

$$M_A = 594.67 \text{ kN-m (Anticlockwise)}$$

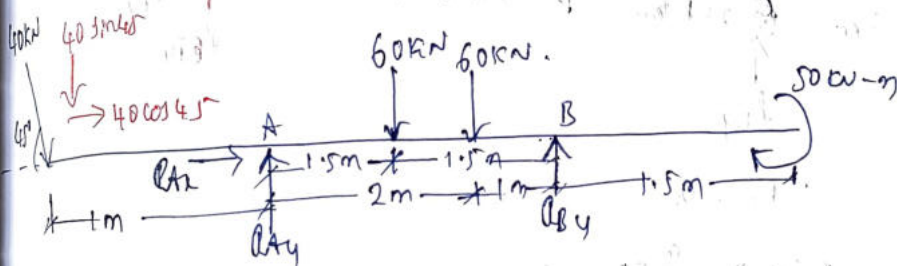
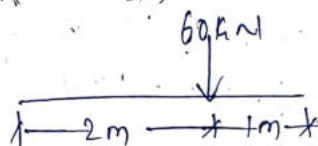
Q.5 Determine the reactions at the supports A & B for a beam loaded as shown in fig.



UDL: $20 \times 3 = 60 \text{ kN}$

URL: $\frac{1}{2} \times 3 \times 40 = 60 \text{ kN}$

$\frac{2}{3} \times 60 = 2 \text{ m}$



$\sum F_x = 0$

$40 \cos 45 + R_{Ax} = 0$

$R_{Ax} = -28.28 \text{ kN}$

$\sum F_y = 0$

$-40 \sin 45 + R_{Ay} - 60 - 60 + R_{By} = 0$

$R_{Ay} + R_{By} = 148.284 \text{ kN} \rightarrow \text{①}$

$\sum M_A = 0$

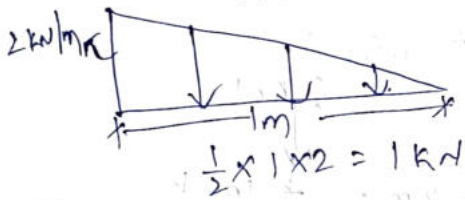
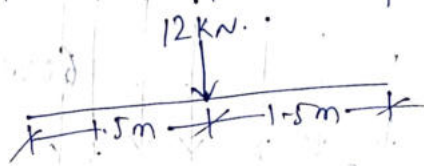
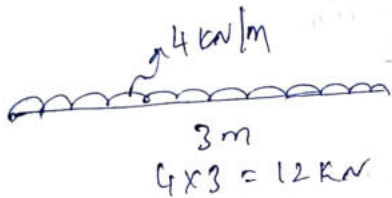
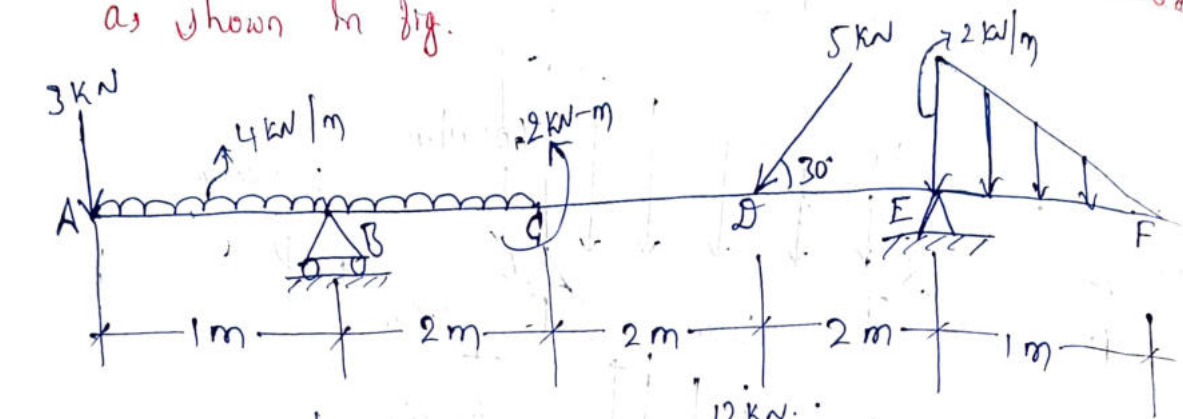
$-40 \sin 45 \times 1 + 60 \times 1.5 + 60 \times 2 - R_{By} \times 3 + 50 = 0$

$R_{By} = 77.23 \text{ kN}$

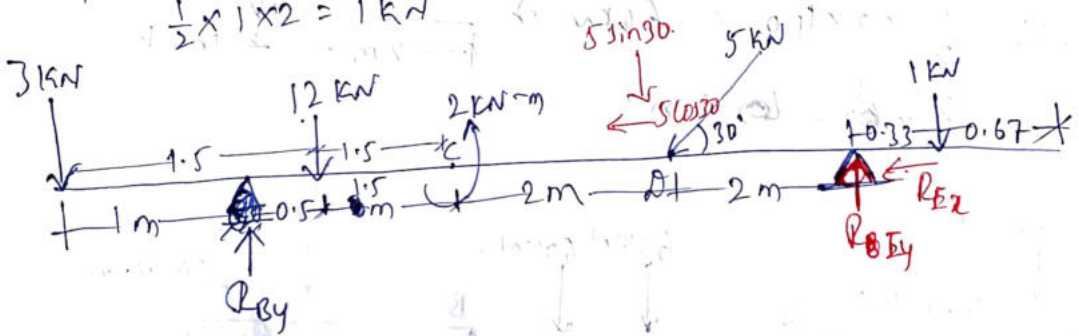
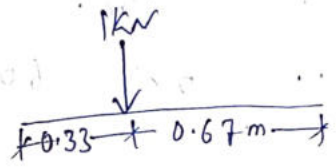
Sub. R_{By} in ①

$R_{Ay} = 71.054 \text{ kN}$

16. Find the support reactions of the beam loaded as shown in fig.



$$\frac{2}{3} \times 1 = 0.67 \text{ m}$$



$$\sum F_x = 0$$

$$-5 \cos 30^\circ - R_{Ex} = 0$$

$$R_{Ex} = -5 \cos 30^\circ$$

$$R_{Ex} = -4.33 \text{ kN}$$

$$\sum F_y = 0$$

$$-3 + R_{By} - 12 - 5 \sin 30^\circ + R_{Ey} - 1 = 0$$

$$R_{By} + R_{Ey} = 18.5 \text{ kN}$$

$$\sum M_B = 0$$

$$-3 \times 1 + 12 \times 0.5 - 2 + 5 \sin 30^\circ \times 2 - R_{Ey} \times 3 + 1 \times (3 + 0.33) = 0$$

$$R_{Ey} = 2.89 \text{ kN}$$

$$R_{By} = 18.5 - 2.89 = 15.61 \text{ kN}$$

M.I. of quadrant of a circle.

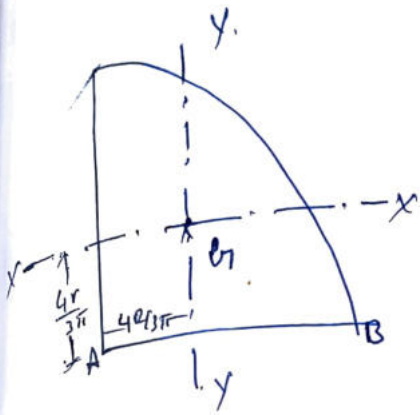


Fig. shows quadrant of a circle of radius 'r'

∴ M.I. of quarter circle about AB = $\frac{1}{4}$ M.I. of circle.

$$\therefore I_{AB} = \frac{1}{4} \left(\frac{\pi r^4}{4} \right)$$

$$I_{AB} = \frac{\pi r^4}{16}$$

To find M.I. about centroidal axis from 11th axis

$$I_{AB} = I_{xx} + A\bar{y}^2$$

$$I_{xx} = I_{AB} - A\bar{y}^2$$

$$= \frac{\pi r^4}{16} - \frac{\pi r^2}{4} \left(\frac{4r}{3\pi} \right)^2$$

$$= \frac{\pi r^4}{16} - \frac{16\pi r^4}{36\pi^2}$$

$$= 0.196 r^4 - 0.1414 r^4$$

$$= 0.055 r^4$$

$$\boxed{I_{xx} = \frac{0.11 r^4}{2}}$$

$$\text{Similarly } \boxed{I_{yy} = \frac{0.11 r^4}{2}}$$